PLANE AND SPHERICAL TRIGONOMETRY

A SERIES OF MATHEMATICAL TEXTS

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TRIGONOMETRY

BY

ALFRED MONROE KENYON

PROFESSOR OF MATHEMATICS, PURDUE UNIVERSITY

AND

LOUIS INGOLD

ASSISTANT PROFESSOR OF MATHEMATICS, THE UNIVERSITY OF MISSOURI

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PREFACE

In Trigonometry, as elsewhere, a motive for the study of each topic is necessary to secure the effective attention of the student.

The knowledge required for the actual solution of triangles — the one motive common to all texts on Trigonometry — is only a fraction of the traditional course, even when the refinements necessary for logarithmic solution are included. Thus, the addition formulas, as such, the solution of trigonometric equations, and all reference to angles larger than 180°, are unnecessary for any process of solution of plane triangles.

In order to share with the student the teacher's knowledge that these other topics are of real importance, other practical problems of an elementary nature are used to introduce them. Thus, composition and resolution of forces is made an introduction to the study of large angles, and is used to illustrate the meaning of the addition formulas. Large angles are also used in problems on rotation and angular speed. Radian measure is shown to be useful in problems on rotation and on mensuration.

Topics for which no wide application exists that is within the student's present grasp — such as De Moivre's theorem and infinite se[illegible] book contains a min-
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tion of triangles and those other processes that deal with speed and accuracy. The arrangement is such that the student makes steady progress in his ability to perform operations and to solve problems that actually occur in practice. The geometric methods of solution of triangles occupies the first five pages; the principles of trigonometric solution of right triangles are completed in the next ten pages; accurate solution of right triangles, the principles of solution of oblique triangles, the detailed logarithmic methods, follow in uninterrupted succession. Thus the student may stop at almost any point with a complete grasp of definite processes whose value is clear to him, to which all that he has studied has contributed.

The number of exercises is very large, and the traditional monotony is broken by illustrations from a variety of topics. Here, as well as in the text, the attempt is often made to lead the student to think for himself by giving suggestions rather than completed solutions or demonstrations.

The text proper is short; what is there gained in space is used to make the tables very complete and usable. Attention is called particularly to the complete and handily arranged table of squares, square roots, cubes, etc.; by its use the Pythagorean theorem and the Cosine Law become practicable for actual computation. The use of the slide rule and of four-place tables is encouraged for problems that do not demand extreme accuracy.

Only a few fundamental definitions and relations in Trigonometry need be memorized; these are here emphasized. The great body of principl[illegible] esses depends upon the[illegible] fundamentals; these [illegible] should be retained, rathe[illegible] on that dependence. Oth[illegible] educational val[illegible] e.

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K, Editor.

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TRIGONOMETRY

CHAPTER I

INTRODUCTION

1. Purpose. The original purpose of Trigonometry was the measurement of distances and angles by *indirect* methods in those cases in which *direct* measurements are impossible or inconvenient; for example, in the determination of the heights of buildings, the widths of rivers, the horizontal width of a hill, and so on. Many other applications have been found, some of which are mentioned in this book.

2. Solution of Triangles. To accomplish the purpose first mentioned, it is easy to see that the propositions regarding triangles which are proved in elementary geometry can be used. Thus the theorem that two triangles are congruent* if two angles and the included side of one of them are equal respectively to the corresponding parts of the other, is frequently used in such problems as that which follows.

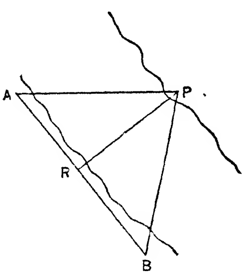


FIG. 1

Example 1. In order to measure the width of a river, for example, it is sufficient to measure the distance AB between two points on the bank and the angles BAP and ABP made by AB with the lines joining A and B, respectively, to any point on the other bank. All of these measurements can be made from one bank of the river. Knowing AB and the angles ABP and BAP the triangle PAB can be drawn to scale by the methods of elementary geometry; then the perpendicular PR from P to AB can be drawn and measured, whence the width PR of the stream can be determined by actual measurement in the figure.

* Two figures are said to be congruent, if by superposition they can be made to coincide in all their parts.

We shall use the following **propositions from geometry**:

All of the parts of a triangle ABC are determined if any one of the following combinations is known:

Either (1) *two sides and the included angle;*

or (2) *two angles and the included side;**

or (3) *three sides;*

or (4) *any two sides, if one angle is a right angle;*

or (5) *two sides and an angle opposite one of them;* but two triangles may be possible in this case. See § 31, p. 41.

When a sufficient number of parts are given, the rest can be found by drawing the triangle as in elementary geometry, and actually measuring the unknown parts, as in Example 1.

To draw the figure, and to make accurate measurements, a ruler marked to scale, a compass, and a protractor are necessary.

The process of finding the unknown parts of a triangle from any such set of given parts is called **solving the triangle.**

3. Preliminary Estimate: Check. In every exercise, the student should first make a preliminary estimate of the size of the unknown parts, and he should bear in mind this crude solution as a guide during his work.

After the unknown parts have been found, the student should use all means within his power to **check** each answer, since even the most experienced persons are liable to errors in reading instruments and in making arithmetical computations.

In figures drawn to scale use these facts as checks:

(*a*) the sum of the angles of a triangle should be 180°.

(*b*) the sum of any two sides should be greater than the third side.

(*c*) the larger of two sides should be opposite the larger of the two opposite angles.

(*d*) the lengths should correspond to the appearance of the figure; thus, if one side appears to be longer than another in the figure, their measures should be unequal in the same sense.

(*e*) the angles should correspond to the appearance of the figure; thus, a right angle is easy to judge by the eye.

* The case in which two angles and *any* side are known is easily reduced to this case, since the sum of all the angles is 180°.

These checks should reveal any *gross* error; but the student should not expect this method of solution (or any other method of computation or measurement) to give *precise* answers in the sense of having no error whatever. The purpose should be to obtain reasonably accurate results and to detect errors that are *unreasonably large.*

EXERCISES I.—GRAPHICAL SOLUTION OF TRIANGLES

1. Solve the following triangles by construction and measurement, and check the answers whenever possible.

(*a*) Two angles are 47° and 53°, the included side is 5.7.

(*b*) Two angles are 43° and 53°, the side opposite the first is 5.7.

(*c*) Two sides are 4.3 and 5.3, the included angle is 57°.

(*d*) The three sides of a triangle are 4.3, 5.3, 6.3.

2. From which of the following sets of given parts is it possible to construct a triangle? In which cases is it impossible? Do any of the sets determine more than one triangle?

(*a*) Two angles are 41° and 59°, the side opposite the latter is 5.1.

(*b*) Two sides are 1.3 and 5.6, the angle opposite the first is 66°.

(*c*) Two angles are 30° and 41°, the included side is 7.

(*d*) Two sides are 7 and 1.1, the included angle is 17°.

(*e*) The three sides are 1.1, 2.3, 3.5.

(*f*) Two sides are 6 and 7, the angle opposite the first is 51°.

3. To determine the width AB of a hill a point C is taken from which the points A and B on opposite sides of the hill are visible. If $AC = 200$ ft., $BC = 223$ ft., and angle $ACB = 62°$, find the width AB.

4. Find the angles which a diagonal of your study table makes with the edges.

5. The steps of a stairway have a tread of 10 in. and a rise of 7 in.; at what angle is the stairway inclined to the floor?

6. To determine without an instrument for measuring angles, the distance between two objects A, B, which are separated by an obstruction, select a point C from which both objects are visible. At C place a smooth board upon a steady support, and mark the precise position of C by sticking a pin in the board. By sighting across this pin locate two other pins, one between A and C, the other between B and C. Draw the lines connecting each of these pins with C. The angle between them is the angle ACB. Finally measure the distances AC and BC. Lay off along the lines CA and CB, on the board, lengths proportional to the actual distances, and thus find AB. (Plane Table Surveying.)

7. A heavy piece of machinery contains a bar AB with an arm AC branching from it. In order to be able to give a mechanic directions for making repairs, the owner measures off on the bar the distance $AB = 10$ in. and on the arm the distance $AC = 10$ in.; he then measures $BC = 8$ in. Make a drawing and find the angle between the arm and the bar.

4. Squared Paper. Rectangular Coördinates. It is often an advantage to draw the figure on paper ruled into squares, called squared paper, or cross-section paper. The location of points is particularly easy on such paper, so that a map, for example, is readily made by using it. By suitably placing the figure, required lengths can frequently be read off at once.

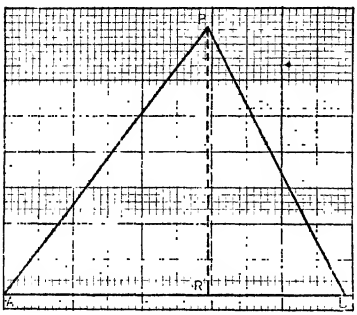


FIG. 2
2 rods = one small space

Thus, in Ex. 1, p. 1, to determine the width of the river, if it is found that $AB = 98$ rods, $\angle A = 51°$, $\angle B = 63°$, the length AB may be laid off on one of the horizontal lines to some convenient scale, and the angles at A and B drawn by means of a protractor. From this figure, the width PR is seen immediately to be about 75 rods.

If any two perpendicular rulings Ox and Oy of the squared paper (see Fig. 3) are selected, the position of any point P in the plane is determined by means of the distances from these two lines to the point P. The paper can be so placed that these distances are horizontal and vertical, respectively; we shall usually suppose the paper in this position.

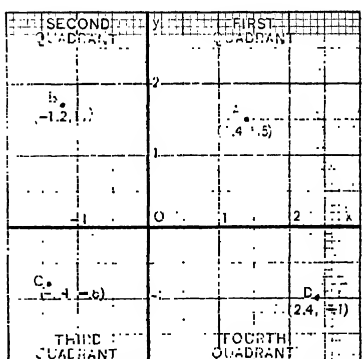


FIG. 3

Thus, in Fig. 3, the horizontal distance from Oy to the point A is 1.4 units and the vertical distance from Ox to A is 1.5 units. To avoid confusion between points at the same distance from Ox but on opposite sides of Oy, it is customary to call distances measured to the right of Oy positive, distances measured to the left negative; thus B, Fig. 3, is said to be -1.2 units from Oy. Similarly, distances measured *downwards* are called *negative*; for example, C in Fig. 3 is said to be $-.8$ units from Ox.

The two distances to any point P from Ox and Oy are called the **rectangular coördinates** of P, and are frequently denoted by the letters x and y, respectively. The horizontal distance x is called the **abscissa** of P; the vertical distance y is called the **ordinate** of P. In giving these distances it is generally understood that the first one mentioned is x, the last y. Thus A, Fig. 3, is briefly denoted by the numbers (1.4, 1.5); B is denoted by $(-1.2, 1.7)$; C by $(-1.4, -.8)$; D by $(2.4, -1)$. The lines Ox, Oy are called the **axes of coördinates,** or simply the **axes.** Ox is called the x-axis, Oy the y-axis. The point O is called the **origin.**

The four portions into which the plane is divided by the axes are called the **first, second, third,** and **fourth quadrants,** as in Fig. 3.

To *locate* a point is to describe its position in the plane in terms of its distances from the coördinate axes; *e.g.* $(-5, 2)$ is a point 5 units to the left of the y-axis and 2 units above the x-axis. To *plot* a point is to mark it in proper position with respect to a pair of axes.

EXERCISES II. — SQUARED PAPER

1. Locate and plot each of the following points with respect to some pair of axes:

(*a*) (1, 2), (*b*) $(2, -3)$, (*c*) $(4, -7)$, (*d*) $(-5, 2)$, (*e*) $(-7, -7)$,
(*f*) (7, 5), (*g*) (5, 12), (*h*) $(8, -3)$, (*i*) $(-5, -5)$, (*j*) $(6, -2)$.

2. Show that the line joining $(5, -4)$ and $(-5, 4)$ is bisected by the origin.

3. On what line do all the points (1, 0), (2, 0), $(-3, 0)$, (1.5, 0) lie? On what line do all the points (0, 0), (0, 1), (0, 2), (0, 5), $(0, -2)$ lie? Make a general statement about such points.

4. Find the distance from the origin to each of the points in Ex. 1, by using the folded edge of another piece of squared paper.

Compute each of the same distances by regarding each of them as the length of the hypotenuse of a right triangle, the lengths of whose sides can be read directly from the figure. Each of these methods can be used as a check on the other.

5. Find the lengths of the sides of a triangle whose vertices are $(0, 0)$, $(2, 3)$, $(1, 5)$, by each of the methods of Ex. 4.

6. From the origin as one vertex construct a triangle two of whose sides are 7 and 13 units long, respectively, with an included angle of $40°$. Measure, by each of the methods of Ex. 4, the length of the unknown side, and measure the angles with a protractor.

7. If a and b are any two numbers positive or negative, but not both zero, show that the line joining (a, b) and $(-a, b)$ is bisected by the y-axis; the line joining (a, b) and $(-a, -b)$ is bisected by the origin.

8. What is the locus of all points in the plane which have the same abscissa? The same ordinate?

9. Two objects A, B in a rectangular field are separated by a thicket, or other obstruction. To determine the distance between them, the lines $AC = 40$ rods, $BC = 30$ rods, are measured parallel to the sides of the field. Find the distance AB, and check the answer, as in Ex. 4.

10. The positions of various objects on a rectangular farm are given by their coördinates in rods, referred to two sides of the farm as axes, as follows: house $(10, 4)$, barn $(6, 4)$, gate of pasture $(60, 20)$. A railroad is constructed through the farm, passing between the house and barn, and a crossing is built at the point $(3, 12)$. Draw a map showing the positions of the various objects. Determine how much farther it is from the house to the barn by way of the crossing than along the straight line connecting them. How much farther is it from the barn to the pasture gate by way of the crossing than along a straight line? Check each answer.

11. A certain city park is bounded by a main street, two cross streets perpendicular to it, and a stream. The distances, in feet, to the stream measured perpendicularly from the main street at 100 ft. intervals are found to be 680, 650, 525, 450, 450, 460, 540.

Draw a map of the park and determine approximately its area by counting the squares inclosed by the figure.

12. To determine the height of a tree OA standing in a level field the distance $OB = 100$ ft. from the base O of the tree to a point B in the field, and the angle $OBA = 37°$, are measured. Find approximately the height by placing the figure on squared paper, after making a preliminary estimate.

CHAPTER II

RIGHT TRIANGLES USE OF TABLES

PART I. FUNDAMENTAL DEFINITIONS AND PRINCIPLES

5. Tables. A method for solving right triangles that is more systematic and more accurate than the method of construction and measurement, consists essentially in making a table of the lengths of the sides and the magnitudes of the corresponding angles of all such triangles. Still the previous methods remain permanently of the utmost importance as a *check.*

It will be shown later that all oblique triangles can be cut up into right triangles in such a way that the same tables can be used in all cases for solving oblique triangles.

Since any triangle can be enlarged (or reduced) in size by drawing it on a larger (or smaller) scale, only **the ratios of the sides** are really important.

For example, it is known by geometry that if one angle of a right triangle is 30°, the side opposite this angle is one half the hypotenuse. Hence if the hypotenuse is given, that side, and hence also the other one, can be determined. If in Fig. 4, $AB = 22.5$, and $\angle A = 30°$, then the side $BC = (1/2)(22.5) = 11.25$.

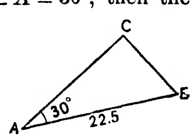


FIG. 4

If, for an acute angle of every right triangle, the ratio of the opposite side to the hypotenuse were known to us, then we could solve every right triangle in the same manner. Tables giving these and other ratios have been constructed.

6. Definitions of the Ratios. As indicated in § 5, the ratio of two sides of a triangle does not depend upon the size of the

triangle, but only upon the angles. Thus in the right triangles MPN, $MP'N'$, $MP''N''$ of Fig. 5, in which PN, $P'N'$, $P''N''$ are perpendicular to MN, the ratios NP/MP, $N'P'/MP'$, $N''P''/MP''$ are all equal. Moreover, if $P'''N'''$ is drawn perpendicular to MP, each of the ratios just mentioned is equal to $N'''P'''/MP'''$. (Why?) These ratios, then, depend *only on the angle α at M.* It is convenient to place the angle on a pair of axes so that the vertex falls at the origin O, one side lies along the x-axis, to the right, and the other side falls in the first quadrant. On this side take any point P at random, except O, and drop the perpendicular PM to the x-axis (see Fig. 6). Let $OP = r$; then by geometry

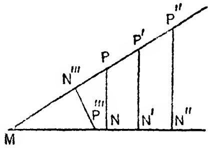


Fig. 5

$$r = \sqrt{x^2 + y^2},^*$$

where x and y are the coördinates of the point P. The various ratios of pairs of the three quantities x, y, r are the same *for all points P* taken in the side OP of the angle α. These are:

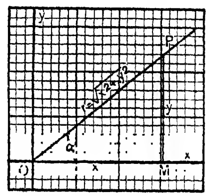


Fig. 6

(1) $\dfrac{y}{r}$, called **the sine of the angle α**, written **sin α**.

(2) $\dfrac{x}{r}$, called **the cosine of the angle α**, written **cos α**.

(3) $\dfrac{y}{x}$, called **the tangent of the angle α**, written **tan α**.

The reciprocals† of these ratios are also often used:

(4) r/y is called the *cosecant of the angle α*, written csc α.

(5) r/x is called the *secant of the angle α*, written sec α.

(6) x/y is called the *cotangent of the angle α*, written ctn α.

These six ratios are collectively called trigonometric ratios or also trigonometric functions of the angle.

* The radical sign is used to denote the *positive* square root.

† The reciprocal of a number is unity divided by the number. The reciprocal of a common fraction is the result of inverting it; thus the reciprocal of y/r is r/y. Every number has a reciprocal except 0, which has not.

Other expressions derived from these are also frequently used; for example, many engineers use the following combinations:

(7) versed sine of $\alpha = 1 - \cos\alpha$, written vers α;

(8) coversed sine of $\alpha = 1 - \sin\alpha$, written covers α;

(9) external secant of $\alpha = \sec\alpha - 1$, written exsec α.

In the right triangle OPM, Fig. 6, y is the side opposite the angle α, x is the side adjacent to α, and r is the hypotenuse. From the definitions (1)–(3), we see that *in any right triangle:*

(10) **The sine of either acute angle** $= \dfrac{\textbf{side opposite}}{\textbf{hypotenuse}}$;

(11) **The cosine of either acute angle** $= \dfrac{\textbf{side adjacent}}{\textbf{hypotenuse}}$;

(12) **The tangent of either acute angle** $= \dfrac{\textbf{side opposite}}{\textbf{side adjacent}}$;

and, after clearing of fractions, we find:

(13) **The side opposite = hypotenuse × sine**
= side adjacent × tangent;

(14) **The side adjacent = hypotenuse × cosine**
= side opposite × cotangent;

(15) **Hypotenuse** $= \dfrac{\textbf{side opposite}}{\textbf{sine}} = \dfrac{\textbf{side adjacent}}{\textbf{cosine}}$.

The student should so thoroughly learn these statements that he can apply them instantly and confidently to any right triangle that he sees, whatever its position in the plane.

The trigonometric functions are connected by many simple relations. Thus:

(16) $\tan\alpha = \sin\alpha/\cos\alpha$, since $y/x = (y/r)/(x/r)$.

Similarly, the student can easily show that

(17) $\text{ctn}\,\alpha = \cos\alpha/\sin\alpha = 1/\tan\alpha$,

(18) $\sec\alpha = 1/\cos\alpha$,

(19) $\csc\alpha = 1/\sin\alpha$.

Other relations will be given later.

7. Applications. The values of these ratios have been computed for all acute angles, and recorded in convenient tables. These tables, together with the formulas just given, enable us to solve all cases of right triangles. The methods are illustrated in the following examples.

Example 1. One angle of a right triangle is 38° and the hypotenuse is 12 ft. Find the lengths of each of the other sides.

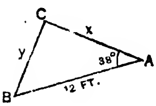


FIG. 7

Draw a figure, mark the given parts, and indicate the parts to be found by suitable letters, say x and y. The sides x and y are then respectively the side adjacent and the side opposite. To find x, note that the hypotenuse is given; hence by (14), § 6,

$$x = 12 \cdot \cos 38° = 12\,(.788) = 9.456\,;$$

and similarly by (13),

$$y = 12 \cdot \sin 38° = 12\,(.616) = 7.392\,;$$

the values $\cos 38° = .788$ and $\sin 38° = .616$ being found in the table printed in § 9, p. 15, where a method for computing such values is explained.

As a check, the Pythagorean theorem may be used, particularly if a table of squares is available. Thus, denoting the hypotenuse by h, we *should* have

$$h = \sqrt{(9.456)^2 + (7.392)^2} = 12.002.$$

This agrees reasonably well with the given value $h = 12$. Another check that is more practical is given by measurement from a good figure.

Example 2. One side of a right triangle is 17 and the angle opposite this side is 27°; what is the length of the hypotenuse? of the other side?

Denote the hypotenuse by u and the unknown side by v. Noting that the side *opposite* the given angle is given, find the *side adjacent*, v, by (14), § 6. To find the hypotenuse, use (15), § 6:

$$v = 17 \cdot \text{ctn}\, 27° = 17\,(1.963) = 33.37.$$

$$u = 17/\sin 27° = 17/.454 = 37.44.$$

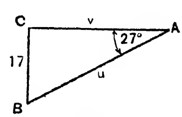


FIG. 8

Check these answers by drawing an accurate figure.

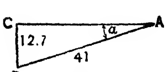


FIG. 9

Example 3. The hypotenuse of a right triangle is 41 and one side is 12.7; find the opposite angle.

Denote the opposite angle by α, then by (10), $\sin \alpha = 12.7/41 = .309$. From the table (p. 15), we see that .309 is the sine of 18°; hence $\alpha = 18°$. Let the student determine the remaining side and angle, and check all answers.

EXERCISES III. — TRIGONOMETRIC RATIOS

1. On cross-section paper construct angles whose sines are, (*a*) 1/5; (*b*) 2/5; (*c*) 3/5; (*d*) 4/5. [HINT. First draw a circle of radius 5.]

2. Is there an acute angle whose sine is any given positive number? Prove that your answer is correct.

3. Construct angles whose tangents are (*a*) 3/10; (*b*) 1/2; (*c*) 2/3; (*d*) 1; (*e*) 10/3; (*f*) 2; (*g*) 7.5.

4. Is there an angle whose tangent is any given positive number?

5. How large, in degrees, is an acute angle whose tangent is 1?

6. How does the angle whose tangent is 2 compare with the angle whose tangent is 1? Check your answer by drawing an accurate figure.

7. One side of a right triangle is 21; the adjacent angle is 42°; determine the remaining side and the hypotenuse. Check.

8. One side of a right triangle is 21 and the opposite angle is 42°; determine the remaining side and hypotenuse. Check.

9. The hypotenuse of a right triangle is 28; one angle is 32°. Determine the two perpendicular sides. Check.

10. What is the angle of inclination of a roof which has half pitch? 1/3 pitch?

[NOTE. The pitch of a roof is equal to the height of the comb above the eaves divided by the total distance between the eaves.]

8. Functions of 0° and 90°. If an angle of 0° (or 90°) be placed on coördinate axes and the construction of p. 8 be made, the point P_0 (or P_{90}) will lie on the x (or y) axis, and we shall have $x = r$, $y = 0$ (or $x = 0$, $y = r$).

The functions sine, cosine, tangent, and secant of 0° are defined by the same ratios as are the corresponding functions of acute angles: $\sin 0° = y/r = 0$, $\cos 0° = x/r = 1$, $\tan 0° = y/x = 0$, $\sec 0° = r/x = 1$. The definitions of cotangent and cosecant given for acute angles cannot be applied to 0°, because $y = 0$ and the divisions x/y and r/y are impossible.

The sine, cosine, cotangent, and cosecant of 90° are defined by the same ratios as are the corresponding functions of acute angles; $\sin 90° = y/r = 1$, $\cos 90° = x/r = 0$, $\text{ctn } 90° = x/y = 0$, $\csc 90° = r/y = 1$. The definitions of tangent and secant given for acute angles cannot be applied to 90°, because $x = 0$ and

the divisions y/x and r/x are impossible. We say that 0° has no cotangent or cosecant, and 90° has no tangent or secant.

9. Construction of Small Tables. Approximate values of the trigonometric functions of a given acute angle may be found by

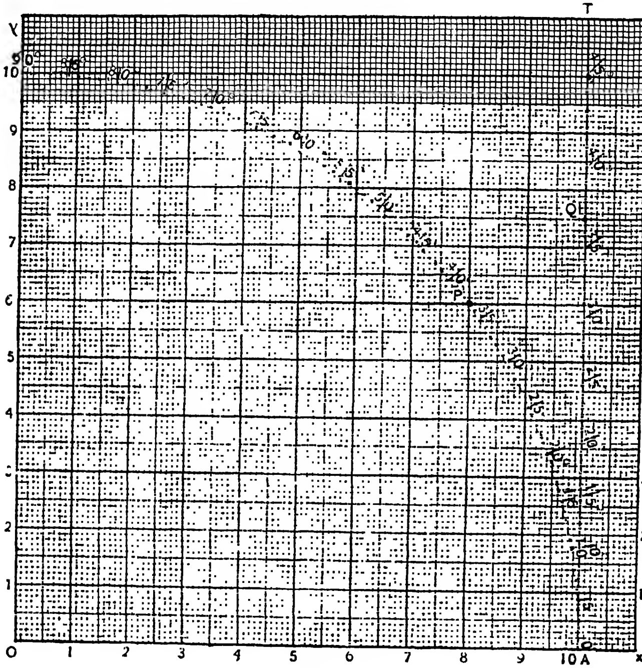


FIG. 10

measurement as follows: On a sheet of squared paper draw Ox and Oy along two of the perpendicular rulings, and in the first quadrant construct a quarter circle, radius $= 10$, center at O. Draw AT perpendicular to Ox and tangent to this circle. Given

* It is often said that the tangent of 90°, for example, is *infinite;* this expression does not give any value to the tangent at 90°, but merely describes the fact that the tangent becomes and remains larger than any number we may name as the angle approaches 90°. Similar statements hold for the others.

now any acute angle, α, lay it off above Ox with its vertex at O. Mark the points where its side crosses the circle and the tangent AT, P and Q, respectively; then the ordinate (y) of the point P can be read to tenths, and this divided by $r = 10$ gives the value of $\sin \alpha$ to two decimal places. Similarly, the abscissa (x) of P can be read to tenths, and this divided by 10 gives $\cos \alpha$. AQ can be read to tenths, and this divided by $OA = 10$ gives $\tan \alpha$. Finally, $\operatorname{ctn} \alpha$, $\sec \alpha$, $\csc \alpha$, can be computed as the reciprocals of $\tan \alpha$, $\cos \alpha$, $\sin \alpha$, respectively. Compute in this way, from Fig. 10, values to fill out the following table.

α	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°
$\sin \alpha$																	
$\cos \alpha$																	
$\tan \alpha$																	
$\operatorname{ctn} \alpha$																	

If all of this table is filled out correctly, it will be found that every number in it occurs twice; once for an angle less than 45° and once for an angle greater than 45°. This is due to the fact that *the sine of any angle is the cosine of its complement; and the tangent of any angle is the cotangent of its complement.*

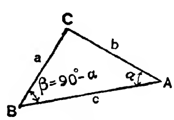


FIG. 11

These relations will now be proved for any acute angle α. Let $\beta = 90° - \alpha$; then α and β are the acute angles of a right triangle. Denote the sides opposite α and β by a and b, respectively; and the hypotenuse by c. Then by § 6, p. 8,

$$\sin \alpha = \text{side opp./hyp.} = a/c; \quad \tan \alpha = \text{side opp./side adj.} = a/b;$$
$$\cos \beta = \text{side adj./hyp.} = a/c; \quad \operatorname{ctn} \beta = \text{side adj./side opp.} = a/b;$$

whence, remembering that $\beta = 90° - \alpha$,

$$(1) \qquad \sin \alpha = \cos \beta = \cos (90° - \alpha),$$

$$(2) \qquad \tan \alpha = \operatorname{ctn} \beta = \operatorname{ctn} (90° - \alpha).$$

In the same way it can be shown that

$$(3) \qquad \sec \alpha = \csc (90° - \alpha).$$

On the opposite page a table of values of the functions is given, for every degree from 0° to 90°, using the preceding facts. The values can be roughly verified by Fig. 10.

EXERCISES IV. — SOLUTION OF RIGHT TRIANGLES

In the solution of triangles, use the following procedure:

(*a*) Draw a diagram approximately to scale, indicating the given parts. Mark the unknown parts by suitable letters, and estimate their values.

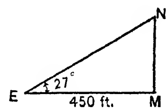
(*b*) *If one of the given parts is an acute angle*, consider the relation of the known parts to the one which it is desired to find, and apply the proper one of formulas (10) ⋯ (15), § 6.

(*c*) *If two sides are given*, and one of the acute angles is desired, think of the definition of that function of the angle which employs the two given sides.

(*d*) Check each result.

1. In the following triangles h denotes the hypotenuse; the angle A is opposite the side a and the angle B is opposite the side b. Use the table to compute the unknown parts from the given parts. Check.

(*a*) $A = 61°$, $b = 41$. (*d*) $A = 32°$, $a = 330$.
(*b*) $a = 421$, $b = 401$. (*e*) $a = 313$, $h = 720$.
(*c*) $a = 62$, $h = 125$. (*f*) $B = 49°$, $h = 24$.



2. Determine the height of a tower MN, if the horizontal distance ME to it is 450 ft. and the *angle of elevation* MEN is 27°. Check.

3. A vertical pole 35 ft. high casts a horizontal shadow 45 ft. long. Determine the angle of elevation of the sun above the horizon. Check.

4. An object known to be 100 ft. in height stands on the bank of a river; from the opposite bank of the river the angle of elevation of the top of the object is found to be 24°; find the width of the river. Check.

5. The radius of a circle is 7 ft. What angle will a chord of the circle 11 ft. long subtend at the center? Check.

6. From the top of a cliff 92 ft. in height the angle of depression of a boat at sea is observed to be 20°. How far out is the boat? Check.

7. To find the distance between two objects A and B, where B is in a swamp, the distance $AC = 350$ ft. is measured at right angles to the line joining them. At C an observer holds an ordinary rake with the end of the handle at his eye and with the center of the rake directed toward A. There appear then to be 6 teeth of the rake between A and B. If the teeth are one inch apart and the handle of the rake is five feet long, determine the distance between A and B.

TRIGONOMETRIC FUNCTIONS TO THREE PLACES OF DECIMALS

α	sin α	covers α	sec α	tan α	ctn α	csc α	vers α	cos α	
0°	.000	1.000	1.000	.000	——	——	.000	1.000	**90°**
1°	.017	.983	[illegible]	.017	57.290	57.299	.000	1.000	89°
2°	.035	.965	1.001	.035	28.636	28.654	.001	.999	88°
3°	.052	.948	1.001	.052	19.081	19.107	.001	.999	87°
4°	.070	.930	1.002	.070	14.301	14.336	.002	.998	86°
5°	.087	.913	1.004	.087	11.430	11.474	.004	.996	**85°**
6°	.105	.895	1.006	.105	9.514	9.567	.005	.995	84°
7°	.122	.878	1.008	.123	8.144	8.206	.007	.993	83°
8°	.139	.861	1.010	.141	7.115	7.185	.010	.990	82°
9°	.156	.844	1.012	.158	6.314	6.392	.012	.988	81°
10°	.174	.826	1.015	.176	5.671	5.759	.015	.985	**80°**
11°	.191	.809	1.019	.194	5.145	5.241	.018	.982	79°
12°	.208	.792	1.022	.213	4.705	4.810	.022	.978	78°
13°	.225	.775	1.026	.231	4.331	4.445	.026	.974	77°
14°	.242	.758	1.031	.249	4.011	4.134	.030	.970	76°
15°	.259	.741	1.035	.268	3.732	3.864	.034	.966	**75°**
16°	.276	.724	1.040	.287	3.487	3.628	.039	.961	74°
17°	.292	.708	1.046	.306	3.271	3.420	.044	.956	73°
18°	.309	.691	1.051	.325	3.078	3.236	.049	.951	72°
19°	.326	.674	1.058	.344	2.904	3.072	.054	.946	71°
20°	.342	.658	1.064	.364	2.747	2.924	.060	.940	**70°**
21°	.358	.642	1.071	.384	2.605	2.790	.066	.934	69°
22°	.375	.625	1.079	.404	2.475	2.669	.073	.927	68°
23°	.391	.609	1.086	.424	2.356	2.559	.079	.921	67°
24°	.407	.593	1.095	.445	2.246	2.459	.086	.914	66°
25°	.423	.577	1.103	.466	2.145	2.366	.094	.906	**65°**
26°	.438	.562	1.113	.488	2.050	2.281	.101	.899	64°
27°	.454	.546	1.122	.510	1.963	2.203	.109	.891	63°
28°	.469	.531	1.133	.532	1.881	2.130	.117	.883	62°
29°	.485	.515	1.143	.554	1.804	2.063	.125	.875	61°
30°	.500	.500	1.155	.577	1.732	2.000	.134	.866	**60°**
31°	.515	.485	1.167	.601	1.664	1.942	.143	.857	59°
32°	.530	.470	1.179	.625	1.600	1.887	.152	.848	58°
33°	.545	.455	1.192	.649	1.540	1.836	.161	.839	57°
34°	.559	.441	1.206	.675	1.483	1.788	.171	.829	56°
35°	.574	.426	1.221	.700	1.428	1.743	.181	.819	**55°**
36°	.588	.412	1.236	.727	1.376	1.701	.191	.809	54°
37°	.602	.398	1.252	.754	1.327	1.662	.201	.799	53°
38°	.616	.384	1.269	.781	1.280	1.624	.212	.788	52°
39°	.629	.371	1.287	.810	1.235	1.589	.223	.777	51°
40°	.643	.357	1.305	.839	1.192	1.556	.234	.766	**50°**
41°	.656	.344	1.325	.869	1.150	1.524	.245	.755	49°
42°	.669	.331	1.346	.900	1.111	1.494	.257	.743	48°
43°	.682	.318	1.367	.933	1.072	1.466	.269	.731	47°
44°	.695	.305	1.390	.966	1.036	1.440	.281	.719	46°
45°	.707	.293	1.414	1.000	1.000	1.414	.293	.707	**45°**
	cos α	vers α	csc α	ctn α	tan α	sec α	covers α	sin α	α

Values of vers $\alpha = 1 - \cos \alpha$ and of covers $\alpha = 1 - \sin \alpha$ are included for completeness.

10. Pythagorean Relations.

The following equation between the abscissa x, the ordinate y, and the radius r is true for every point in the plane:

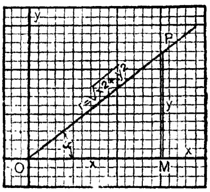


FIG. 12

(1) $$x^2 + y^2 = r^2.$$

Dividing by r^2, we obtain

$$x^2/r^2 + y^2/r^2 = 1;$$

but by § 6, at least when α is acute, $x/r = \cos\alpha$, $y/r = \sin\alpha$; hence

(2) $$\sin^2\alpha + \cos^2\alpha = 1;$$

i.e. the sum of the squares of the sine and cosine of any acute angle is equal to unity.†

Dividing (1) by x^2, and then by y^2, we obtain respectively:

(3) $$1 + \tan^2\alpha = \sec^2\alpha,$$

(4) $$1 + \operatorname{ctn}^2\alpha = \csc^2\alpha.$$

These formulas and those of § 6 are often useful in simplifying expressions or in verifying equations. Other interesting relations are given in exercises that follow.

Example 1. To show that $\sin^4\alpha - \cos^4\alpha = \sin^2\alpha - \cos^2\alpha$.

The expression on the left is the difference of two squares and can therefore be factored; hence we have $\sin^4\alpha - \cos^4\alpha = (\sin^2\alpha + \cos^2\alpha)(\sin^2\alpha - \cos^2\alpha)$ which is equal to $\sin^2\alpha - \cos^2\alpha$, since $\sin^2\alpha + \cos^2\alpha = 1$.

The formulas may also be used to compute the value of one of the trigonometric functions from that of another.

Example 2. Given $\tan\theta = 5/12$, to find $\cos\theta$.

Analytic Method. By (2), $1 + \tan^2\theta = \sec^2\theta$; hence, $\sec^2\theta = 1 + 25/144 = 169/144$, or $\sec\theta = 13/12$. Hence, $\cos\theta = 12/13$, since $\cos\theta = 1/\sec\theta$.

Geometric Method. The following method is much more practical, and is easily applied to any example of this sort.

Draw a right triangle whose base is 12 and whose altitude is 5. The hypotenuse is easily found to be 13. It follows that

$$\cos\theta = \text{side adjacent/hypotenuse} = 12/13.$$

---

* Formulas (2), (3), and (4) are called the **Pythagorean relations** because they are obtained from this equation, which is the Pythagorean theorem of plane geometry.

† This statement, as well as (3) and (4) below, will later be found to hold for all angles, for the general definitions of sine and cosine.

EXERCISES V.—PYTHAGOREAN RELATIONS IDENTITIES

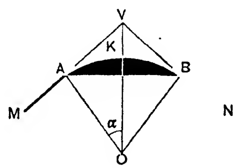
1. In exercises (*a*) – (*i*) determine the values of the remaining functions of the acute angle θ by each of the methods of Example 2, p. 16.

(*a*) $\sin\theta = 3/5$.	(*b*) $\sin\theta = 1/3$.	(*c*) $\cos\theta = 1/3$.
(*d*) $\sin\theta = 5/13$.	(*e*) $\tan\theta = \sqrt{3}$.	(*f*) $\tan\theta = 3/4$.
(*g*) $\tan\theta = 1/m$.	(*h*) $\sin\theta = b/c$.	(*i*) $\sec\theta = 2$.

Prove the following relations for any acute angle θ:

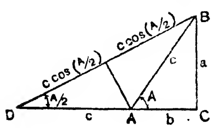
2. $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$.
3. $\cos\theta\tan\theta = \sin\theta$.
4. $\tan\theta + \operatorname{ctn}\theta = \sec\theta\csc\theta$.
5. $\sin\theta\sec\theta = \tan\theta$.
6. $(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$.
7. $(\sin^3\theta + \cos^3\theta) = (\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta)$.
8. $\cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$.
9. $\sec^2\theta\csc^2\theta = \tan^2\theta + \operatorname{ctn}^2\theta + 2$.

10. If a and b are the sides of a right triangle, c the hypotenuse, and A the angle opposite a, show that the area of the triangle is equal to either of the expressions $(ac\cos A)/2$, $(bc\sin A)/2$.



11. Two straight pieces of railroad track MA and NB are to be connected by a circular track AKB with a radius of 500 ft. and center O, tangent to MA and NB. The straight portions of the track produced intersect at a point V at an angle of 100°. (*a*) How far back from V should the track begin to turn? (*b*) How far from V along the bisector OV of the angle AVB is the center O? (*c*) Find the shortest distance from V to the curved portion.

12. If, in a figure similar to that of Ex. 11, $\angle AVO$ is any angle, and $\angle VOA$ is denoted by α, and $OA = r$, show that (*a*) $AV = r\tan\alpha$; (*b*) $KV = r \operatorname{exsec}\alpha$; (*c*) $AB = 2r\sin\alpha$.



13. The side b of the triangle in Ex. 10 is extended beyond A to a point D, making $AD = c$, so that ABD is isosceles. Show that

(*a*) $\angle ADB = A/2$; (*b*) $BD = 2c\cos(A/2)$.

(*c*) From the right triangles DCB and ACB, show that $c\sin A = a = 2c\cos(A/2)\sin(A/2)$;

hence $\sin A = 2\sin(A/2)\cos(A/2)$;

(*d*) Likewise, show that $c\cos A = b = 2c\cos^2(A/2) - c$;

hence $\cos A = 2\cos^2(A/2) - 1 = \cos^2(A/2) - \sin^2(A/2)$.

PART II. ACCURACY OF SOLUTIONS—APPLICATIONS

11. The Question of Greater Accuracy. The degree of accuracy of the results obtained by using the values of the trigonometric functions to three places of decimals, while sufficient for many ordinary applications, is not satisfactory for some purposes; for example, in extended surveys, in astronomy, and in any work for which the data must be determined by using instruments of precision.*

More accurate values have been calculated. The values for angles at intervals of $1'$ are given to five decimal places in the Tables (Table II). Still more accurate values are available in separately printed tables.

12. Functions of 30°, 45°, 60°. To determine the functions of the angle 45° construct an isosceles right triangle having each of its equal sides m units in length. Each of the equal angles is 45°. The hypotenuse is $m\sqrt{2}$; hence $\sin 45° = m/(m\sqrt{2}) = 1/\sqrt{2} = \sqrt{2}/2 = .7071^+$. Compute the values of the other functions of 45° in a similar manner.

To determine the functions of 30° and 60°, draw an equilateral triangle of side m and drop a perpendicular from one vertex to the opposite side. The acute angles of each of the right triangles thus formed are 30° and 60°. Find the hypotenuse and the sides of one of these right triangles in terms of m, and compute the values of each of the functions for each of these angles.

The values in the following table should be memorized:

	0°	30°	45°	60°	90°	$\sqrt{2} = 1.414$
sin	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1	$\sqrt{3} = 1.732$
cos	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0	$1/\sqrt{2} = \sqrt{2}/2$
tan	0	$\sqrt{3}/3$	1	$\sqrt{3}$		$1/\sqrt{3} = \sqrt{3}/3$

* Thus it is obviously unwise to use values to more than three decimal places in reducing any large measurements on the earth's surface unless a standard steel tape and other standard surveying instruments are available.

13. Use of the Large Tables. Five-place tables are used in precisely the same manner as the small table of p. 15.

Example 1. One angle of a right triangle is 42° 20′ and the hypotenuse is 28 ft. 6 in. long. Find the remaining sides and the other angle. Draw a diagram to illustrate the problem, indicating the given parts. Denote the unknown parts by the letters a and b, as in Fig. 13.

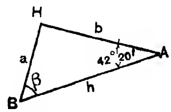


FIG 13

To find b, note that it is the *side adjacent* to the given angle, and that the hypotenuse is given. Hence, by (14), § 6,

$$b = 28.5 \cos 42° 20' = 28.5 \times .73924 = 21.07.$$

Note that a is opposite the given angle; hence by (13), § 6,

$$a = 28.5 \sin 42° 20' = 28.5 \times .67344 = 19.19;$$

the sine and the cosine of 42° 20′ being found in a table.

To find β, note that it is the complement of 42° 20′; hence $\beta = 47° 40'$.

Example 2. The perpendicular sides of a right triangle are 22 ft. 6 in. and 54 ft., respectively. Find the hypotenuse and the angles. Draw a diagram, indicating the given parts and lettering the parts to be found, as in Fig. 14. To find α, note that the given parts are the sides opposite and adjacent to it; hence by the definition of *tangent*, we may write

$$\tan \alpha = 22.5 \div 54 = .41667.$$

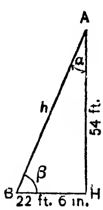


FIG. 14

From the tables $\tan 22° 37' = .41660$;

hence $\alpha = 22° 37'+$ and $\beta = 67° 23'-$.

Another method is the following: By the Pythagorean theorem of plane geometry, using a table of squares and square roots or by direct calculation,

$$h^2 = \overline{54}^2 + \overline{22.5}^2 = 3422.25.$$

Hence $h = 58.5$. But this method is arduous without a table of squares. See Table VI.

Having found $\alpha = 22° 37'$ we might find h as follows. By (15), § 6, $h = 54/(\cos 22° 37') = 54/.92310 = 58.498$. This method is no shorter than the one used above, and is open to the objection that any error made in computing α vitiates the resulting value found for h. In general, compute each unknown part from the given parts; *i.e. do not use computed parts as data if it can be avoided.*

As in these Examples, observe the procedure suggested on p. 14, Exercises IV, in solving any triangle.

EXERCISES VI. — RIGHT TRIANGLES LARGE TABLES

1. Solve the following right triangles. The hypotenuse is denoted by h, other sides by other small letters, and any angle by the capital letter corresponding to the small letter that denotes the side opposite it.

(a) $A = 61° 17'$, $b = 1.4$. (d) $M = 49° 49'$, $h = 24.6$. (g) $p = 18.2$, $q = 50$.
(b) $A = 32° 31'$, $a = 33$. (e) $b = 4.848$, $h = 10$. (h) $u = 11.65$, $h = 25$.
(c) $a = 62.12$, $h = 254$. (f) $U = 63° 2'$, $u = 40$. (i) $m = 34.2$, $h = 100$.

2. The base of an isosceles triangle is 324 ft., the angle at the vertex is 64° 40′. Find the equal sides and the altitude.

3. A chord of a circle is 21.5 ft., the angle which it subtends at the center is 41°. Find the radius of the circle.

4. To determine the width BA of a river, a line BC 100 rods long is laid off at right angles to a line from B to some object A on the opposite bank visible from B. The angle BCA is found to be 43° 35′. Find AB.

5. The shadow of a tower 200 ft. high is 252.5 ft. long. What is the angle of elevation of the sun?

6. Two ships in a vertical plane with a lighthouse are observed from its top, which is 200 ft. above sea level. The angles of depression of the two ships are 15° 17′ and 11°22′. Find the distance between the ships.

14. Projections. The **projection** of a line segment AB upon a line l is defined to be the portion MN of the line l between perpendiculars drawn to it from A and B, respectively. The length of this projection is easily found if the length of AB and the angle α which the line AB makes with l are known. For, draw a parallel to l through A, meeting BN at C. Then ACB is a right triangle and the angle at A is α; hence by (14), § 6,

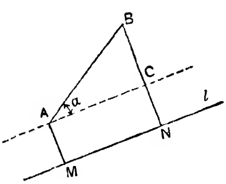


FIG. 15

$$MN = AB \cos \alpha,$$

or, *the projection of a segment upon a given line is equal to the product of the length of the segment and the cosine of the angle the segment makes with the given line.*

The projections of a segment upon the coördinate axes are frequently used. If the segment makes an angle α with the horizontal, the projections on the x and y axes are, respectively,

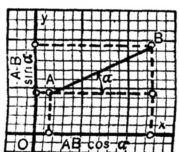


FIG. 16

(1) $\text{Proj}_x AB = AB \cos \alpha,\quad \text{Proj}_y AB = AB \sin \alpha,$

where $\text{Proj}_x\ AB$ and $\text{Proj}_y\ AB$ denote the projections of AB on the x-axis and the y-axis, respectively.

15. Applications of Projections. In mechanics and related subjects, forces and velocities are represented graphically by line segments. A force, say of 10 lb., is represented by a segment 10 units in length in the direction of the force. A velocity of 20 ft. per sec. is represented by a segment 20 units in length in the direction of motion.

The projection upon a given line l, of a segment representing a force, represents the effective force in the direction l; this is called the *component* of the given force in the direction l.

Example 1. A weight of 50 lb. is placed upon a smooth plane inclined at an angle of 27° with the horizontal. What force acting directly up the incline will be required to keep the weight at rest?

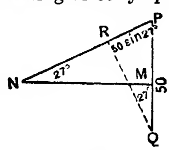


FIG. 17

Draw to some convenient scale a segment 50 units in length directly downward to represent the force exerted by the weight. Project this segment upon a line inclined at an angle of 27° with the horizontal. The length of this projection is $50 \sin 27° = 22.7$ nearly. This represents the component of the force down the plane. Therefore, a force of 22.7 lb. acting up the plane will be required to keep the weight at rest.

EXERCISES VII. — PROJECTIONS

1. Find the horizontal and vertical projections of the segments:

(*a*) length 42, making an angle of 37° with the horizontal.

(*b*) length 5.5, making an angle of 50° with the vertical.

2. A straight railroad crosses two north and south roadways a mile apart. The length of track between the roadways is $1\frac{1}{4}$ mi. A train travels this distance in 2 min. Find the components of the velocity of the train parallel to the roadways and perpendicular to them. Find the angle between the track and either roadway.

3. The eastward velocity of a certain train is 24 mi. per hour. The northward velocity is 32 mi. per hour. Find its actual velocity along the track and the angle the track makes with the east and west direction.

4. A car is drawn by means of a cable. If a force of 5000 lb. exerted along the track is required to pull the car, what force will be required when the cable makes an angle of 15° with the track?

5. Find the horizontal and vertical components of a force represented by a segment 30 units long at an angle of 40° with the horizontal.

16. The Use of Logarithms. Logarithms may be used to shorten computations involving multiplications, divisions, raising to powers or extracting roots, but not involving additions or subtractions. In much of the numerical work which follows, the use of logarithms is very advantageous in saving time and labor, but the student should bear in mind that logarithms are not necessary. They are merely convenient, and they belong no more to trigonometry than to arithmetic. One of the questions which a computer has to decide is whether or not it will be advantageous to use logarithms in a given problem.

Tables usually contain (1) tables of logarithms; (2) tables of the trigonometric functions; (3) tables of logarithms of the trigonometric functions. The notation log tan 61° means the logarithm of the tangent of 61°; *i.e.* the tangent of 61° is a number, 1.804^+, and the logarithm of this number 1.804^+ is .25625.

A formula which has been arranged so as to involve only products and quotients of powers and roots of quantities either known or easily computed is said to be *adapted to logarithmic computation.*

Thus the formula $h = \sqrt{a^2 + b^2}$, which gives the hypotenuse h of a right triangle in terms of the sides a and b, is not adapted to logarithmic computation. On the other hand, the formula $b = \sqrt{h^2 - a^2} = \sqrt{(h + a)(h - a)}$ which gives one side in terms of the hypotenuse and the other side is adapted to logarithmic computation because $(h + a)$ and $(h - a)$ are easily obtained from h and a. Thus, if the hypotenuse is 17.34 and one side is 12.27, the other side is

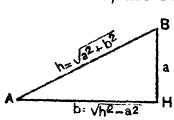


FIG. 18

$$x = \sqrt{(5.07)(29.61)}.$$

From Table I:

$$\begin{aligned} \log 5.07 &= 0.70501 \\ \log 29.61 &= 1.47144 \\ \log x^2 &= 2.17645 \\ \log x &= 1.08822 \\ x &= 12.252 \end{aligned}$$

Again, such an expression as $12.5 \sin 42° 37'$, which might occur in solving a triangle, would be computed as follows:

$$\begin{aligned} \log 12.5 &= 1.09691 && \text{[Table I]} \\ \log \sin 42° 37' &= 9.83065 - 10 && \text{[Table III]} \\ \log [12.5 \sin 42° 37'] &= 10.92756 - 10 && \\ &= 0.92756 && \end{aligned}$$

Hence $\quad 12.5 \sin 42° 37' = 8.4637$ [Table I]

17. Products with Negative Factors. To find by use of logarithms the product of several factors some of which are negative, the product of the same factors, all taken positively, is first obtained, and the sign is then determined in the usual way by counting the number of negative signs.

Thus, to obtain the product of the four factors -115, 23.41, $-.6422$, and $-.1123$, we write $x = (115)(23.41)(.6422)(.1123)$; then

$$\begin{aligned}
\log 115 &= 2.06070 && \text{(n)}\\
\log 23.41 &= 1.36940 &&\\
\log .6422 &= 9.80767 - 10 && \text{(n)}\\
\log .1123 &= \underline{9.05038 - 10} && \text{(n)}\\
\log x &= 2.28815 \qquad \text{hence} && x = 194.15
\end{aligned}$$

Here the fact that the first factor and the last two are negative is indicated by writing an (n) in parenthesis to the right of the corresponding logarithms. The product of the given factors all taken positively is 194.15; since the number of negative factors is odd the product is really -194.15.

For other processes, see the Explanation of the Tables.

EXERCISES VIII. — RIGHT TRIANGLES — MISCELLANEOUS

1. Solve by means of logarithms the following right triangles, where h denotes the hypotenuse, other small letters the sides, and the corresponding capital letters the angles opposite those sides.

(a) $A = 63°$; $h = 28.54$.	(b) $a = 735.1$; $h = 846.2$.
(c) $P = 65° 25'.2$; $p = 69.25$.	(d) $r = 9.328$; $s = 6.302$.
(e) $A = 28° 25'$; $h = 29.36$.	(f) $a = 59.68$; $h = 69.27$.
(g) $V = 28° 40'.4$; $v = 20.71$.	(h) $G = 36° 21'$; $h = 41.376$.

2. A tree stands on the opposite side of a small lake from an observer. At the edge of the lake the angle of elevation of the top of the tree is found to be 30° 58′. The observer then measures 100 ft. directly away from the tree and finds the angle of elevation to be 18° 26′. Find the height of the tree and the width of the lake.

3. From a point 250 ft. from the base of a tower and on a level with the base the angle of elevation of the top is 62° 32′. Find the height.

4. To determine the height of a tower, its shadow is measured and found to be 97.4 ft. long. A ten-foot pole is then held in vertical position and its shadow is found to be 5.5 ft. Find the height of the tower and the angle of elevation of the sun.

5. Find the length of a ladder required to reach the top of a building 50 ft. high from a point 20 ft. in front of the building. What angle would the ladder in this position make with the ground?

6. The width of the gable of a house is 34 ft.; the height of the house above the eaves is 15 ft. Find the length of the rafters and the angle of inclination of the roof. Find the pitch of the roof. (Ex. 10, p. 11.)

7. A kite string is 250 ft. long and makes an angle of 40° with the level ground. Find approximately the height of the kite above the ground, neglecting the sag in the string.

8. One bank of a river is a bluff rising 75 ft. vertically above the water. The angle of elevation of the top of the bluff from the water's edge on the opposite bank is 20° 27′; find the width of the river.

9. A taut rope 100 ft. long is attached to the top of a building. The free end reaches the ground 24 ft. 7 in. away from the base of the building. Find the height of the building and the angle which the rope makes with the ground.

10. Find the angles which the diagonal of a rectangle 12 ft. wide and 17 ft. long makes with the sides.

11. A chord of a circle is 100 ft. long and subtends an angle of 40° 42′ at the center. Find the radius of the circle.

12. A hill rises 8 ft. vertically in a horizontal distance of 40 ft. Find the angle of inclination of the hill with the horizontal. What is the difference in elevation of two points that are 500 ft. apart measured up the hill?

13. Find the length of a side of an equilateral triangle circumscribed about a circle of radius 15 inches.

14. Devise a formula for solving an isosceles triangle when the base and the base angles are given; when the base and one of the equal sides are given; when one of the equal sides and one of the base angles are given.

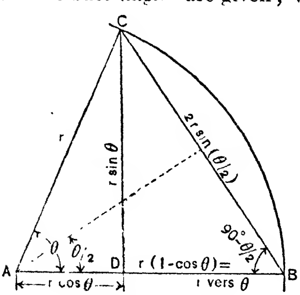


FIG. FOR EX. 17

15. The base of an isosceles triangle is 245.5 and each of the base angles is 68° 22′. Find the equal sides and the altitude.

16. The altitude of an isosceles triangle is 32.2 and each of the base angles is 32° 42′. Find the sides of the triangle.

17. In the accompanying figure show that $\angle BCD = \theta/2$; that $DB = r(1 - \cos\theta) = r \operatorname{vers}\theta$; and that $CB = 2r\sin(\theta/2)$.

18. From the same figure show that $\sin\theta = 2\sin(\theta/2)\cos(\theta/2)$.

[HINT. Find the area of $\triangle ABC$, using first AB, then BC, as base.]

19. Show that $\tan(\theta/2) = (1 - \cos\theta)/\sin\theta$.

CHAPTER III

SOLUTION OF OBLIQUE TRIANGLES

PART I. FUNCTIONS OF OBTUSE ANGLES

18. Obtuse Angles. The solution of oblique triangles involves obtuse* as well as acute angles. Let an obtuse angle α be placed on the coördinate axes with the vertex at the origin and one side along the x-axis to the right; then the other side will fall in the second quadrant. The ratios $\sin\alpha$, $\cos\alpha$, etc., are defined in terms of x, y, and $r=\sqrt{x^2+y^2}$ precisely as for acute angles. (See § 6.) It should be noticed, however, that since x is negative while y and r are positive, every ratio which involves x is negative for an obtuse angle; thus $x/r=\cos\alpha$, $y/x=\tan\alpha$, and their reciprocals, $\sec\alpha$ and $\operatorname{ctn}\alpha$, are all negative for obtuse angles.

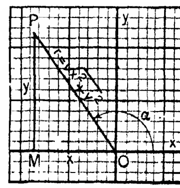


FIG. 19

We have seen how it is possible to find the ratios of angles greater than 45° from a table extending no farther than 45°, by means of the relations $\sin(90°-\alpha)=\cos\alpha$, etc., which were proved on p. 13. By means of similar relations it is possible to find the values of the ratios for obtuse angles from the same table.

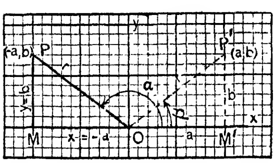


FIG. 20

Let α be placed on coördinate axes as described above, and let the supplement of α be denoted by β (which is an acute angle). Lay off β from Ox so that the other side of β falls in the first quadrant. From a point P in the side of α (in second quadrant) and a point P' in the side of β (in first quadrant) at the same distance r from the origin, draw the

* An obtuse angle is an angle which is greater than 90° and less than 180°.

perpendiculars PM, $P'M'$, as in Fig. 20. The value of x for the point P will be negative since P is in the second quadrant. Let its coördinates be $(-a, b)$; then, since the triangles OPM, $OP'M'$ are symmetric, the coördinates of P' are (a, b). As in § 6, we have

$$\sin\alpha = \frac{b}{r} = \sin\beta, \qquad \cos\alpha = -\frac{a}{r} = -\cos\beta,$$

or, since $\beta = 180° - \alpha$,

(1) $$\sin\alpha = \sin(180° - \alpha);$$

(2) $$\cos\alpha = -\cos(180° - \alpha).$$

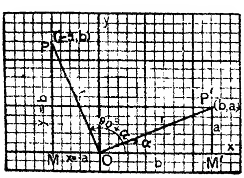
In a similar manner it can be shown that

(3) $$\tan\alpha = -\tan(180° - \alpha).$$

It follows that if α is an obtuse angle *we find its sine by looking for the sine of its supplement, which is an acute angle,* and similarly for the other functions, always having regard for the proper sign. The relations just found, together with those of § 9, enable us to find the values of the functions for any angle which can occur in a triangle, from a table which gives them from 0° to 45°.

EXERCISES IX.—FUNCTIONS OF OBTUSE ANGLES

1. From the accompanying figure prove the following relations:



(*a*) $\sin(90° + \alpha) = \cos\alpha$;
(*b*) $\cos(90° + \alpha) = -\sin\alpha$;
(*c*) $\tan(90° + \alpha) = -\operatorname{ctn}\alpha$.

2. Construct obtuse angles whose functions have the following values:

(*a*) $\sin\theta = 1/3$; (*b*) $\tan\theta = -3/4$;
(*c*) $\cos\theta = -3/5$; (*d*) $\sin\theta = 1/2$;
(*e*) $\sin\theta = \sqrt{2}/2$; (*f*) $\sin\theta = \sqrt{3}/2$.

3. Find the values of the remaining functions of the angles of Ex. 2.

4. Express the following as functions of an angle less than 45°, and look up their values in a table.

(*a*) sin 121°; (*b*) cos 101°; (*c*) tan 168°;
(*d*) sin 99°; (*e*) ctn 178°; (*f*) cos 154°;
(*g*) cos 133° 11′; (*h*) tan 144° 38′; (*i*) sin 92° 3′.

PART II. FUNDAMENTAL PRINCIPLES OF SOLUTION

19. General Method for Oblique Triangles. In the solution of oblique triangles the following cases arise:

Case I. *Given two angles and a side.*

Case II. *Given two sides and the included angle.*

Case III. *Given the three sides.*

Case IV. *Given two sides and an angle opposite one of them.*

A general method for solving oblique triangles in all of these cases consists in dividing the triangle into two right triangles by a perpendicular from a vertex to the opposite side; these right triangles are then solved by the methods of the previous chapter. In all cases except the three side case the perpendicular can be drawn in such a manner that one of the resulting right triangles contains two of the given parts.

This method applied in the various cases leads to formulas for the solution if letters are employed for the sides and angles.

20. Case I: Given Two Angles and a Side. In this case it is immaterial which side is given, since the third angle can be found immediately from the fact that the sum of the three angles is 180°. Drop the perpendicular from either extremity of the given side.

Example 1. An oblique triangle has one angle equal to 43°, another equal to 67°, and the side opposite the unknown angle equal to 51. Determine the remaining parts.

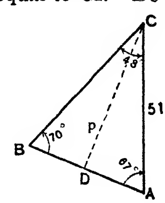


FIG. 21

It is immediately seen that the third angle is $180° - (43° + 67°) = 70°$. To solve this triangle draw the figure approximately to scale and drop the perpendicular $CD = p$ from one extremity C of the known side to AB, the side opposite C. Denote the unknown side CB by a. In the right triangle ACD, the hypotenuse and one angle are known; hence by (13), § 6, $p = 51 \sin 67° = 46.95$. An angle and the side opposite, in the right triangle BCD, are now known; hence by (15), § 6, $a = p/\sin 70° = 46.95/.9397 = 49.96$.

The side AB may be found in the same manner. Check as in § 3, p. 2.

21. Case II: Given Two Sides and the Included Angle. The triangle can be divided into two right triangles, one of which contains two of the known parts, by a perpendicular drawn from either extremity of the unknown side to the side opposite.

Example 1. Two sides of a triangle are 26.5 and 32.8; the included angle is 52° 18′. Find the remaining parts.

In the figure let $AB = 32.8$, $AC = 26.5$, and the angle at $A = 52° 18'$. Drop a perpendicular p from B to the opposite side. Denote the unknown side by a and the segments of AC by x and y as in Fig. 22; then p, x, y, a, can be computed in the following order:

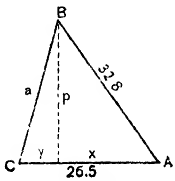


FIG. 22

$$p = 32.8 \sin 52° 18' = (32.8)(.79122) = 25.952.$$
$$x = 32.8 \cos 52° 18' = (32.8)(.61153) = 20.058.$$
$$y = 26.5 - 20.058 = 6.442.$$
$$a^2 = p^2 + y^2 = \overline{25.952}^2 + \overline{6.442}^2 = 715 \text{ nearly}.$$
$$a = \sqrt{715} = 26.74.$$

The student may determine the angles at B and C.

22. Case III: Given the Three Sides. In this case it is not possible to divide the triangle into two right triangles in such a way that one of them contains two of the given parts; however, if a perpendicular is dropped to the longest side from the vertex of the angle opposite, the segments into which this side is divided by the perpendicular are easily computed, as in the example below. There is one and only one solution, provided no side is greater than the sum of the other two.

Example 1. The sides of a triangle are $a = 36.4$, $c = 50.8$, and $b = 72.5$. Determine the angles.

Draw a figure and drop a perpendicular from B upon AC. Denote the segments of the base by x and y as in Fig. 23; then

$$p^2 = \overline{50.8}^2 - x^2 = \overline{36.4}^2 - y^2;$$

hence $$x^2 - y^2 = \overline{50.8}^2 - \overline{36.4}^2 = 1255.68;$$

that is, $$(x - y)(x + y) = 1255.68.$$

Since $$x + y = b = 72.5,$$

we have $$x - y = 1255.68 \div 72.5 = 17.32;$$

whence, adding, $$x = 44.91,$$

and, subtracting, $$y = 27.59.$$

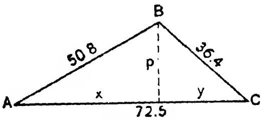


FIG. 23

Since we now know x and y, the angles at A and C are easily found. The student may complete the solution. See also § 25.

23. Case IV: Given Two Sides and the Angle Opposite One of Them. The triangle is easily solved by the general method, dropping the perpendicular from the vertex of the angle included by the given sides.

Example 1. One angle of a triangle is $37° 20'$; one side adjacent is 25.8 and the side opposite is 20.8. Solve the triangle.

First construct the given angle A and on one side of A lay off $AB = 25.8$. With B as center and radius $= 20.8$ describe an arc of a circle meeting the opposite side in two points C and C'. Either of the triangles ABC, ABC' satisfies the given conditions; the case is on this account called **the ambiguous case.**

The student should note that the triangle BCC' is isosceles and that the interior angle of ABC at C is equal to the exterior angle of ABC' at C'; hence the interior angles C and C' are supplementary. To solve ABC draw the perpendicular $BD = p$ from B; then determine p from the right triangle ABD.

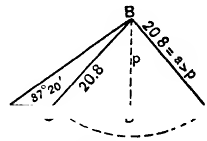


FIG. 24

$$p = 25.8 \sin 37° 20' = 15.6464.$$

Next determine C from the right triangle BDC;

$$\sin C = \frac{p}{a} = \frac{15.6464}{20.8} = .75223;$$

hence C is the *acute* angle whose sine is .75223; *i.e.* $C = 48° 47'.05$.

The student can complete the solution as follows:

$$AC = AD + DC; \; B = 180° - (A + C).$$

Also for triangle ABC',

$$C' = 180° - C; \; B' = 180° - (A + C'); \; AC' = AD - CD.$$

EXERCISES X. — SOLUTION OF TRIANGLES

Find the remaining parts of the following triangles by suitably dividing each into two right triangles. Capital letters represent angles; small letters the sides opposite them.

1. (*a*) $A = 17° 17'$, $B = 37° 37'$, $c = 174$;
   (*b*) $A = 24° 14'$, $C = 43° 13'$, $c = 240$;
   (*c*) $L = 28°$, $M = 51°$, $l = 6.3$.

2. (*a*) $a = 41$, $b = 51$, $C = 62°$;
   (*b*) $b = 3.5$, $c = 2.6$, $A = 33°$;
   (*c*) $u = 22$, $v = 12$, $W = 42°$.

3. (*a*) $a = 7$, $b = 12$, $c = 15$;
   (*b*) $l = 10$, $m = 14$, $n = 20$;
   (*c*) $u = 3$, $v = 4$, $w = 5$.

4. (*a*) $a = 50.8$, $b = 35.9$, $A = 64°$
   (*b*) $g = 6.22$, $k = 7.48$, $G = 26°$
   (*c*) $b = 23.4$, $q = 19.8$, $B = 109°$;
   (*d*) $a = 213$, $b = 278$, $B = 100°$.

5. To determine the distance from a point A to an inaccessible object B, a base line $AC = 300$ ft. and the angles $BAC = 40°$, $BCA = 50°$ are measured. Find the distance AB.

6. To determine the distance between two trees A, B on opposite sides of a hill, a point C is chosen from which both trees are visible; the distances $AC = 400$ ft., $BC = 361$ ft., and the angle $ACB = 55°$ are then measured. What is the distance between the trees?

7. The sides of a triangular field are 43 rods, 48 rods, and 57 rods, respectively; determine the angles between the sides.

8. A 50 ft. chord of a circle subtends an angle of 100° at the center. A triangle is to be inscribed in the larger segment having one of its sides 40 ft. long. How long is the other side? Is there only one solution?

9. A triangle having one of its sides 60 ft. long is to be inscribed in the segment of Ex. 8. Determine the remaining side. How many solutions are there in this case?

24. The Law of Sines. In Example 1, § 20, it may be observed that it was not really necessary to calculate p numerically in order to find a, for we might have written

$$a = \frac{p}{\sin 70°} = \frac{51 \sin 67°}{\sin 70°}$$

from which a could have been found by the use of logarithms in one calculation.

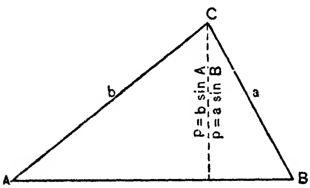


FIG. 25 *a*

A formula which can be used for all examples of this case can be obtained in a similar manner. Let us denote the sides opposite A, B, C, by a, b, c, respectively. The adjoining figures represent two cases: in one the triangle has all its angles acute; in the other, one angle, B, is obtuse. In either figure by (13), § 6,

$$p = b \sin A.$$

Moreover, in the first figure $p = a \cdot \sin B$. In the second figure, denote the angle DBC by B'; then B' is the supplement of B; hence by (1), § 18, $\sin B = \sin B'$. But by (13), § 6, $p = a \sin B' = a \sin B$; hence, in any case,

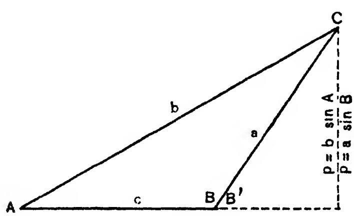


FIG. 25 *b*

$$p = a \sin B.$$

It follows that

$$b \sin A = a \sin B,$$

or, dividing first by the product $\sin A \sin B$, and second by $a \sin A$,

$$(1) \qquad \frac{a}{\sin A}=\frac{b}{\sin B}, \qquad \text{or} \qquad \frac{b}{a}=\frac{\sin B}{\sin A}.$$

If the perpendicular is drawn from one of the other vertices, say from B, the above procedure leads to the equation

$$\frac{a}{\sin A}=\frac{c}{\sin C}, \qquad \text{or} \qquad \frac{c}{a}=\frac{\sin C}{\sin A}.$$

Combining this with formula (1) we have

$$(2) \qquad \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}.$$

This result, known as the **law of sines**, may be stated as follows:

In any triangle any two sides are proportional to the sines of the angles opposite them.

By using this law any example under Case I can be solved as in the following example. There is always one and only one solution if the sum of the given angles is less than 180°.

Example 1. Given one side of a triangle $a = 2.903$ and two of the angles $B = 79° 40'$, $C = 33° 15'$; find the remaining parts.

We first obtain the angle A opposite the given side a and then apply the law of sines.

$$A = 180° - (B + C) = 180° - (79° 40' + 33° 15') = 67° 5'.$$

By the law of sines

$$\frac{b}{2.903}=\frac{\sin 79° 40'}{\sin 67° 5'}, \qquad \text{or} \qquad b=\frac{2.903 \sin 79° 40'}{\sin 67° 5'}.$$

The computation by logarithms follows:

$a = 2.903$	$\log a = 0.46285$
$B = 79° 40'$	$\log \sin B = 9.99290 - 10$
$A = 67° 5'$	$\operatorname{colog} \sin A = 0.03571$
	$\log b = 0.49146$
	$b = 3.1007$

The side c is found similarly from the equation

Thus: $$\frac{c}{2.903}=\frac{\sin 33° 15'}{\sin 67° 5'}.$$

Check by the facts mentioned in § 3, p. 2.

---

* The fact that a proportion $x/a = b/c$ gives $x = ab/c$ should be memorized by some device. Then $\log x = \log a + \log b + \operatorname{colog} c$.

25. The Law of Cosines. In Case II also, it is possible to eliminate the auxiliary parts and express the unknown side directly in terms of the given parts. Denote, as before, the sides and angles by a, b, c, A, B, C; and let b, c, and A be the given parts. Drop a perpendicular p from B to the opposite side, and denote the segments of the opposite side by x and y.

By (13), § 6, we have, in Fig. 26,

$$p = c \sin A, \quad x = c \cos A, \quad y = b - x = b - c \cos A,$$

$$a^2 = y^2 + p^2 = (b - c \cos A)^2 + c^2 \sin^2 A$$

$$= b^2 - 2\,bc \cos A + c^2(\sin^2 A + \cos^2 A) = b^2 - 2\,bc \cos A + c^2;$$

or,

$$(1) \qquad \boldsymbol{a^2 = b^2 + c^2 - 2\,bc \cos A.}$$

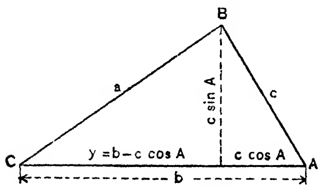


FIG. 26

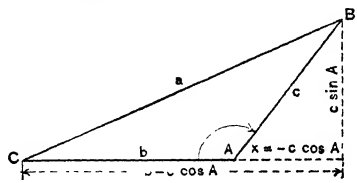


FIG. 27

If the side a to be found is opposite an obtuse angle A, $y = b + x$; but by (2) § 18, $x = c \cos(180° - A) = - c \cos A$; hence $y = b - c \cos A$ and $p = c \sin(180° - A) = c \sin A$, exactly as in the case considered above.

This result, called the **law of cosines**, may be stated as follows:

In any triangle, the square of any side is equal to the sum of the squares of the other two sides minus twice their product into the cosine of their included angle.

Example 1. Two sides of a triangle are 2.1 and 3.5 and the included angle is 53° 8′. Determine the remaining parts. Draw the figure, and denote the unknown side by a; then, using the table of squares,

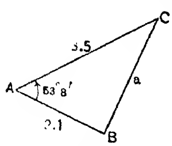


FIG. 28

$$a^2 = \overline{2.1}^2 + \overline{3.5}^2 - 2(2.1)(3.5) \cos 53° 8'$$

$$= 4.41 + 12.25 - (14.70)(.59995)$$

$$= 4.41 + 12.25 - 8.82 = 7.84$$

Hence $a = \sqrt{7.84} = 2.8$

Complete the solution as in § 24; and check as in § 3, p. 2.

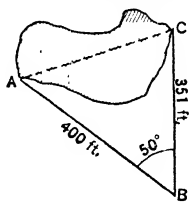
Case III (§ 22) also may be treated by the *law of cosines*; for, if a, b, c are known, $\cos A$ can be found from (1).

EXERCISES XI. — SINE LAW — COSINE LAW

1. Solve the following triangles by using the law of cosines:

(*a*)	$a = 22$,	$b = 12$,	$C = 42°$.	(*e*)	$a = 2.2$,	$b = 4.2$,	$c = 5.5$.
(*b*)	$a = 14$,	$c = 16$,	$B = 52°$.	(*f*)	$l = 13$,	$m = 16$,	$n = 20$.
(*c*)	$l = 28$,	$m = 36$,	$N = 125°$.	(*g*)	$u = 41$,	$v = 51$,	$W = 61°$.
(*d*)	$a = 21$,	$b = 24$,	$c = 28$.	(*h*)	$b = 3.5$,	$c = 2.6$,	$A = 33°$.

2. To determine the width AC of a lake, the distances $AB = 400$ ft., $BC = 351$ ft., and the angle $ABC = 50°$, are measured. Find the width.

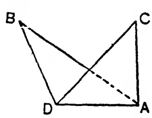


3. Obtain the remaining parts of the following triangles by using the law of sines with logarithms.

(*a*)	$A = 51° 47'$,	$B = 66° 20'$,	$c = 337.6$.
(*b*)	$A = 48° 10'$,	$B = 54° 10'$,	$c = 38.7$.
(*c*)	$B = 38° 12'$,	$C = 61° 10'$,	$a = 70.12$.
(*d*)	$U = 46° 36'$,	$V = 124° 18'$,	$w = 1001$.
(*e*)	$B = 21° 16'$,	$C = 113° 34'$,	$d = 20.93$.
(*f*)	$a = 39.75$,	$B = 62° 42'$,	$M = 52° 22'$.

4. The side of a hill is inclined at an angle of 22° 37′ to the horizon. A flagstaff at the top of the hill subtends an angle of 13° 17′ from a point at the foot of the hill, and an angle of 18° 2′ from a point 100 ft. directly up the hill. Find the height of the flagstaff.

5. To determine the elevation of an inaccessible object C above the point D, a base line $BD = 250$ ft., and the angles $ADC = 51° 16'$, $DBA = 37° 24'$, and $ADB = 124° 41'$ are measured, where A is vertically beneath the object C in a horizontal plane with B and D. How high is C above the point A?



[Note. Such angular measurements are obtained in practice by means of a *transit*; in this exercise the point A need not be visible, since the leveling device enables one to measure the angles mentioned without actually sighting on A.]

6. To find the distance from a station A to an inaccessible point B, a base line $AC = 500$ ft., and the angles $ACB = 68° 18'$ $CAB = 58° 28'$ are measured. Find the distance AB.

7. To find the height of an inaccessible object AB, a base line $CD = 250$ ft. is measured directly toward the object: also the angles of elevation $ADB = 48° 20'$ and $ACB = 38° 40'$. Find the height AB.

[Hint. First find DB by solving the triangle CDB; then find the height by solving the right triangle DAB.]

PART III. SPECIAL LOGARITHMIC METHODS

26. The Law of Tangents. In § 25, we used the Law of Cosines for the solution of triangles in Case II; but logarithms could not be used advantageously, and the computations without logarithms are tedious. We shall now obtain a formula for Case II that is adapted to logarithmic computation.

In this case, since one angle is given, the sum of the other two can easily be found. The law of tangents is a formula for obtaining the difference of the unknown angles. When the sum and the difference are known, the angles themselves can be found immediately by addition and subtraction.

Let ABC be any triangle having two sides a and b unequal, say $a > b$; the included angle C may be acute, right, or obtuse.

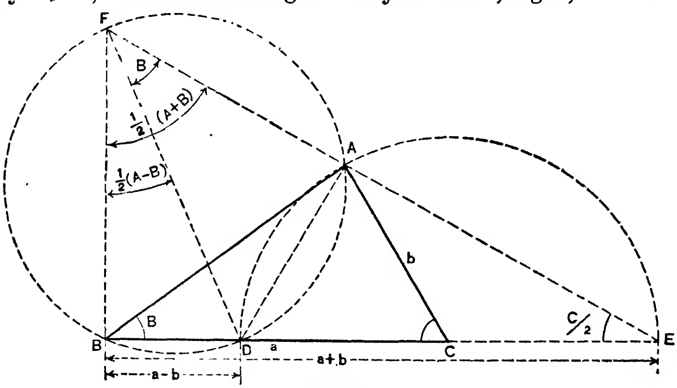


FIG. 29

With a radius b, the shorter of the given sides, and center C, the vertex of the included angle, describe a circle through A which cuts the side CB in a point D between B and C and also at a second point E beyond C. Draw EA, and at B erect a perpendicular which meets EA produced at F. On DF as a diameter construct a circle; this circle will pass through A and B, for FAD is a right angle since it is the supplement of DAE which is inscribed in a semicircle, and FBD is a right angle by construction. This construction is possible for any triangle in which $a > b$.

The angle $BFE = (A+B)/2$, since it is the complement of $CEA = C/2$, and $(A+B+C)/2 = 90°$. The angle DFA is equal to B, since both angles intercept the same arc AD; hence, $BFD = BFE - DFA = (A-B)/2$.

In the right triangles DBF and EBF, by (13), § 6, $a-b = BF \tan[(A-B)/2]$, and $a+b = BF \tan[(A+B)/2]$. Therefore,

$$\frac{a-b}{a+b} = \frac{\tan[(A-B)/2]}{\tan[(A+B)/2]}. \tag{1}$$

This formula is still true, but is trivial, if $a=b$, since in that case both sides are zero; if $a<b$ the result would obviously be

$$\frac{b-a}{b+a} = \frac{\tan[(B-A)/2]}{\tan[(B+A)/2]}.$$

This completes the proof of the **law of tangents**:

In any plane triangle the difference of any two sides is to their sum as the tangent of one half the difference of the angles opposite is to the tangent of one half their sum.

Since $(A+B)/2$ is the complement of $C/2$ the formula may be written

$$\tan\frac{1}{2}(A-B) = \frac{a-b}{a+b}\operatorname{ctn}\frac{C}{2}, \tag{2}$$

a form which is adapted to logarithmic computation, and which enables us to find $(A-B)/2$ directly from the two given sides a and b and their included angle C.

27. The Tangents of the Half-angles. In this article we shall obtain certain useful formulas for the tangents of the half-angles of a triangle. Let us draw the bisectors of the angles of the triangle ABC and denote their point of intersection by I. By geometry, this point is known to be the center of the inscribed circle. Denote the radius of this circle by r. Draw the radii ID, IE, IF, perpendicular to the sides. By geometry

the tangents to the inscribed circle from any vertex of the triangle are equal. Let us use the notation

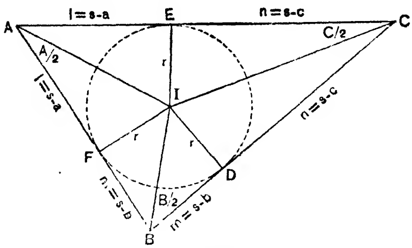


FIG. 30

$$l = AE = AF;$$
$$m = BF = BD;$$
$$n = CD = CE;$$
$$2s = a + b + c.$$

Then we have
$$2s = 2l + 2m + 2n,$$
or $\quad s = l + m + n,$

and since $\quad a = m + n,$
$$b = n + l,$$
$$c = l + m,$$

we obtain $\qquad s - a = l, \quad s - b = m, \quad s - c = n.$

We have then from the right triangles AFI, BDI, and CEI:

$$(1) \qquad \tan\frac{A}{2} = \frac{r}{s-a}, \qquad \tan\frac{B}{2} = \frac{r}{s-b}, \qquad \tan\frac{C}{2} = \frac{r}{s-c}.$$

Eliminating r from the last two by division, we have

$$(2) \qquad \frac{\tan(B/2)}{\tan(C/2)} = \frac{s-c}{s-b} = \frac{a+b-c}{a-b+c}.$$

28. Second Proof of the Law of Tangents. The formula (2) of § 27 enables us to give another proof of the law of tangents. In the triangle ABC, suppose $A > B$. From B draw BA', making $\angle ABA'$ equal to A, and meeting AC produced in A'; then $\angle CBA' = B' = A - B$, and $\angle BCA' = C' = 180° - C$.

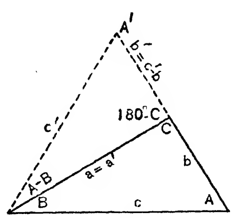


FIG. 31

In the triangle BCA' denote the side opposite C' by c', and the side opposite B by b'. Since the triangle ABA' is isosceles, $A'A = c'$, and therefore $b' = c' - b$. If we apply (2), § 27, to the triangle BCA', we find

$$\frac{\tan(B'/2)}{\tan(C'/2)} = \frac{a'+b'-c'}{a'-b'+c'}$$

or,
$$\frac{\tan[(A-B)/2]}{\tan[90° - C/2]} = \frac{a+c'-b-c'}{a-c'+b+c'} = \frac{a-b}{a+b},$$

and this reduces to the law of tangents:

$$\tan\frac{1}{2}(A-B) = \frac{a-b}{a+b}\operatorname{ctn}\frac{C}{2}.$$

Let us now apply the law of tangents to the solution of an example in Case II. There is always one and only one solution in this case if the given angle is less than 180°.

Example 1. Two sides of a triangle are 22.531 and 34.645; the included angle is 43° 31′. Determine the remaining parts.

Let $a = 34.645$, $b = 22.531$, $C = 43° 31'$, and denote the unknown parts by A, B, c. The sum $A + B$ of the unknown angles is $180° - C = 180° - 43° 31' = 136° 29'$. The difference $A - B$ is obtained as follows:

$$C/2 = 21°\ 45'.5;\quad a - b = 12.114;\quad a + b = 57.176.$$

$\log(a-b) = 1.08328$	whence	$(A-B)/2 =$	$27°\ 57'.6$
$\log\operatorname{ctn}(C/2) = 0.39888$	as above,	$(A+B)/2 =$	$68°\ 14'.5$
$\operatorname{colog}(a+b) = \underline{8.24279 - 10}$	adding,	$A =$	$96°\ 12'.1$
$\log\tan[(A-B)/2] = 9.72495 - 10$	subtracting,	$B =$	$40°\ 16'.9$

Check by using the law of sines.

EXERCISES XII. — LOGARITHMIC SOLUTION OF CASE II

1. Solve each of the following triangles, using logarithms:

(*a*)	$a = 52.8$,	$b = 25.2$,	$C = 124°\ 34'$.
(*b*)	$b = 55.1$,	$c = 45.2$,	$A = 16°\ 16'$.
(*c*)	$l = 131$,	$m = 72$,	$N = 39°\ 46'$.
(*d*)	$a = 35$,	$b = 21$,	$C = 48°\ 48'$.
(*e*)	$u = 604$,	$v = 291$,	$W = 106°\ 19'$.
(*f*)	$a = 23.45$,	$b = 18.44$,	$D = 81°\ 50'$.
(*g*)	$u = .6238$,	$v = .2347$,	$C = 108°\ 30'$.

2. To determine the distance between two objects A and B separated by a hill, the distances $AC = 300$ ft., $BC = 277$ ft., and the angle $ACB = 65°\ 47'$, are measured. From these measurements find the distance AB.

3. Two objects, A, B, are separated by an impassable swamp. A station C is selected from which distances in a straight line can be measured to each of the objects. These distances are found to be $CA = 341$ ft. 7 in., $CB = 237$ ft. 5 in., and the angle ACB is found to be 53° 11′. Find the distance AB.

4. Two objects, A, B, are separated by a building. To determine the direction of the line joining them, a point C is taken from which both A and B are visible and the distances $AC = 200$ ft., $BC = 137$ ft. 9 in., and the angle $ACB = 52°\ 25'$ are measured. Determine the angle which AB makes with AC. Also the distance AB.

EXERCISES XIII. — LOGARITHMIC SOLUTION OF CASE III

1. Solve each of the following triangles, using logarithms:

(a) $a = 22.2,\ b = 31.82,\ c = 40.64.$ (f) $p = 38.2,\ b = 45.36,\ d = 26.54.$
(b) $a = 27.53,\ b = 18.93,\ c = 30.14.$ (g) $m = .126,\ n = .3226,\ c = .253.$
(c) $a = 523.8,\ b = 566.2,\ c = 938.4.$ (h) $a = .0506,\ b = .1234,\ c = .0936.$
(d) $l = 3.171,\ m = 5.331,\ n = 5.101.$ (i) $u = 167,\ v = 321,\ w = 231.$
(e) $u = 40.04,\ v = 50.56,\ w = 70.12.$ (j) $u = 196.1,\ v = 264.1,\ w = 135.4.$

2. To determine without an instrument for measuring angles the angle ACB between two lines meeting at C, the distances $CA = 500$ ft. and $CB = 700$ ft. are measured; AB is then found to be 633 ft. Find $\angle ACB$.

3. Three objects, A, B, C, are situated on the edge of a lake. It is desired to determine the angles between the lines joining them. The distances AB, BC, and CA are found to be 321 ft., 472 ft., and 511 ft., by a process similar to that of Ex. 2, p. 33. Find the angles.

31. Logarithmic Solution of Case IV. The Ambiguous Case. Since in this case a side and its opposite angle are known, the triangle can be solved by the sine law.

Example 1. Two sides of a triangle are $c = .35211$ and $a = .30135$, and the angle opposite a is $A = 33° 17'$. Determine the remaining parts.

Denote the unknown side by b and the unknown angles by B and C. By the law of sines $\sin C/\sin A = c/a$. Since the angle C is determined by its sine, there may be two angles less than 180° (see § 23 and (1), § 18) which satisfy this equation and the conditions of the problem. The work may be arranged as follows:

$$c = .35211 \qquad \log c = 9.54668 - 10$$
$$A = 33° 17' \qquad \log \sin A = 9.73940 - 10$$
$$a = .30135 \qquad \operatorname{colog} a = 0.52093$$
$$\log \sin C = 9.80701 - 10$$

$C_1 = 39° 53'$	$C_2 = 140° 7'$
$B_1 = 106° 50'$	$B_2 = 6° 36'$
$\dfrac{b_1}{a} = \dfrac{\sin B_1}{\sin A}$	$\dfrac{b_2}{a} = \dfrac{\sin B_2}{\sin A}$
$\log a = 9.47907 - 10$	$\log a = 9.47907 - 10$
$\log \sin B_1 = 9.98098 - 10$	$\log \sin B_2 = 9.06046 - 10$
$\operatorname{colog} \sin A = 0.26060$	$\operatorname{colog} \sin A = .26060$
$\log b_1 = 9.72065$	$\log b_2 = 8.80013 - 10$
$b_1 = .52559$	$b_2 = .063114$

Check by the law of cosines or by the law of tangents.

The number of solutions of a problem belonging to Case IV can be determined by an easy calculation without actually solving the triangle. If

the given angle A is acute, the perpendicular $p = c \sin A$ is first computed, where c is the given side adjacent to the given angle. If the given side a opposite A is less than this perpendicular, it is clear from Fig. 34 that there can be no solution. If a is just equal to p, it is easily seen from Fig. 35 that there is just one solution. If a is greater than p and less than c, there are two solutions, as in Fig. 24, p. 29. If a is greater than c, there is only one solution, as shown in Fig. 36; for if a is put in the position BC', the angle A fails to lie within the triangle, and the conditions of the problem are not satisfied.

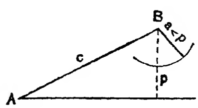


FIG. 34

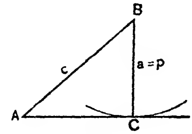


FIG. 35

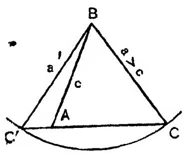


FIG. 36

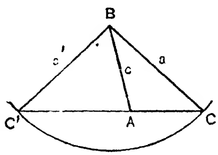


FIG. 37

If the angle $A \geqq 90°$, there is no solution if a is equal to or less than c, because A must then be the larger angle of the triangle, and by geometry the greater side lies opposite the greater angle. If a is greater than c, there is just one solution, as in Fig. 37. These results are tabulated below.

I. $A < 90°$

$a < c \sin A$	no solution
$c \sin A < a < c$	two solutions
$a = c \sin A$ or $a \geqq c$	one solution

II. $A \geqq 90°$

$a \leqq c$	no solution
$a > c$	one solution

EXERCISES XIV. — LOGARITHMIC SOLUTION OF CASE IV

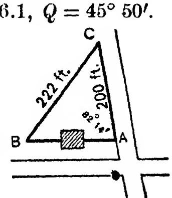
1. Solve each of the following triangles, using logarithms; if two solutions exist, obtain both of them.

(a) $a = 17.16$, $b = 14.15$, $B = 42°$. (d) $l = 281$, $m = 152$, $L = 103°$.

(b) $a = 54$, $b = 48.6$, $A = 31° 14'$. (e) $b = 13.12$, $c = 7.22$, $B = 39° 54'$.

(c) $u = 971$, $v = 1191$, $U = 51° 15'$. (f) $p = 48$, $q = 36.1$, $Q = 45° 50'$.

2. In a certain town the streets intersect at an angle of 82° 14′. It is desired to know the distance between two objects, A and B, which lie on a line parallel to one set of streets and which are separated by a large building. A line $AC = 200$ ft. is measured along a side line parallel to the other set of streets, and $CB = 222$ ft. is then measured. Determine AB.



3. The pilot of a ship S sees a lighthouse H on the shore ; by measuring the angle of elevation of the top of the lighthouse, and knowing its height, he determines that it is 8950 ft. from his ship. At the ship an angle of $2° 40'$ is subtended by a line connecting the lighthouse with a light L on the shore known to be 575 ft. from the lighthouse. Find the angle SLH and thus determine exactly the position of the ship with reference to the shore. Practically how may he tell which of the two possible solutions is actually correct ?

PART IV. APPLICATIONS PROBLEMS

32. Areas of Triangles. In elementary geometry it is shown that the area $\boldsymbol{A}$ of a triangle* is one-half the product of any side and the altitude from the opposite vertex to that side, *i.e.* $\boldsymbol{A} = p \cdot c/2$. By applying this result we obtain formulas for the following cases :

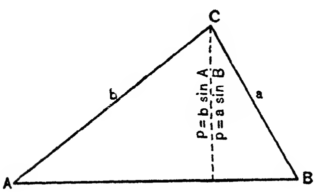


FIG. 38

(1) *Given two sides b, c, and their included angle A.*

If the perpendicular is drawn as in Fig. 38, we have

$$p = b \sin A,$$

and $\boldsymbol{A} = (1/2)pc = (1/2)bc \sin A,$

i.e. *the area of a triangle is equal to one half the product of any two sides and the sine of their included angle.*

(2) *Given two angles A, C, and their included side b.*

Solve the triangle by Case I to find B and one side, say $c = b \sin C/\sin B$; then,

$$\boldsymbol{A} = \frac{bc \sin A}{2} = \frac{b^2 \sin C \sin A}{2 \sin B}.$$

(3) *Given the three sides.*

Divide the triangle into three triangles AIC, CIB, BIA by lines from the vertices to the center of the inscribed circle, using b, a, c, respectively, as the bases of these triangles. The altitude of each is equal to r; hence

$$\boldsymbol{A} = \frac{ar}{2} + \frac{br}{2} + \frac{cr}{2} = \frac{(a+b+c)r}{2} = rs,$$

or, substituting for r its value, § 29,

$$\boldsymbol{A} = s\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

$$= \sqrt{s(s-a)(s-b)(s-c)}.$$

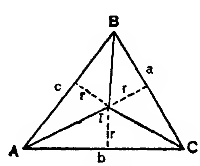


FIG. 39

* The area is denoted by the boldface type $\boldsymbol{A}$ in distinction from the angle A.

33. Composition and Resolution of Forces and Velocities. We saw in § 15 that forces and velocities may be represented graphically by straight line segments. The length of such a segment represents the magnitude of the force or velocity, and its direction the direction of the force or velocity.

To find the effect of two simultaneous velocities, let us suppose that a body moves along a straight track with a velocity of 4 units per second and that each point of the track moves with a velocity of 3 units per second along a line making an angle of 60° with the track. What is the position of the body at the end of 1 second? To answer this question draw a segment 4 units long to represent the magnitude and direction of the velocity of the body along the track, and from the ends of this segment draw segments AC, BD, each 3 units in length and making an angle of 60° with AB to represent the magnitude and direction of the velocity of the ends of the track. The track will then take the position CD at the end of 1 second. But since the body moves along the track at the rate of 4 units per second, it will reach the point D at the end of 1 second. That is, it will reach the same point as if it had moved along the diagonal AD with a speed represented by the length of the diagonal. The velocity represented by AD is called the **resultant** of the velocities represented by AB and AC. AB and AC are called **components.** The length of AD can be computed by solving the triangle ABD, of which we know two sides and the included angle.

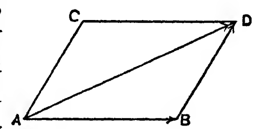


FIG. 40

The resultant of any two velocities may be found by drawing from a common point A, segments AB, AC to represent the given velocities in magnitude and direction and then completing the parallelogram $ABCD$. The diagonal AD represents the resultant. This fact is often called the **parallelogram law.**

The resultant of two forces is found by a similar construction. This diagram is known as the **parallelogram of forces.** We proceed to show how to find the resultant from the components, and to solve other problems.

Example 1. The angle between the directions of two forces of 19 lb. and 26 lb. is 54°. Find the magnitude and direction of their resultant.

The forces may be represented by segments 19 units long and 26 units long, respectively, and making an angle of 54° with each other. If the parallelogram is completed which has these segments for two of its intersecting sides, the diagonal extending from their intersection to the opposite corner will represent the resultant both in magnitude and in direction. This diagonal is a side of a triangle having two sides equal to 19 and 26, respectively, with an included angle of 126° (the supplement of 54°). Hence we can find the diagonal and the angle which it makes with either side, that is, the magnitude and direction of the resultant.

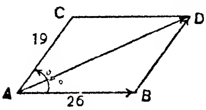


FIG. 41

Example 2. Two forces of 51 lb. and 73 lb. have a resultant of 80 lb. Find the angle between them.

In this case, in the parallelogram of forces, the diagonal and two intersecting sides are known; the angle opposite the diagonal is determined by Case III. The required angle is the supplement of this one.

EXERCISES XV. — AREAS FORCES VELOCITIES

1. Complete the solution of Examples 1 and 2 above.

2. Three forces of 13 lb., 22 lb., and 28 lb., respectively, are in equilibrium. Determine the angles which they make with one another.

[HINT. If the forces are in equilibrium, the resultant of any two of them must equal the third in magnitude but must be opposite to it in direction; hence regard one of the forces as the resultant of the other two and find the angle between these as in Example 2 above.]

3. Find the resultant of two forces of 30 lb. and 40 lb. acting at an angle of 60° with each other.

4. A ball rolls along the diagonal of the floor of a car from the back to the front with a speed of 30 ft. per second. The car is moving forward with a speed of 40 ft. per second. Find the actual speed of the ball if the car is 7 ft. wide and 30 ft. long.

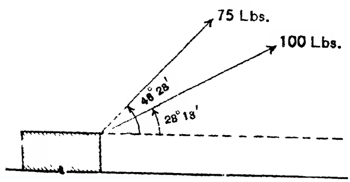


FIG. FOR EX. 5

5. Two forces are acting on a block resting on the ground as shown in the figure. What horizontal force do they exert?

6. A point is kept at rest by forces of 6, 8, 11 lb. Find the angle between each pair.

7. A boat is rowed across a river at the rate of 3.5 mi. per hour; the river flows at the rate of 4.8 mi. per hour. Find the speed of the boat and the direction of its motion.

8. A ship is sailing 10 mi. per hour and a sailor climbs the mast 200 ft. high in 30 sec. Find his speed relative to the earth, and the direction of his motion.

9. A train is going 15 mi. per hour northward; a man crosses the car eastward 2 ft. per second. Find his speed relative to the ground, and his direction.

10. A ball rolling along the floor 10 ft. per second is struck so that its speed is increased 2 ft. per sec., and the direction of motion is changed 45°. What speed and direction of motion is due to the stroke alone?

11. A river flows 4 mi. per hour, and a motor boat goes 9 mi. per hour. In what direction must the boat be pointed to go straight across the river, and what will be its speed?

12. Determine the areas of the following triangles:

(a) $a = 829$, $b = 592$, $C = 62°$. (b) $a = 713$, $b = 987$, $c = 1255$.
(c) $B = 25°$, $C = 68°$, $b = 392$. (d) $a = 231$, $q = 195$, $A = 47°$.
(e) $u = 8$, $v = 5$, $W = 60°$. (f) $a = 72.3$, $A = 52° 35'$, $M = 63° 17'$.
(g) $l = .582$, $m = .601$, $n = .427$. (h) $b = 21.5$, $c = 30.456$, $D = 41° 22'$.

13. Find the area of a triangular field having one of its sides 15 rods in length, and the two adjacent angles, respectively, 70° and 69° 40′.

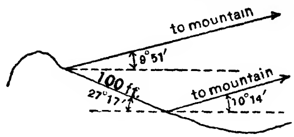
EXERCISES XVI. — MISCELLANEOUS EXERCISES

1. Solve the following triangles:

(a)	$a = 10.34$,	$B = 5° 7'.6$,	$C = 19° 49'$.
(b)	$a = 36.423$,	$b = 14.578$,	$C = 68° 14'$.
(c)	$l = 14.236$,	$m = 13.761$,	$N = 45° 11'$.
(d)	$a = 734.34$,	$B = 108° 6'$,	$C = 61° 7'$.
(e)	$u = 32.19$,	$v = 69.182$,	$U = 69° 17'$.
(f)	$a = .75632$,	$b = .62751$,	$C = 84° 48'$.
(g)	$c = 454.72$,	$J = 11° 11'$,	$C = 57° 37'$.
(h)	$a = 474.17$,	$b = 1008.8$,	$c = 940.25$.
(i)	$a = 100.37$,	$c = 95.376$,	$B = 100° 58'$.
(j)	$d = 391.68$,	$D = 25° 36'$,	$B = 68° 13'$.
(k)	$a = 622.02$,	$b = 293.22$,	$A = 100°$.
(l)	$u = 375.64$,	$v = 438.79$,	$w = 133.94$.
(m)	$a = .010231$,	$c = .0047233$,	$A = 44° 58'$.
(n)	$a = 476.53$,	$P = 40° 17'$,	$A = 39° 14'$.
(o)	$b = 94.961$,	$a = 88.234$,	$C = 12°$.
(p)	$b = .43124$,	$a = .53467$,	$A = 99° 59'$.
(q)	$l = .021467$,	$m = .019407$,	$n = .034354$.

2. A balloon is observed from the ground and from an upper window of a building 60 ft. directly above. The angles of elevation are found to be $10° 42'$ and $9° 58'$. Find the distance from each point to the balloon.

3. The angles of elevation of an inaccessible mountain peak from two stations on a neighboring hill are $10° 14'$ and $9° 51'$. The stations are in a vertical plane with the mountain peak, they are 100 ft. apart, and a line joining them makes an angle of $27° 17'$ with the horizontal. Determine the height of the mountain and the distance of the peak from each station.

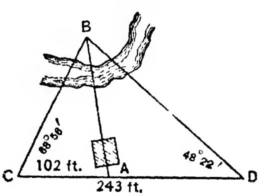


4. The diagonals of a parallelogram are 22 ft. and 31 ft., and the angle between them is $51° 12'$. Determine the sides of the parallelogram.

5. To determine the distance between two objects A and B that have a barrier between them, a distance $AC = 200$ ft. is measured to a point C from which both objects are visible. The distance $BC = 321$ ft. and the angle $ACB = 68° 41'$. Find the distance AB.

6. The sides of a triangular field are 82.7 rods, 91.4 rods, and 104.3 rods. Determine the area of the field, and the angles between the sides.

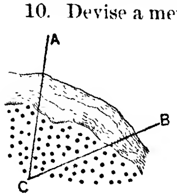
7. To find the distance between two objects A and B situated on opposite sides of a lake, the distance $AC = 250$ ft. and the angles $CAB = 44° 13'$, $ACB = 51° 9'$, are measured. Find the distance AB.



8. An object B is wholly inaccessible and is invisible from a certain point A. To find the distance AB, two points C and D from which B can be seen are selected on a line through A. If $CD = 243$ ft., $CA = 102$ ft., $\angle DCB = 68° 56'$, $\angle CDB = 48° 22'$, find AB.

9. It is desired to know the height of an object AB. A line $CD = 250$ ft. in a horizontal plane with the base A of the object, is measured, also the angle of elevation $ACB = 13° 22'$, and the angles $DCA = 35° 37'$ and $CDA = 64° 28'$. Determine the height AB.

10. Devise a method for finding the distance between two objects when there is only one accessible point from which both are visible. Assume certain measurements and solve.



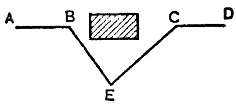
[Hint. In the figure, C is supposed to be the only point from which both A and B are visible; make use also of two other points D and E, from which A and B, respectively, are visible.]

11. A tall building stands at the foot of a hill. From a point on the

side of the hill the angle of depression of the base of the building is observed to be $14° 36'$, and the angle of elevation of the top is $21° 43'$. A level line from the instrument meets the building 19 ft. 7 inches above the base. Find the height of the building.

12. A balloon is observed, at the moment it passes over a level road, from two points in the road an eighth of a mile apart. The angles of elevation from the two points are $33° 17'$ and $42° 6'$. Find the distances of the balloon from the two observers.

13. In surveying, it is sometimes desired to extend such a line as AB in the figure beyond an obstacle. Show that this could be done by means of a broken line $ABECD$. What measurements would be necessary to determine the distance BC and the angle ECD?



14. How far to the side of a target 1300 ft. away should a gunner aim from a ship going 15 mi. per hour, if the speed of the bullet is 2000 ft. per second and he fires at the instant he is directly opposite?

15. From a railway train going 50 mi. per hour a bullet is fired 1000 ft. per second at an angle of $75° 28'.3$ with the track ahead. Find its speed and direction.

16. A man in a railway car going 45 mi. per hour observes the raindrops falling at an angle of $10°$ with the horizontal. Assuming that the raindrops are actually falling vertically, find their speed.

17. The resultant of two forces is 10 lb.; one of the forces is 8 lb. and makes an angle of $36°$ with the resultant. Find the magnitude of the other force.

18. A horse pulls a canal boat by a rope which makes an angle of $25° 35'$ with the tow path. What size of engine would propel the boat at the same speed? (Assume that the horse is doing one "horse power.")

19. A man climbs a hill inclined (on the average) $32°$ with the horizontal. His pocket barometer shows that at the end of $2\frac{1}{2}$ hours he has increased his elevation 2750 ft. Find his average speed up the slope.

20. Find the areas of triangles which have the following given parts:

(a)	$a = 116.082$,	$b = 100$,	$C = 118° 15' 41''$.
(b)	$b = 100$,	$A = 76° 38' 13''$,	$C = 40° 5'$.
(c)	$u = 31.325$,	$V = 13° 57' 2''$,	$U = 53° 11' 18''$.
(d)	$a = 408$,	$b = 41$,	$c = 401$.
(e)	$a = .9$,	$b = 1.2$,	$c = 1.5$.
(f)	$a = 63.89$,	$b = 138.24$,	$c = 121.15$.

21. Find the area of a triangular piece of ground having two angles, respectively, $73° 10'$ and $90° 50'$, and the side opposite the latter 150.6 rods.

CHAPTER IV

DIRECTED LINE SEGMENTS AND ANGLES

PART I. GENERAL DEFINITIONS AND PRINCIPLES

34. Directed Lines and Segments. Vectors. Such expressions as north and south, right and left, up and down, call attention to the necessity of distinguishing between the two directions of a straight line in order to express our ideas with precision. It is often convenient to select one direction on a straight line as the *positive direction;* the other is then called the *negative direction.* Thus, if two forces act along the same line, but in opposite directions, it is convenient to call one positive and the other negative. A line on which a choice of directions has been made is called a directed line.* In drawings, the positive direction of a directed line is indicated by an arrow head. A portion of a line between two of its points, A, B, is called a *segment.* We distinguish between the two possible directions of a segment as follows: The notation AB means the segment whose initial point is A and whose terminal point is B, while BA means the segment whose initial point is B and whose terminal point is A.

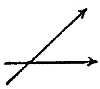


FIG. 42

A *force* is indicated graphically by such a *directed line segment,* whose direction indicates the direction of the force, and whose length indicates the intensity or amount of the force. Velocities are represented in the same manner. See p. 43.

A segment is said to be *positive* if its direction coincides with the positive direction of the line on which it lies; otherwise it

* The assignment of a positive direction on one line does not determine the positive direction on any other line, but it is often convenient in the case of parallel lines to choose the same direction on each as the positive direction. In what follows this choice will be understood unless the contrary is specified.

is a *negative* segment. In Fig. 43, AB, FG, etc., are positive segments; DA and FE are negative. Two segments are said to have the *same sense* if they lie on the same line or on parallel lines, and if both are positive or both are negative. Two segments are said to be of *opposite sense* if they lie on the same line or on parallel lines, and if one is positive and the other is negative. Two segments having the same length and the same sense are *equal;* if they have the same length and are of opposite sense, each is the *negative* of the other. Thus, in Fig. 43, $AB = EF$, while $AC = - GE$ and $CB = - FG$.

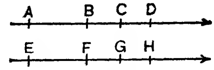


FIG. 43

The numerical measure of a directed segment is the number of units in its length with the sign $+$ or $-$, according as the segment is positive or negative.

With the agreements of this article, a directed segment may be substituted for any parallel segment of the same magnitude. Such a movable directed segment is often called a **vector.**

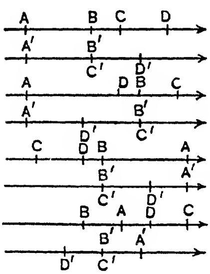


FIG. 44

35. Geometric Addition of Line Segments. Given two line segments AB and CD, of the same or opposite sense; to add the second to the first, place the initial point of the second on the terminal point of the first; then the segment from the initial point of the first to the terminal point of the second is their sum. Thus, lay off $A'B' = AB$ and $B'D' = CD$; then $A'D' = AB + CD$ (Fig. 44). This sum in the case of forces or velocities is a special case of the resultant, defined on p. 43, when the forces (or velocities) lie on the same lines.

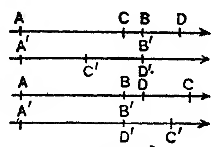


FIG. 45

36. Subtraction of Line Segments. Given two segments AB and CD of the same or opposite sense, to subtract the second from the first, add the negative of the second

to the first;* in other words, place the terminal point of the subtrahend on the terminal point of the minuend; then the segment from the initial point of the minuend to the initial point of the subtrahend is the required difference.

EXERCISES XVII. — ADDITION AND SUBTRACTION OF SEGMENTS

1. By laying them off on a directed line with some convenient unit, add the segments whose numerical measures are 3 and 4; 2 and -4; -4 and 5; -2 and -3.

2. Find the sum, or *resultant*, of two forces that act in the same line whose intensities (measured in pounds) are -5 and $+10$, respectively. Draw a figure to represent the solution.

3. If three forces of intensities $+7$, -15, $+2$ (lb.), respectively, act on a body in the same line, find the resultant force. Draw a figure.

4. If a man walks with a speed of 4 mi. per hour toward the rear of a train going 35 mi. per hour, find his actual speed. Draw a figure.

5. A man's gains and losses (indicated by $-$) in business in successive months are \$250, $-$\$118, \$35, \$712, $-$\$15. Find the total gain, and the average gain per month. Draw a figure.

6. The gains and losses in the population of a city in successive years are 3500, -1100, -2300, $+600$, $+2800$. Find the total gain, and the average gain per year.

7. Verify that $AB + CD = CD + AB$ by laying off segments.

8. Verify that $(AB + BC) + DE = AB + (BC + DE)$.

9. Add BA to AB. What is the result?

[NOTE. A segment whose end points coincide is called a zero segment, or simply, zero.]

10. Verify that if A, B, C, are any three points on a line, then no matter what the order of the points, or which is the positive direction of the line, always $AB + BC = AC$ (six cases).

11. Lay off the segments $AB = 8$, $CD = 10$, $BC = 6$, $DE = 5$, and perform geometrically the following operations, checking each by the corresponding numerical equation.

(a) $AB + BC - DE$. (c) $CA - DB + DE + BC$.
(b) $AD - BE - AB$. (d) $AB - CE + DC + BD$.

12. By n times a segment, or $n \cdot AB$, we mean the segment

$$\overset{(1)}{AB} + \overset{(2)}{AB} + \overset{(3)}{AB} + \cdots + \overset{(n)}{AB}$$

where n is a positive integer. If $AB = 4$, construct $3 \cdot AB$.

---

* For it is required to find a segment which can be substituted for x in the equation $x + CD = AB$, and $AB + DC$ is the unique solution of this equation

13. Construct 5 times a segment whose numerical measure is $-3/5$.

14. If $AB = n \cdot CD$, we say that $CD = AB \div n$. By the well-known method of plane geometry we can find points P_1, P_2, $P_3, \cdots$, P_{n-1}, such that $AP_1 = P_1P_2 = P_2P_3 = \cdots = P_{n-1}B = AB/n$, where n is any positive integer. Draw a segment at random and divide it into fifths.

37. Rotation. Directed Angles. In describing rotation, it is convenient to regard angles as positive or negative in the following manner: an angle is thought of as generated by the rotation of one of its sides about the vertex as center; its first position is called the *initial side*, the final position is called the *terminal side*. An angle generated by a rotation opposite to the motion of the hands of a clock (*counterclockwise*), is said to be *positive;* an angle generated by a *clockwise* rotation, is said to be *negative.**

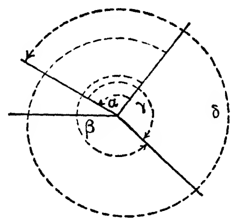


FIG. 46

Thus, in Fig. 46, α, β, δ, are positive angles; γ is negative.

Logically, the familiar units of angle used in elementary geometry and thus far in this book may be defined as follows: if the terminal side of the angle rotates in the positive direction until it coincides (for the first time), (a) with the perpendicular to the initial line at the vertex, the angle is a *right angle;* (b) with the prolongation of the initial side through the vertex, the angle is a *straight angle;* (c) with the initial line itself, the angle is a *revolution* (or a *perigon*). A *degree* (°) is defined in terms of any one of these by the equation $360^\circ = 1$ rev. $= 2$ str. $\measuredangle = 4$ rt. $\measuredangle$; it is divided into *minutes* ($'$) and *seconds* ($''$), so that $1^\circ = 60' = 3600''$. An *acute* angle is a positive angle less than a right angle. An *obtuse* angle is greater than a right angle and less than a straight angle.

Angles may be of any magnitude, positive or negative. Thus, in Fig. 46, for example, β is greater than a straight angle; and δ is greater than 360°, or a complete revolution. In rotating parts of machinery, such angles have a very vivid meaning. Thus, a wheel which rotates 370° per second has a very different speed from that of a wheel which rotates 10° per second.

* Either of these directions may of course be chosen as the positive direction of rotation, the other is then the negative direction. The choice here made is the customary one for angles; but in many kinds of machinery, the other sense of rotation is considered positive, as in the case of a clock.

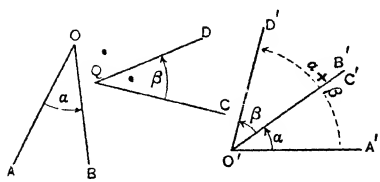


FIG. 47

38. Geometric Addition and Subtraction of Directed Angles. To add two angles, place the initial side of the second in the terminal side of the first and make their vertices coincide, then the angle from the initial side of the first to the terminal side of the second is their sum. To subtract an angle α from an angle β, place the terminal side of α on the terminal side of β; the angle from the initial side of β to the initial side of α is their difference $(\beta - \alpha)$. Compare these with the rules for adding and subtracting segments (§§ 35, 36). In Fig. 47 a positive angle β is added to a positive angle α. In Fig. 48 a negative angle β is added to a positive angle α. In Fig. 49 a positive angle α is subtracted from a positive angle β.

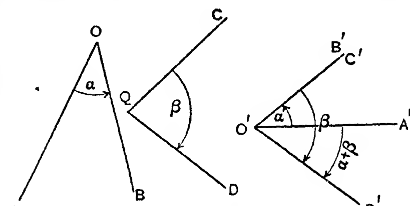


FIG. 48

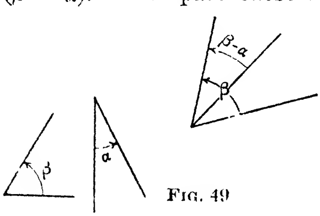


FIG. 49

39. Placing Angles on Rectangular Axes. To place any given angle on a pair of rectangular axes in the plane of the angle, put the vertex at the origin and the initial side on the x-axis extending to the right; the terminal side will then fall in one of the four quadrants (or, if the angle is a multiple of a right angle, on one of the axes). If the terminal side falls in the first quadrant, the angle is said to be *an angle in the first quadrant*, etc. In Fig. 50, α is a positive angle in the first quadrant, β is a negative angle in the fourth quadrant, δ is a positive angle in the fourth quadrant.

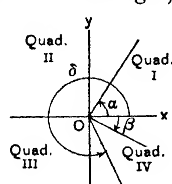


FIG. 50

EXERCISES XVIII. — DIRECTED ANGLES

1. What angle will the minute hand of a clock generate in 2 hr. 24 min. 10 sec.?

2. A flywheel is running steadily at the rate of 450 revolutions per min. What angle does one of its spokes generate in 2 sec.? In 1.2 sec.?

3. By means of a ruler and a protractor, construct the following angles and their sums; check by adding their numerical measures.

(*a*) $-75°$ and $125°$. (*b*) $66°$ and $-30°$. (*c*) $45°$ and $30°$, and $70°$.
(*d*) $-60°$ and $-36°$. (*e*) $485°$ and $55°$. (*f*) $-750°$ and $30°$.

4. With some two of the angles just given verify $\alpha + \beta = \beta + \alpha$.

5. (*a*) Construct $27° + 85° + (-45°) + 135°$.
(*b*) Construct $-150° + 96° + 24° + (-80°)$.

6. If a wheel is rotating $120°$ per sec., how many revolutions does it make per minute? how many per hour? How many degrees does it turn through per minute?

7. Express an angular speed of 2.5 revolutions per second in degrees per second; in revolutions per minute; in degrees per minute.

8. A flywheel rotates at the rate of 40 revolutions per minute. Through what angle does one of its spokes turn in a second?

9. Reduce an angular speed of 3.4 revolutions per second to degrees per second; to degrees per minute; to revolutions per minute.

10. Find the angular speed of the rotation of the earth on its axis (*a*) in revolutions per minute; (*b*) in degrees per second.

11. Construct a right triangle whose sides are 3 and 4; construct an angle which is 3 times the smaller angle of this triangle.

12. Construct a right triangle with hypotenuse $= 12$ and one side $= 6$; construct an angle equal to one fourth of the larger acute angle of this triangle.

13. Construct an angle 3.5 times the smallest angle of the triangle in Ex. 12.

14. Every acute angle is a positive angle in the first quadrant; construct and place on the axes a positive angle in the first quadrant that is not acute.

15. Every obtuse angle is a positive angle in the second quadrant; construct and place on the axes a positive angle in the second quadrant that is not obtuse.

16. Construct the following angles and place them on the axes: (*a*) $-150°$; (*b*) $285°$; (*c*) $480°$; (*d*) $570°$; (*e*) $-225°$; (*f*) $-450°$.

17. In what quadrant is each of the following angles: $459°$, $682°$, $725°$, $-100°$, $-1090°$, $\pm 85°$, $\pm 95°$, $\pm 175°$, $\pm 185°$, $\pm 265°$, $\pm 275°$, $\pm 355°$?

40. Congruent Angles. If the difference of two angles, α and β, is n times $360°$ (where n is one of the numbers 0, 1, 2, 3, ... etc.), they are said to be **congruent** angles and we write $\alpha \cong \beta$; read: α is congruent to β. Thus $15° \cong 375°$, $-172° \cong 188°$, etc. If two angles γ and δ are not congruent, they are said to be incongruent, and we write $\gamma \not\cong \delta$, $\delta \not\cong \gamma$. Thus, $45° \not\cong 400°$. To prove that two angles are congruent it is necessary and sufficient to show that their difference is either 0 or a multiple of $360°$; that is, if $\alpha \cong \beta$, $\alpha - \beta = \pm n \cdot 360°$; and conversely.

41. Properties of Congruent Angles. If two congruent angles are placed on the same pair of axes, their terminal sides will coincide. For example, any two of the angles $50°$, $410°$, $-310°$ are congruent; when placed on the same axes their terminal sides all coincide. If two incongruent angles be placed on the same axes, their terminal sides will not coincide.* This is the geometric equivalent of § 40.

(1) Every angle obtained by putting $n = 0, 1, 2, 3$, etc. in the formula $\alpha \pm n\,360°$ is congruent to α; conversely every angle congruent to α is found in this set. (Use § 40.)

(2) If α is any angle whatever, there is one and only one angle between $0°$ and $360°$ ($0°$ included, $360°$ excluded) which is congruent to α. For if α is an angle of any size, the addition to α and subtraction from α of successive multiples of $360°$ ($360°$, $720°$, $1080°$, etc.) will give all angles congruent to α, and obviously one and only one of these lies between $0°$ and $360°$.

(3) If $\alpha \cong \gamma$ and if $\beta \cong \gamma$, then $\alpha \cong \beta$. That is, *if each of two angles is congruent to a third angle they are congruent to each other.* Proof: $\alpha - \gamma = \pm m \cdot 360°$, and $\gamma - \beta = \pm n \cdot 360°$; whence, adding: $\alpha - \beta = (\pm m \pm n)\,360°$, that is, $\alpha \cong \beta$.

* The word *congruent* is thus equivalent to the word *superposable* as used in geometry; but we must remember that two angles are superposable if and only if it is possible to make them coincide vertex with vertex, initial side with initial side, terminal side with terminal side. That such angles are not identical is evident in such practical instances as rotating machinery; the motion of a flywheel, $30°$ per second, differs essentially from that of a wheel turning $410°$ (or from that of one turning $-310°$ per second). See § 37, p. 51.

(4) *If the same angle be added to (or subtracted from) each of two congruent angles, the results will be congruent angles.*

Given $\alpha \cong \beta$, to prove: (*a*) $\alpha + \gamma \cong \beta + \gamma$, (*b*) $\alpha - \gamma \cong \beta - \gamma$.

(*a*) $(\alpha + \gamma) - (\beta + \gamma) = \alpha - \beta = \pm n \cdot 360°$; hence $\alpha + \gamma \cong \beta + \gamma$

(*b*) $(\alpha - \gamma) - (\beta - \gamma) = \alpha - \beta = \pm n \cdot 360°$; hence $\alpha - \gamma \cong \beta - \gamma$.

(5) *The negatives of congruent angles are congruent.*

Given $\alpha \cong \beta$, to prove that $(-\alpha) \cong (-\beta)$.

Proof: $(-\beta) - (-\alpha) = \alpha - \beta = \pm n \cdot 360°$; hence $(-\beta) \cong (-\alpha)$.

By (4) the transposition of a term from one side of a congruence to the other with change of sign is permissible; *e.g.* from $45° - 350° \cong 55°$ follows $45° \cong 55° + 350°$; from $\alpha + 150° \cong \beta + 180°$ follows $\alpha - \beta \cong 30°$.

By (5) it is permissible to change the signs of all terms of a congruence; *e.g.* from $2x - 18° \cong 3x - 63°$ follows $-x \cong -45°$ and $x \cong 45°$.

It is not ordinarily permissible to multiply or divide both sides of a congruence by any number (except to multiply by an integer); *e.g.* from $30° \cong 390°$ it does not follow that $10° \cong 130°$.

EXERCISES XIX. — CONGRUENT ANGLES

1. Draw figures on polar coördinate paper to illustrate (4), § 41, when (*a*) $\alpha = 30°$, $\beta = 390°$, $\gamma = 20°$; (*b*) $\alpha = 90°$, $\beta = -270°$, $\gamma = 45°$; (*c*) $\alpha = 72°$, $\beta = 432°$, $\gamma = 72°$.

2. Taking $\alpha = 60°$, $\beta = -300°$, $\gamma = -50°$, $\delta = 310°$, draw a figure showing that (*a*) $\alpha \cong \beta$; (*b*) $\gamma \cong \delta$; (*c*) $\alpha - \gamma \cong \beta - \delta$.

3. Find the angle between 0° and 360° which is congruent to each of the following: (*a*) $-42°\ 13'$; (*b*) $-842°$; (*c*) $364°\ 23'$; (*d*) $360°$; (*e*) $-90°$; (*f*) $420°$; (*g*) $2700°$.

4. Solve for x the congruence $27° - x \cong 360° + 2x$.
Ans. $x = 9° \pm n \cdot 120°$.

5. Find 3 values for x which satisfy the congruence $3x - 70° \cong 150° - x$.
Ans. $x = -35°$, $55°$, $145°$.

6. Find the smallest positive value of x which satisfies the congruence: $x + 200° \cong 40° - 3x$. *Ans.* $x = 50°$.

7. Prove that the sum of the interior angles of a convex polygon is congruent to 0° or 180° according as the number of sides is even or odd.

8. Compare a rotational speed of 30° per sec. with a speed of 390° per sec.

9. Reduce an angular speed of 390° per second to revolutions per second; to revolutions per minute.

10. If $\alpha \cong \beta$ and $\gamma \cong \delta$, prove that $\alpha + \gamma \cong \beta + \delta$, and $\alpha - \gamma \cong \beta - \delta$. Compare (4), § 41.

PART II. ANGULAR SPEED — RADIAN MEASURE

42. Measurement of Angles. An angle may be named and used before it is expressed in any system of measurement. Thus, we may refer to an angle A of a right triangle whose perpendicular sides are 16 in., and 24 in., respectively; and we can compute $\tan A = 24/16 = 1.5$, etc., without measuring A in terms of any unit angle. General theorems like the law of sines remain true in any system of measurement.

The measure of an angle — say $36°$ — consists of two distinct ideas: the *unit angle* (in this example, one degree) and the *abstract number* (here 36) which expresses the numerical measure of the angle in terms of the chosen unit. The elementary units are defined in § 37. For many purposes it is convenient to use another unit angle called the *radian.*

43. Radian Measure of Angles.* A **radian** is a positive angle such that when its vertex is placed at the center of a circle, the intercepted arc is equal in length to the radius.

This unit is thus a little less than one of the angles of an equilateral triangle; in fact it follows from the geometry of the circle, since the length of a semicircumference is πr, that

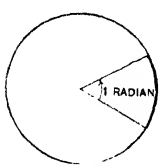


FIG. 51

(1) $\boldsymbol{\pi}$ **radians** $= \mathbf{180°}$, where $\boldsymbol{\pi} = \mathbf{3.14159}$,

whence 1 radian $= 57°\ 17'\ 44''.806$, or $57°.3$ approximately. Inversely $1° = .01745$ radians.

It is easy to change from degrees to radians and *vice versa* by means of relation (1), which should be remembered. Conversion tables for this purpose are printed in Table IV.

44. Use of Radian Measure. It is shown in geometry that two angles at the center of a circle are to each other as their intercepted arcs; therefore if an angle at the center is measured in radians and if the radius and the intercepted arc are measured in terms of the same linear unit, their numerical measures satisfy the simple relation:

$$\textbf{arc} = \textbf{angle} \times \textbf{radius}.$$

* Sometimes also called circular measure.

In other words, the number of linear units in the arc is equal to the product of the number of radians in the angle by the number of linear units in the radius.

Example 1. Find the difference in latitude of two places on the same meridian 200 mi. apart, taking the radius of the earth as 4000 mi.

Angle = arc/radius = 1/20 in radians = $2° 51' 53''$, approximately.

45. Angular Speed. In a rotating body a point P, which is at a distance r from the axis of rotation, moves through a distance $2\pi r$ during each revolution or through a distance r while the body turns through an angle of one radian. Therefore if v is the linear (actual) speed of P (in linear units per time unit, *e.g.* feet per second), and if ω is the angular speed of the rotating body (in radians per time unit, *e.g.* radians per second), then their numerical measures satisfy the relation

$$v = r \cdot \omega;$$

hence *the angular speed of a rotating body is numerically equal to the actual speed of a point one unit from the axis of rotation.*

Engineers usually express the angular speed of the rotating parts of machinery in revolutions per minute (R. P. M.) or revolutions per second (R. P. S.). These are easily reduced to radians per minute (or per second) by remembering that one revolution equals 2π radians.

Example 1. A flywheel of radius 2 ft. rotates at an angular speed of 2.5 R. P. S. Find the linear speed of a point on the rim.

In radians per second, $\omega = 2.5 \times 2\pi = 5\pi$, and for a point 2 ft. from the axis of rotation $v = 2 \times 5\pi = 31.416$ ft. per second.

Example 2. Find the angular speed of a 34-in. wheel on an automobile going 20 mi. per hour.

Every time the wheel turns through a radian the car goes forward 17 in. (the length of the radius), and 20 mi. per hour = 352 in. per sec.; therefore the wheel turns through 352/17 = 20.7 . . . radians per second.

46. Notation. In measuring angles in radian measure we shall adopt the practice universal in advanced work and write only the *numerical measure* of the angle in terms of the unit *one radian.* Thus in the expression tan x, the letter x will denote a *number* (the numerical measure of an angle), rather than the angle itself.

When necessary, to call attention to the fact that radian measure is intended, the symbol ($^{(r)}$) is appended to the numerical measure, thus:

$$1^{(r)} = 1 \text{ radian} = 57° \, 17' \, 44''.8,$$
$$2^{(r)} = 2 \text{ radians} = 114° \, 35' \, 29''.6,$$
$$\pi^{(r)} = \pi \text{ radians} = 180° = 2 \text{ rt. } \measuredangle\text{s},$$
$$(\pi/2)^{(r)} = \pi/2 \text{ radians} = 90° = 1 \text{ rt. } \angle,$$

and so forth.

As it happens that the acute angles whose trigonometric functions are most easily recalled without consulting tables are simple fractional parts of a straight angle, the number π often appears as a factor of the numerical measure of angles. In this system, for example, $\sin(\pi/2) = 1$, $\cos(\pi/3) = 1/2$, $\tan(\pi/4) = 1$, etc.

The use of pure numbers, such as 2 or π in place of an angle, is precisely similar to the use of 10 for 10 ft. or 10 inches in expressing lengths. The student should supply the *unit of measurement* (radians, or feet or inches), and should not confuse the number π ($= 3.14159 \cdots$) with the angle whose measure is π *radians*, as he should not confuse the number 10 with the distance 10 *feet*.

47. Relations between Angular Units. The units of angle mentioned thus far are degree, minute, second, right angle, straight angle, revolution (or perigon), radian. The relations between these units is given in the following table:

	°	′	″	RT. ∠	REV.	RADIANS
$1° =$	1	60	3600	1/90	1/360	$\pi/180$
$1' =$	1/60	1	60	1/5400	1/21600	$\pi/10800$
$1'' =$	1/3600	1/60	1	1/324000	1/1296000	$\pi/648000$
1 rt. ∠ =	90	5400	324000	1	1/4	$\pi/2$
1 str. ∠ =	180	10800	648000	2	1/2	π
1 rev. =	360	21600	1296000	4	1	2π
1 radian =	$180/\pi$	$10800/\pi$	$648000/\pi$	$2/\pi$	$1/(2\pi)$	1

Another unit frequently used in France and occasionally elsewhere is the *grade*, which is 1/100 of a right angle.

EXERCISES XX. — ANGULAR SPEED — RADIAN MEASURE

1. Express the following angles in degrees, minutes, and seconds:

   (*a*) $\pi^{(r)}/4$; (*b*) $\pi^{(r)}/6$; (*c*) $2\,\pi^{(r)}/3$; (*d*) $3^{(r)}$.

2. Express the following angles in radians:

   (*a*) 25°; (*b*) 30°; (*c*) 35°; (*d*) 28° 39′; (*e*) 114° 35′.

3. How far short of one revolution is $6^{(r)}$?

4. To gain ability to judge the size of angles in circular measure, express approximately (to within 1°) angles whose sizes are $1^{(r)}$, $4^{(r)}$, $5^{(r)}$, $8^{(r)}$. Draw an angle which is about your impression of an angle of $2^{(r)}$, and measure it with a protractor: do not revise your figures.

5. If a vehicle moves at the rate of 15 ft. per sec., through what angle does one of its wheels, 3 ft. in diameter, revolve in one second?

6. If the linear speed of a vehicle is 30 mi. per hour, what is the angular speed of one of its wheels which is 4 ft. in diameter?

7. A wheel 5 ft. in diameter is connected by a belt 40 ft. in length with a wheel 4 ft. in diameter. If the large wheel makes 30 revolutions per minute, how often does the seam of the belt pass this wheel? What is the angular speed of the smaller wheel?

8. Find the angular distance on the earth between two points whose distance from each other, on the arc of a great circle, is 800 miles. [Take the radius of the earth to be 4000 miles.]

9. Find the distance in miles between two points on the earth's surface whose angular distance is 1°; between two points whose angular distance is 0.25 radians.

10. Find the length of the subtended arc of an angle of 3.46 radians at the center of a circle of radius 5.

11. Find the length of the subtended arc of an angle of 55° at the center of a circle of radius 3. (Compare the work of Exs. 10 and 11.)

12. Find the angle at the center which subtends an arc of 3 ft. on a circle of radius 4 ft. Express the angle in radians and in degrees, and compare the work done in the two cases.

13. Reduce to radian measure by means of the Tables:

    (*a*) 23° 40′; (*b*) 68° 45′ 20″; (*c*) 138° 35′ 15″.

14. Reduce to degree measure by means of the Tables:

    (*a*) $3.46^{(r)}$; (*b*) $.256^{(r)}$; (*c*) $.0127^{(r)}$; (*d*) $8.24^{(r)}$.

15. Reduce the following angular speeds to degrees per second; to revolutions per second; to revolutions per minute:

    (*a*) $4.5^{(r)}$ per sec.; (*b*) $2.48^{(r)}$ per sec.; (*c*) $10.54^{(r)}$ per sec.

CHAPTER V

FUNCTIONS OF ANY ANGLE

PART I. DEFINITIONS. READING OF TABLES

48. Resolution of Forces. Projections. In § 33, p. 43, we saw how to find the components of a force, or a velocity, on any line, as the projection of the force on that line; and we saw that the components of a force F on each of two perpendicular axes, even when the angle α is obtuse, are

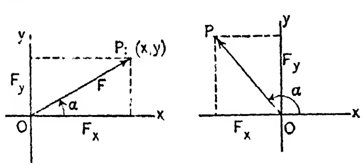


FIG. 52

(1) $$F_x = \text{Proj}_x F = F\cos\alpha,$$
$$F_y = \text{Proj}_y F = F\sin\alpha.$$

If several forces occur in the same problem, some of them may make an angle α greater than 180° with the positive direction Ox. It is convenient to *define* $\cos\alpha$ and $\sin\alpha$ for angles greater than 180° so that the equations (1) remain true.

If we do so, the projection on the two axes of any directed line r joining the origin O to a point P are, respectively,

(2) $$\boldsymbol{x = \text{Proj}_x r = r\cos\alpha, \qquad y = \text{Proj}_y r = r\sin\alpha,}$$

where α is the angle between the positive direction Ox and the positive direction OP, and may be an angle of any size, positive or negative.

Hence the desired definitions are:

(3) $$\boldsymbol{\cos\alpha = \frac{x}{r}, \qquad \sin\alpha = \frac{y}{r}.}$$

These definitions are consistent with those already given, §§ 6, 18, for the sine and the cosine; *i.e.* in case $0° \leqq \alpha \leqq 180°$, they determine the same values as the earlier definitions.

49. General Definitions. Trigonometric Functions of any Angle. The definitions of $\sin\alpha$ and $\cos\alpha$ given in § 48 have, of course, no necessary dependence upon forces. Each is a number which depends only on the magnitude and sign of the angle. A purely

geometric definition of these and of the other trigonometric functions of any angle α, consistent with the definitions of §§ 6, 18, and with the fundamental relations between them, such as $\tan\alpha = \sin\alpha/\cos\alpha$, $\sin^2\alpha + \cos^2\alpha = 1$, the reciprocal relations, etc., may be made as follows:

Place the given angle on a pair of rectangular axes, and select any point P whose coördinates are (x, y) on the terminal side at a distance $r > 0$ from the origin. Then

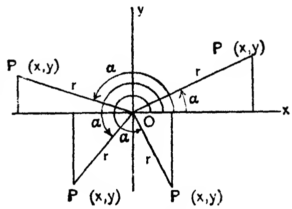


FIG. 53

(1) $$\sin\alpha = \frac{y}{r} = \frac{\text{ordinate}}{\text{radius}},$$

(2) $$\cos\alpha = \frac{x}{r} = \frac{\text{abscissa}}{\text{radius}},$$

(3) $$\tan\alpha = \frac{y}{x} = \frac{\text{ordinate}}{\text{abscissa}}, \quad \text{provided } x \neq 0;^*$$

(4) $$\operatorname{ctn}\alpha = \frac{x}{y} = \frac{\text{abscissa}}{\text{ordinate}}, \quad \text{provided } y \neq 0;$$

(5) $$\sec\alpha = \frac{r}{x} = \frac{\text{radius}}{\text{abscissa}}, \quad \text{provided } x \neq 0;$$

(6) $$\csc\alpha = \frac{r}{y} = \frac{\text{radius}}{\text{ordinate}}, \quad \text{provided } y \neq 0.$$

Three additional functions sometimes used are:

(7) The versed sine of α: $\operatorname{vers}\alpha = 1 - \cos\alpha$.

(8) The coversed sine of α: $\operatorname{covers}\alpha = 1 - \sin\alpha$.

(9) The external secant of α: $\operatorname{exsec}\alpha = \sec\alpha - 1$.

By these definitions every angle has a sine and a cosine, because in the ratios y/r and x/r the denominator r is never zero. There is no secant or tangent† for 90°, or for 270°, or for any

* The exceptions noted are based on the general principle that a fractional expression does not represent a number if its denominator is zero.

† To say that 90° has no tangent does not mean that the tangent of 90° is zero. When we say that an article has no value we mean that it has a value and that value is zero. Not so here. Since the general definition of tangent does not apply to 90°, we could, if we found it convenient, define tan 90°, but we do not; we leave it undefined. Often it is said $\tan 90° = \infty$, but this does not mean that 90° has a tangent; it means that as an angle α increases from 0° to 90°, $\tan\alpha$ increases without limit, *and that before α reaches 90°*.

angle whose terminal side coincides with either the positive or negative end of the y-axis, because the denominator x in the ratios r/x, y/x, is zero. Similarly, there is no cosecant or cotangent for $0°$ or for $180°$, or for any angle whose terminal side coincides with the positive or negative end of the x-axis. There exists a tangent, cotangent, secant, and cosecant for every angle except those just mentioned.

The values of the same trigonometric function of two congruent angles (§ 40) are equal; that is, if α is congruent to β, then $\sin\alpha = \sin\beta$, $\cos\alpha = \cos\beta$, $\tan\alpha = \tan\beta$; and likewise for all the other trigonometric functions. In other words, *any angle obtained by adding to a given angle, or subtracting from it, a multiple of 360°, has the same functions as the given angle.*

For example: $\sin(-295°) = \sin 65°$, $\cos(-315°) = \cos 45°$, $\tan 1476° = \tan 36°$, $\sin(\theta - 180°) = \sin(180° + \theta)$, $\cos(x - 90°) = \cos(270° + x)$, $\tan(360° - y) = \tan(-y)$.

50. Algebraic Signs of Trigonometric Functions. The sine of any angle in the first or second quadrant is positive, because the ordinate of any point above the x-axis is positive; the sine of any angle in the third or fourth quadrant is negative, because the ordinate of any point below the x-axis is negative.

The cosine of any angle in the first or fourth quadrant is positive, because the abscissa of any point to the right of the y-axis is positive; similarly, the cosine of any angle in the second or third quadrant is negative.

Similarly, the signs of $\tan\alpha$, $\text{ctn}\,\alpha$, $\sec\alpha$, $\csc\alpha$, etc., may be determined directly from a figure; they are as follows:

Quadrant	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$	$\text{ctn}\,\alpha$	$\sec\alpha$	$\csc\alpha$
1st	+	+	+	+	+	+
2d	+	−	−	−	−	+
3d	−	−	+	+	−	−
4th	−	+	−	−	+	−

NOTE. (1) $\tan\alpha$ is positive (negative) when $\sin\alpha$ and $\cos\alpha$ have like (unlike) signs; (2) reciprocals have the same sign.

EXERCISES XXI.—FUNCTIONS OF THE GENERAL ANGLE

1. By placing the angles on the axes, show from the definitions that

(*a*) $\sin 225° = -\sqrt{2}/2$, $\cos 225° = -\sqrt{2}/2$.
(*b*) $\sin 150° = 1/2$, $\cos 150° = -\sqrt{3}/2$.
(*c*) $\sin 330° = -1/2$, $\cos 330° = \sqrt{3}/2$.
(*d*) $\sin(-315°) = \sqrt{2}/2$, $\cos(-315°) = \sqrt{2}/2$.
(*e*) $\sin(-1020°) = \sqrt{3}/2$, $\cos(-1020°) = 1/2$.
(*f*) $\sin 180° = 0$, $\sin(n \cdot 180°) = 0$; for $n = \pm 1, \pm 2, \pm 3, \cdots$.
(*g*) $\cos 90° = 0$, $\cos[(2n-1)90°] = 0$; for $n = \pm 1, \pm 2, \pm 3, \cdots$.

2. Which of the following are positive and which negative? $\sin 72°$, $\sin 352°$, $\sin 850°$, $\tan 128°$, $\sec 260°$, $\sin(-20°)$, $\cos(-380°)$, $\sin(-260°)$, $\cos 160°$, $\text{ctn } 280°$, $\cos 33°$, $\csc 91°$, $\cos(-40°)$, $\tan(-140°)$, $\cos(-400°)$.

3. Prove for *any angle* α that $\sin^2 \alpha + \cos^2 \alpha = 1$. [Use $x^2 + y^2 = r^2$.]

Prove each of the other Pythagorean relations for *any angle* α:

$1 + \tan^2 \alpha = \sec^2 \alpha$, if $\cos \alpha \neq 0$; $1 + \text{ctn}^2 \alpha = \csc^2 \alpha$, if $\sin \alpha \neq 0$.

4. Prove that $\text{ctn } \alpha$, $\sec \alpha$, $\csc \alpha$, are the reciprocals of $\tan \alpha$, $\cos \alpha$, $\sin \alpha$, respectively, for all values of α for which both are defined.

5. (*a*) Prove that the sine of any angle in the first or second quadrant is between 0 and 1. (*b*) Prove that the cosine of any angle in the 1st or 4th quadrant is between 0 and 1.

6. Prove that if an angle is not an odd multiple of a right angle its sine is between -1 and $+1$; and conversely. For what angles is $\sin \alpha = +1$; $\sin \alpha = -1$; $\cos \alpha = +1$?

7. Show that $\tan \alpha = \sin \alpha / \cos \alpha$ for all values of α, if $\cos \alpha \neq 0$.

8. Show that $\tan \alpha$ and $\text{ctn } \alpha$ may have any values whatever.

9. Show that $\text{vers } \alpha$ and $\text{covers } \alpha$ are always positive or zero.

10. If an angle α starts at 0° and gradually increases to 360°, show that the behavior of $\sin \alpha$ and $\cos \alpha$ will be as indicated in this table:

α	0°	$0° < \alpha < 90°$	90°	$90° < \alpha < 180°$	180°	$180° < \alpha < 270°$	270°	$270° < \alpha < 360°$	360°
$\sin \alpha$	0	increases to	1	decreases to	0	decreases to	-1	increases to	0
$\cos \alpha$	1	decreases to	0	decreases to	-1	increases to	0	increases to	1

11. By placing the angles indicated on rectangular axes determine the numbers to fill the blanks in the following table:

α	30°	45°	60°	120°	135°	150°	210°	225°	240°	300°	315°	330°
$\sin \alpha$												
$\cos \alpha$												

12. Assuming that the sun passes directly overhead, trace the change in the length of the shadow of an object from dawn to sunset. Which trigonometric function do you think of in this problem?

13. Assuming the results of Exs. 10 and 11, derive from them the variation of the tangent from 0° to 360° and its values at each of the angles mentioned in Ex. 11. Do the same for ctn α, sec α, csc α.

51. Reading of Tables. Sine and Cosine of $-\theta$ and $90° + \theta$. In order to find the sine (or any other trigonometric function) of an angle we consult the tables. In the tables the values of the sine, for example, are printed only up to 45°. To find the sine of an acute angle greater than 45° we make use of the relation $\sin\alpha = \cos(90° - \alpha)$. The tables are arranged to facilitate this by having the angles above 45° printed at the bottom of the page, and the column headings changed from sine to cosine, etc. (See *Tables*, p. 22.)

If we wish to find the sine of an angle greater than 90°, we must find a way to express the sine in terms of some function of an acute angle less than 45°. In Chapter III this has been done for obtuse angles. The purpose of what follows is to make a similar reduction for *any* angle, positive or negative.

The following four relations are true for every angle θ (positive, negative, zero):

(1) $\sin(-\theta) = -\sin\theta$, (3) $\sin(90° + \theta) = \cos\theta$,

(2) $\cos(-\theta) = \cos\theta$, (4) $\cos(90° + \theta) = -\sin\theta$.

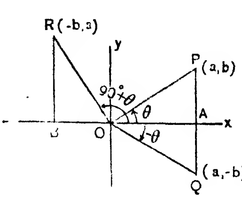


FIG. 54

Proof: (*a*) When $\theta = 0°$ the formulas are easily verified.

(*b*) When $0° < \theta < 90°$, place the angles θ, $-\theta$, and $90° + \theta$ on the same coördinate axes, with the origin as center and a convenient radius $r > 0$, and describe a circle cutting the terminal sides of θ, $-\theta$, $90° + \theta$, in P, Q, R. Let the coördinates of P be (a, b); then those of Q are $(a, -b)$, and those of R are $(-b, a)$, since the right triangles OAP, OAQ, RBO are congruent. Then, by § 49, $\sin\theta = b/r$, $\cos\theta = a/r$; $\sin(-\theta) = -b/r$, $\cos(-\theta) = a/r$; and $\sin(90° + \theta) = a/r$,

$\cos(90° + \theta) = -b/r$ Therefore $\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = \cos\theta$, $\sin(90° + \theta) = \cos\theta$, $\cos(90° + \theta) = -\sin\theta$.

(*c*) When $\theta = 90°, 180°, 270°$, verify by direct substitution.

(*d*) When $90° < \theta < 180°$, or $180° < \theta < 270°$, or $270° < \theta < 360°$, the formulas are proved by a figure drawn as in (*b*).

The formulas (1)–(4) are thus proved for all values of θ between 0° and 360° (0° included).

Finally, if α is any angle whatever, there is one angle β between 0° and 360° (0° included) such that $\beta \cong \alpha$. (See § 41.) We have just seen that formulas (1)–(4) hold for β; but since $\alpha \cong \beta$, we have also $-\alpha \cong -\beta$, $90° + \alpha \cong 90° + \beta$. (See § 41.) It follows that (1)–(4) hold for α, since any trigonometric function has the same value for any two congruent angles.

52. Reading of Tables. Sine and Cosine of $180° \pm \theta$ and $270° \pm \theta$. *The relations tabulated below are true for every angle* $\boldsymbol{\theta}$.

	$-\theta$	$90° - \theta$	$90° + \theta$	$180° - \theta$	$180° + \theta$	$270° - \theta$	$270° + \theta$
sin	$\boldsymbol{-\sin\theta}$	$\cos\theta$	$\boldsymbol{\cos\theta}$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$
cos	$\boldsymbol{\cos\theta}$	$\sin\theta$	$\boldsymbol{-\sin\theta}$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$

Proof: The four relations in black-face type have been proved above; to prove the others, we proceed as follows:

(1) Let $\alpha = -\theta$, then $90° - \theta = 90° + \alpha$, and $\sin(90° - \theta) = \sin(90° + \alpha) = \cos\alpha = \cos(-\theta) = \cos\theta$.

(2) $\text{Cos}(90° - \theta) = \cos(90° + \alpha) = -\sin\alpha = -\sin(-\theta) = \sin\theta$.

(3) Let $\alpha = 90° - \theta$, then $180° - \theta = 90° + \alpha$, whence $\sin(180° - \theta) = \sin(90° + \alpha) = \cos\alpha = \cos(90° - \theta) = \sin\theta$; and $\cos(180° - \theta) = \cos(90° + \alpha) = -\sin\alpha = -\sin(90° - \theta) = -\cos\theta$.

(4) Let $\alpha = 90° + \theta$ and make use of the formulas for $90° + \alpha$ to obtain formulas for $180° + \theta$.

(5) Let $\alpha = 180° - \theta$ and make use of the formulas for $90° + \alpha$ to obtain formulas for $270° - \theta$.

(6) Let $\alpha = 180° + \theta$ and make use of the formulas for $90° + \alpha$ to obtain formulas for $270° + \theta$.

53. Extension to Other Functions. *The relations tabulated below are true for every angle θ, except as noted:*

	$-\theta$	$90° - \theta$	$90° + \theta$	$180° - \theta$	$180° + \theta$	$270° - \theta$	$270° + \theta$
sin	$\mathbf{-\sin\theta}$	$\mathbf{\cos\theta}$	$\mathbf{\cos\theta}$	$\mathbf{\sin\theta}$	$\mathbf{-\sin\theta}$	$\mathbf{-\cos\theta}$	$\mathbf{-\cos\theta}$
cos	$\mathbf{\cos\theta}$	$\mathbf{\sin\theta}$	$\mathbf{-\sin\theta}$	$\mathbf{-\cos\theta}$	$\mathbf{-\cos\theta}$	$\mathbf{-\sin\theta}$	$\mathbf{\sin\theta}$
tan *	$-\tan\theta$	$\text{ctn}\,\theta$	$-\text{ctn}\,\theta$	$-\tan\theta$	$\tan\theta$	$\text{ctn}\,\theta$	$-\text{ctn}\,\theta$
ctn †	$-\text{ctn}\,\theta$	$\tan\theta$	$-\tan\theta$	$-\text{ctn}\,\theta$	$\text{ctn}\,\theta$	$\tan\theta$	$-\tan\theta$
sec *	$\sec\theta$	$\csc\theta$	$-\csc\theta$	$-\sec\theta$	$-\sec\theta$	$\csc\theta$	$\csc\theta$
csc †	$-\csc\theta$	$\sec\theta$	$\sec\theta$	$\csc\theta$	$-\csc\theta$	$-\sec\theta$	$-\sec\theta$

Proof: The relations in black-face type have been proved above; to make the others depend on these, use the relation $\tan\alpha = \sin\alpha \div \cos\alpha$ and the fact that the last three functions are the reciprocals of the first three in reverse order.

The whole body of relations in this table, which have now been proved for all angles θ, except as noted below, ‡ may be remembered by the two following "rules of thumb."

1. *Determine the sign by the quadrant in which the angle would lie if θ were acute;* the result holds whether θ is acute or not.

2. *In case of $90° \pm \theta$ or $270° \pm \theta$ change the name of the function to the cofunction; in case of $-\theta$ or $180° \pm \theta$ do not change the name of the function.*

Example 1. $\sin(175°) = \sin(180° - 5°) = +(\text{rule 1})\sin(\text{rule 2})5°$.

Example 2.

$\cos 175° = \cos(90° + 85°) = -(\text{rule 1})\sin(\text{rule 2})85° = -\sin 85°$.

Example 3.

$\tan 300° = \tan(270° + 30°) = -(\text{rule 1})\text{ctn}(\text{rule 2})30° = -\sqrt{3}$.

Example 4.

$\tan 300° = \tan(180° + 120°) = +(\text{rule 1})\tan(\text{rule 2})120° = -\sqrt{3}$.

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* When θ = an odd multiple of $\pm 90°$ it has no tangent or secant.

† When θ = an even multiple of $\pm 90°$ it has no cotangent or cosecant.

‡ The tabulated relations are all true if θ is not a multiple of 90°; they fail only in the cases mentioned in the preceding footnotes. See §§ 8, 49.

EXERCISES XXII. — READING OF TABLES — REDUCTION TO FUNCTIONS OF ACUTE ANGLES

1. Express the following as functions of acute angles not greater than 45°. Make use of congruent angles whenever advantageous:

(*a*) $\sin 150° 21'$. (*b*) $\cos 125° 15'$. (*c*) $\tan 283° 45'$.
(*d*) $\text{ctn}(-36° 16')$. (*e*) $\sec 460°$. (*f*) $\csc(-210° 20')$.
(*g*) $\sin(-943° 24')$. (*h*) $\cos 551° 23'$. (*i*) $\tan(-546° 28')$.

2. Show directly from a figure that

(*a*) $\sin(118° 26') = \sin 61° 34' = \cos 28° 26'$.
(*b*) $\cos(118° 26') = -\cos 61° 34' = -\sin 28° 26'$.
(*c*) $\tan(118° 26') = -\tan 61° 34' = -\text{ctn}\, 28° 26'$.
(*d*) $\sin 312° 18' = -\sin 47° 42' = -\cos 42° 18'$.

3. Make a tabular form of 5 columns and 16 rows, and at the head of the columns, beginning with the 2d, enter the words sine, cosine, tangent, cotangent. In the first column, beginning with the 2d line, enter the following angles: $172° 26'$, $-153° 18'$, $253° 12'$, $-208° 25'$, $285° 32'$, $-312° 18'$, $389° 15'$, $-416° 27'$, $462° 50'$, $-502° 11'$, $552° 37'$, $-618° 42'$, $650° 14'$, $-700° 24'$, $1000° 10'$. Use a table of trigonometric functions.

4. Reduce the following to functions of acute angles as in Ex. 2: (*a*) $\sin 164° 22'$. (*b*) $\cos 348° 12'$. (*c*) $\tan 264° 46'$. (*d*) $\text{ctn}\, 128° 14'$. (*e*) $\sec 222° 45'$. (*f*) $\csc 305° 42'$. (*g*) $\sin 142° 25'$. (*h*) $\cos 275° 23'$.

5. From the tables find the values of the following logarithms:

(*a*) $\log(-\cos 161° 11')$. (*b*) $\log \sin 161° 11'$.
(*c*) $\log(-\sin 217° 17')$. (*d*) $\log(-\cos 252° 48')$.

[Note that the numbers in parentheses in (*a*), (*c*), and (*d*) are positive; if the minus sign were absent, each of them would be negative. Negative numbers have no real logarithms.]

6. Compute the values of the following expressions by logarithms:

(*a*) $2.35 \sin 148° 23'$. (*b*) $24.8 \cos 160° 40'$. (*c*) $16.2 \cos 320° 45'$.

7. Find the components on the axes of a force of magnitude 5.74 (lb.) which makes an angle of $215° 20'$ with the positive end of the x-axis.

8. A force is indicated by stating its magnitude (in pounds) and its direction (*i.e.* the angle it makes with the positive end of the x-axis). Find the components on the axes of the forces indicated below:

(*a*) (4.17, 128°). [This means magnitude 4.17, angle 128°.]
(*b*) (24.8, 250° 10′). (*c*) (5.72, 310° 35′).
(*d*) (51.4, 141° 25′). (*e*) (40.5, − 23° 40′).

9. Find the magnitude and the direction of a force whose components on two perpendicular axes are $F_x = 25.46$, $F_y = 38.72$.

10. Find the magnitude and the direction of a force whose components are $F_x = -12.8$, $F_y = 6.45$.

PART II. GRAPHS OF TRIGONOMETRIC FUNCTIONS

54. Plotting of Values. A table of a few values, such as those of Exs. 10 and 11, p. 63, furnishes enough data to construct a fair graph of the functions sine and cosine. The table on p. 15 also may be used; it contains more than is really needed except for exceedingly accurate drawing. For example if x denotes the angle and y denotes its sine, we have to plot the curve represented by the equation $y = \sin x$.

On a sheet of cross section paper draw a pair of rectangular axes Ox and Oy. Before plotting any points it is necessary

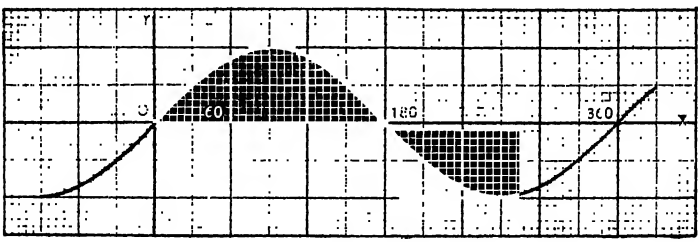


FIG. 55

to decide how many spaces shall represent a unit length and how many degrees shall be represented by the unit length, in order to get all the desired points on the paper.

In Fig. 55, one unit on the horizontal scale is chosen to represent 60°, which is convenient to the size of the paper; but any other unit might as well have been chosen.

We then plot the points corresponding to the angles indicated, and draw through them a smooth curve, keeping in mind the general behavior of the sine as given in the first of the tables of Ex. 10, p. 63.

In plotting curves it is of advantage in many ways to make the horizontal and vertical scale units the same, and this should be done if not too inconvenient.

* If we were to take the two scale units the same in plotting the curve $y = \sin x$ where the unit angle is the degree, one arch of the curve would be 180 units long and only one unit high.

55. Graphs in Radian Measure. A very convenient unit angle for many such graphs is the *radian* (§ 43). We shall agree to use the radian as a unit in trigonometric graphs, unless something is said to the contrary. The graph of $y = \sin x$,

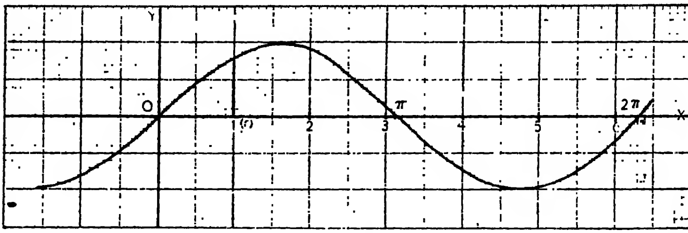


FIG. 56

drawn on this scale, is shown in Fig. 56; it resembles Fig. 55 very closely, since 1 radian $= 57°.3$ is very close to $60°$, which was used in § 54.

The use of the radian as the unit angle both in such graphs and in all other connections, is universal in the Calculus and in other advanced mathematical topics.

56. Mechanical Construction for $\sin x$. Instead of computing values of y for certain values of x as in the preceding table and plotting these for points on the curve $y = \sin x$, we can shorten the work materially by the following graphical method. Construct a pair of rectangular axes and choose a scale unit; for the sake of fixing our ideas let us suppose that this unit is one inch. At C, a convenient point on the x-axis as center, construct a unit circle. Choose some number for x, and lay off $ACB = x$ radians (above the x-axis if $x > 0$, below if $x < 0$). On the x-axis lay off $OD =$ arc $AB = x$ in. (to the right if $x > 0$, to the left if $x < 0$). Then the abscissa of D, and therefore of every point on the vertical line through D, is x; it is obvious from the construction that the ordinate of B, and therefore of every point on the horizontal line through B, is $\sin x$. Therefore the coördinates of the point P where these lines intersect are $(x, \sin x)$, and P is a point on the curve $y = \sin x$.

This method may be used to plot the curve $y = \sin x$ as fol-

lows. Suppose it is desired to plot the curve from $x = -\pi/2$ to $x = \pi/2$. Choose a scale unit and lay off on the x-axis $OE = 1$ unit, $OH = (\pi/3)$ units $= 22/21$ of OE, approximately. Divide OH into a convenient number of parts (say 4), and then mark the points $-\pi/2$, $-5\pi/12$, $-\pi/3$, $-\pi/4$, $-\pi/6$, $-\pi/12$, $\pi/12$, $\pi/6$, $\pi/4$, $\pi/3$, $5\pi/12$, $\pi/2$, on the x-axis. At C, a convenient point on the x-axis, construct a circle with radius

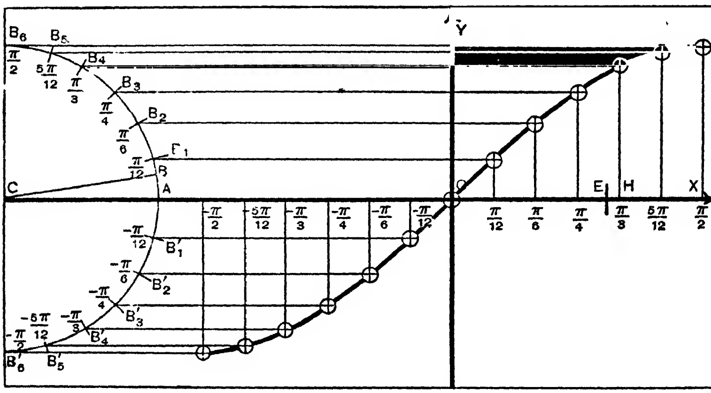


FIG. 57

$= OE$. With a protractor (or by bisection of arcs in this case) lay off arcs AB_1, AB_2, etc. which subtend angles $\pi/12$ radians, $\pi/6$ radians, etc. at the center C; also, AB_1', AB_2', etc., subtending angles $-\pi/12$ radians, $-\pi/6$ radians, etc. Intersections of corresponding horizontal and vertical lines give points on the curve. Proceed similarly for any value of x.

EXERCISES XXIII. — GRAPHS OF THE TRIGONOMETRIC FUNCTIONS

1. From the values of $\cos x$ in Exs. 10 and 11, p. 63, plot the graph of the equation $y = \cos x$, choosing the units as in § 54.

2. Draw the curve $y = \cos x$ by a mechanical construction similar to that of § 56, with the scale used in §§ 55–56.

3. Plot the graphs of each of the equations

$$(a)\ y = \tan x, \quad (b)\ y = \operatorname{ctn} x,$$

on the scale of § 54, or on the scale of § 55.

[NOTE. The scale of § 55 (radian measure) is preferable. See Table V.]

4. Trace the variations of $\cos x$ as x increases from 0 to 2π radians, from the curve of Ex. 2.

5. Trace the variation of $\tan x$ as x varies from 0 to 2π, from the curve of Ex. 3 (*a*). Trace the variation of $\operatorname{ctn} x$.

6. Plot the graphs, preferably on the radian scale, of the equations:

$$(a)\ y = \sec x, \quad (b)\ y = \csc x.$$

7. Show that the graph of $y = \sin x + \cos x$ can be constructed by mechanically adding the ordinates of the two curves $y = \sin x$ and $y = \cos x$.

8. By analogy to Ex. 7 show how to draw mechanically each of the following curves:

(*a*) $y = \sin x - \cos x$.
(*b*) $y = 2 \sin x + 3 \cos x$.
(*c*) $y = \tan x - 2 \sin x$.
(*d*) $y = -\cos x$.
(*e*) $y = \operatorname{vers} x = 1 - \cos x$.
(*f*) $y = \operatorname{exsec} x = \sec x - 1$.

9. Show that the graph of $y = x + \sin x$ can be constructed mechanically. (Use radian measure.)

10. Show how to construct the graph of $y = \sin 2x$ mechanically by shortening the horizontal lengths in the graph of $y = \sin x$ in the ratio $1 : 2$.

11. By analogy to Exs. 7, 9, 10, draw mechanically each of the following curves from graphs previously drawn:

(*a*) $y = \cos 3x$.
(*b*) $y = \sin x - 3 \cos 2x$.
(*c*) $y = x - \cos x$.
(*d*) $y = \tan x + \sin 2x$.

12. Show that the graph of $y = \sec x$ can be drawn mechanically from that of $y = \cos x$ by means of the relation $\sec x = 1/\cos x$.

13. By analogy to Ex. 12 show how to draw, from the graphs of $\sin x$ and $\cos x$, the graphs of each of the following curves:

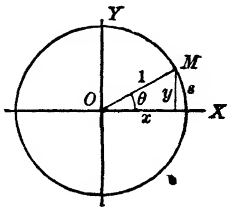
(*a*) $y = \csc x$. (*b*) $y = \tan x$. (*c*) $y = \operatorname{ctn} x$. (*d*) $y = \operatorname{vers} x$.

14. Plot each of the curves (*a*) $y = \sin(x/2)$, (*b*) $y = \cos(x/2)$.

15. Show that the graph of $y = \sin(x - \pi/6)$ can be drawn by moving the graph of $y = \sin x$ to the right by an amount $\pi/6$.

16. By analogy to Ex. 15 draw mechanically the following curves:

(*a*) $y = \cos(x - \pi/6)$.
(*b*) $y = \sin(x + \pi/3)$.
(*c*) $y = 2 \tan(x - \pi/4)$.
(*d*) $y = 2 \sin(4x - \pi/3)$.



17. Show how to draw mechanically the graphs of each of the following curves:

(*a*) $y = \sin^2 x$. (*b*) $y = \cos^2 x$. (*c*) $y = \tan^2 x$.

18. If a point M moves in a circular path of unit radius with a constant angular speed 1 radian per second, show that the angle $\theta = t$ (radians); hence show that the coördinates (x, y) of M are: $x = \cos t$, $y = \sin t$.

PART III. APPLICATIONS OF LARGE ANGLES

57. Composition and Resolution of Forces. As in § 48, the components on the axes of any force of magnitude F which makes an angle α with the positive end of the x-axis, are

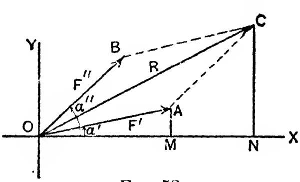


FIG. 58

(1) $$F_x = F\cos\alpha,\quad F_y = F\sin\alpha.$$

Given two forces, F', F'' which make angles α', α'', respectively, with the positive end of the x-axis, we may find the components of each of them on each of the axes. The *sum* of the two x-components is $F'_x + F''_x = F'\cos\alpha' + F''\cos\alpha''$, and is equal to the x-component of the resultant R of F' and F'', as is evident from a figure, since

$$\text{Proj}_x\, OC = ON = OM + MN = \text{Proj}_x F' + \text{Proj}_x F''.$$

Hence the x-component R_x of R is:

(2) $$R_x = F'\cos\alpha' + F''\cos\alpha'',$$

and in like manner the y-component R_y of R is

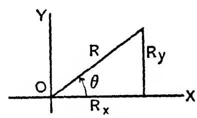


FIG. 59

(3) $$R_y = F'\sin\alpha' + F''\sin\alpha''.$$

These results hold, by (1), when F' and F'' lie in any positions.

From (2) and (3) the magnitude R of the resultant and the angle θ which it makes with the positive x-axis are given by

(4) $$R = \sqrt{R_x^2 + R_y^2},\qquad \tan\theta = R_y \div R_x;$$

where, in case of ambiguity, the quadrant in which θ lies is determined by the signs of R_x and R_y in an obvious manner.

58. The Projection Theorem. We can now generalize the preceding results and prove the following important theorem:

The sum of the projections on any straight line l, of a broken line whose segments are taken in order so that the terminal point of each segment is the initial point of the next, is equal to the projection on l of a line segment joining the initial point of the first segment of the broken line to the terminal point of the last segment.

Proof. (1) The theorem is true when the broken line consists of two segments, for using the notation of § 14, in any

figure, $\text{Proj}_l\, AB = A'B'$, $\text{Proj}_l\, BC = B'C'$, $\text{Proj}_l\, AC = A'C'$ and $A'B + B'C' = A'C'$, whatever the order of points $A'B'C'$.

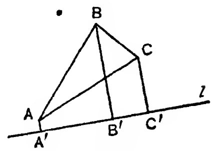


FIG. 60

(2) If the theorem is true for $n-1$ segments it is also true for n segments. Let A_1A_n be the straight line joining the initial point of the first segment with the terminal point of the $(n-1)$th segment. Then by hypothesis:

$$\text{Proj}_l\, A_1A_2 + \text{Proj}_l\, A_2A_3 + \cdots + \text{Proj}_l\, A_{n-1}A_n = \text{Proj}_l\, A_1A_n.$$

Now by (1), $\text{Proj}_l\, A_1A_n + \text{Proj}_l\, A_nA_{n+1} = \text{Proj}_l\, A_1A_{n+1}$; hence,

$$\text{Proj}_l\, A_1A_2 + \cdots + \text{Proj}_l\, A_{n-1}A_n + \text{Proj}_l\, A_nA_{n+1} = \text{Proj}_l\, A_1A_{n+1}.$$

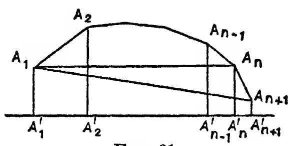


FIG. 61

(3) The theorem is true for 2 segments; hence it is true for 3 segments, consequently for 4 segments, etc., for any number.

59. Application to Forces and Velocities. The results of § 58 apply to forces and velocities, since the resultant of any number of forces (or velocities) is found by forming a broken line whose sides are equal and parallel to the given forces (or velocities), the initial point of each force (or velocity) being placed at the terminal point of the preceding one. The resultant is represented by the directed line segment connecting the initial point of the first to the terminal point of the last.

The result of § 58, applied to forces, exactly as in § 57, gives the components of the resultant R on the two axes, in terms of given forces F', F'', F''', etc., which make angles α', α'', α''', etc., with the positive end of the x-axis, as follows:

(1) $R_x = F'_x + F''_x + F'''_x + \cdots = F' \cos \alpha' + F'' \cos \alpha'' + \cdots,$

(2) $R_y = F'_y + F''_y + F'''_y + \cdots = F' \sin \alpha' + F'' \sin \alpha'' + \cdots.$

The magnitude R of the resultant and the angle θ which it makes with the positive x-axis are given by

(3) $R = \sqrt{R^2_x + R^2_y}, \qquad \tan \theta = R_y \div R_x.$

The signs of R_x and R_y determine the quadrant in which θ lies.

EXERCISES XXIV. — COMPOSITION AND RESOLUTION OF VECTORS

1. Find the components R_x and R_y of the resultant of two forces $(F' = 12, \alpha' = 30°)$ and $(F'' = 20, \alpha'' = 60°)$.

2. Find the magnitude R and the direction θ of the resultant of Ex. 1.

3. Find the resultant (R, θ) of three forces $(100, 350°)$, $(150, 490°)$, $(200, 720°)$, where (F, α) indicates a force of magnitude F and direction α.

4. Find the resultant (R, θ) of three velocities $(25, 20°)$, $(10, 210°)$, $(18, 325°)$, where (v, α) indicates a velocity of magnitude v and direction α.

5. If a force of intensity F makes an angle α with any line in space, show that the component of F along that line is $F \cos \alpha$. Draw a figure.

6. A force is often indicated by stating its two components, in the order (F_x, F_y), in parentheses, separated by a comma. Find the magnitude and the direction of each of the following forces:

(*a*) $(5, -10)$. (*b*) $(-42.5, 25.64)$. (*c*) $(-6, -8)$.
(*d*) $(-48.6, -72.9)$. (*e*) $(50.8, -32.9)$. (*f*) $(-42.2, -54.6)$.

7. Show that, in general, the magnitude F and the direction α of a force whose components on the axes are F_x and F_y are given by

$$F = \sqrt{F_x^2 + F_y^2}, \qquad \tan \alpha = F_y \div F_x,$$

while the quadrant in which F lies is given by the algebraic signs of $\sin \alpha = F_y \div F$ and $\cos \alpha = F_x \div F$.

8. If a force of intensity 12 makes angles whose cosines are 2/3, 1/3, and 2/3 with three mutually perpendicular lines Ox, Oy, and Oz, respectively, show that the components on these three lines are 8, 4, 8, respectively.

9. If the cosines of the angles which a force F makes with three mutually perpendicular lines are $\cos \alpha$, $\cos \beta$, and $\cos \gamma$, respectively, show that the components of F on those lines are $F \cos \alpha$, $F \cos \beta$, $F \cos \gamma$, respectively.

10. If the components of a force on three mutually perpendicular lines in space are 2, 3, and 6, respectively, show that the intensity of their resultant is represented by the diagonal of the rectangular parallelopiped determined by the components, and compute its value.

11. Show that the cosine of the angle made by the force of Ex. 10 with the first of the three mutually perpendicular lines is 2/7. Find the cosines of the angles which the force makes with each of the other two perpendicular lines.

12. If a force has components A, B, and C on three mutually perpendicular lines Ox, Oy, Oz, show that the intensity of the force is $R = \sqrt{A^2 + B^2 + C^2}$. Show that the cosines of the angles which the force makes with Ox, Oy, and Oz are A/R, B/R, and C/R, respectively.

60. Uniform Circular Motion. The importance of the functions of large angles and of negative angles is well illustrated by the simple problem of uniform rotation. Let M be a point of a rotating body, at a distance $a = OM$ (in feet) from the axis of rotation. We have seen (§ 45) that it is convenient to measure the angle θ, between the initial position OA and the position OM, in *radians*. Then the arc s which subtends θ is

(1) $s = a\theta$, (θ in radians, a and s in feet);

and θ is proportional to the time t (in seconds) after M was at A:

(2) $\theta = kt$, whence $s = akt$, (k a constant),

where the constant k is the angle formed in one second, *i.e.* the angular speed is k radians per second.

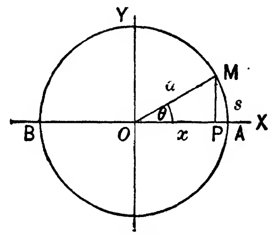


FIG. 62

The values of the coördinates, x and y, of M, are

(3) $x = a\cos\theta, \quad y = a\sin\theta$;

or,

(4) $x = a\cos kt, \quad y = a\sin kt.$

It is evident that in any circular motion, the value of θ will exceed π as soon as t exceeds π/k; hence large angles occur very naturally in (3) and (4).

If the rotation is clockwise, it is counted (§ 37) as negative; and all values of θ and the number k are negative. Then the angles in (3) and (4) are negative.

61. Period. Amplitude. The total time T for one revolution is called the **period** of the rotation. Since the angular speed is k radians per second, and since one revolution is 2π radians, $T = 2\pi/k$, in seconds.

If t is increased by the amount T, the angle θ is increased by one revolution or 2π radians. Hence, by (3), x and y are not changed since both the sine and the cosine have the same values for any two congruent angles.

The quantity a, the radius of the circular path, is called the **amplitude**.

62. Vibration. Simple Harmonic Motion. A point on a vibrating stretched cord moves back and forth in a manner similar

to the motion of the projection of M, § 60, on the x-axis. It is assumed in Physics that any point of such a steadily vibrating body actually moves precisely as this projection:

(1) $$x = a \cos kt.$$

The motion of the projection of M on the y-axis is precisely similar to the preceding; it is

(2) $$y = a \sin kt.$$

The kind of motion described by (1) or by (2) is called **simple harmonic motion** (S. H. M.). As in § 61, the quantity a is called the **amplitude**, and $T = 2\pi/k$ is called the **period** of the S. H. M. The moving point returns to its original position every $2\pi/k$ seconds, as in § 61; *i.e.* $T = 2\pi/k$ is the time of one complete vibration.

EXERCISES XXV. — CIRCULAR MOTION VIBRATION

1. Show that the coördinates (x, y) of the point M of § 60, for a uniform rotation of angular speed 1, are $x = a \cos t$, $y = a \sin t$. See Ex. 18, p. 71.

2. Show that the period of the rotation of Ex. 1 is 2π.

3. Find the values of x and y in Ex. 1 with $a = 10$, when $t = 1$, 2, $\pi/2$, 3, π, 4, 2π, 7. Plot these pairs of values of x and y, and show that the points lie on the circular path.

4. A recording instrument often used in Physical laboratories consists of a cylinder which is covered with a paper painted with lampblack. A fine needle is attached to a vibrating body, such as a tuning fork, and this needle is allowed to touch the lampblacked surface while the cylinder is slowly rotated.

If the apparatus is adjusted so that the needle would trace an element of the cylinder if the cylinder were at rest, show that the curve actually traced on the moving cylinder resembles the curve $y = \sin x$.

5. If the cylinder of Ex. 4 rotates so that a point on its surface travels a distance of k units per second, show that the curve traced on the blackened paper is precisely the curve $y = a \sin kt$, where $2\pi/k$ is the period of the tuning fork, if the amplitude of the vibration remains constant.

6. If a tuning fork makes 256 complete vibrations per second, show that its period is $T = 1/256$.

7. Show that the motion of a point on the tuning fork of Ex. 6 is described by the equation $y = a \sin kt$ where $k = 2\pi/T = 512\pi$, *i.e.* by the equation $y = a \sin 512\pi t$.

8. Plot the curve which represents the equation $y = a \sin 512\,\pi t$ for the value $a = 1/100$ (ft.) $= 1$ large unit on the y-axis.

9. If two vibrations in the same line take place simultaneously, the displacement of a point in the vibrating body is the sum of the displacements due to the two vibrations taken separately.

Show that the equation $y = a \sin kt + b \cos kt$ represents such a compound vibration.

10. Plot the curve which represents each of the following compound vibrations:

(a) $y = 2\cos 3\,t + 3\sin 3\,t$; (b) $y = \sin 2\,t - \cos 3\,t$;
(c) $y = \cos 5\,t - 2\sin 2\,t$; (d) $y = 2\sin 3\,t + 5\cos 2\,t$.

11. The **pitch** of a screw is the distance it moves parallel to its axis when the head makes one complete turn; *i.e.* the distance between two turns of the screw thread.

Find the distance the screw moves when the head turns through an angle of 230°, if the pitch is 1/20 in.

[Note. Instruments of precision for measuring distances accurately are made on this principle, and are called **micrometers.**]

12. Find the pitch of a screw that moves through a distance 1/8 in. in turning through an angle of 1200°.

13. A spiral stairway has a railing similar to the thread of a screw. If the pitch is 10 ft., find the angular width of steps that rise 7 in. How large must the inner radius of the base be made to make the net width of the inner tread 5 in.?

14. If the base of a spiral similar to a screw thread has its center on the origin of a pair of rectangular axes Ox and Oy, and passes through a point of Ox at a distance a from O, and if the vertical distance from this base is denoted by h, show that

$$x = a\cos\theta, \quad y = a\sin\theta, \quad h = p\theta/360,$$

where p is the pitch and θ is the angle, in degrees, through which the point (x, y) turns.

15. If the point of Ex. 14 moves with uniform speed, so that $\theta = kt$, where k is a constant, show that

$$x = a\cos kt, \quad y = a\sin kt, \quad h = pkt/360.$$

CHAPTER VI

THE ADDITION FORMULAS

63. Reduction of $A \cos \alpha \pm B \sin \alpha$. — Formulas. Such expressions as $A \cos \alpha \pm B \sin \alpha$ arise in various connections: thus a combination of two vibrations gives this kind of a form.

Another very different connection in which such expressions arise is in resolution of forces. If a force of magnitude A makes an angle α with the positive x-axis, while another force B makes an angle of $\alpha + 90°$ with the x-axis, the x-component R_x of their resultant R is

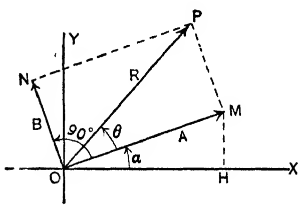


FIG. 63

(1) $R_x = A \cos \alpha + B \cos (\alpha + 90°)$
$= A \cos \alpha - B \sin \alpha;$

and the y-component R_y of R is

(2) $R_y = A \sin \alpha + B \sin (\alpha + 90°)$
$= A \sin \alpha + B \cos \alpha.$

In both these cases, it is possible, and advantageous, to express the sum of the two terms as the product of a single number times the cosine (or the sine) of a single angle. In the case of forces this is obvious; for in Fig. 63 we have, by § 48,

(3) $\quad R_x = R \cos (\alpha + \theta), \qquad R_y = R \sin (\alpha + \theta),$

where θ is the angle between A and R. Inserting these values in (1) and (2), we find:

(4) $\quad \boldsymbol{R \cos (\alpha + \theta) = A \cos \alpha - B \sin \alpha;}$

(5) $\quad \boldsymbol{R \sin (\alpha + \theta) = A \sin \alpha + B \cos \alpha.}$

Moreover, from the figure, $A = R \cos \theta$, $B = R \sin \theta$.

Substituting these values in (4) and (5) and dividing through by R, we find:

(6) $\quad \boldsymbol{\cos (\alpha + \theta) = \cos \alpha \cos \theta - \sin \alpha \sin \theta.}$

(7) $\quad \boldsymbol{\sin (\alpha + \theta) = \sin \alpha \cos \theta + \cos \alpha \sin \theta.}$

The formulas (6) and (7) are often called the **addition formulas.***

To reduce $A \cos \alpha - B \sin \alpha$ to the form $R \cos(\alpha + \theta)$, R and θ are found from the relations $R = \sqrt{A^2 + B^2}$, $\tan \theta = B/A$. Likewise $A \sin \alpha + B \cos \alpha$ reduces to $R \sin(\alpha + \theta)$ by (5).

64. Illustrative Examples.

Example 1. To reduce $2 \cos \alpha - 3 \sin \alpha$ to the form $R \cos(\alpha + \theta)$, we may use that part of Fig. 63 which shows $A (= 2)$, $B (= 3)$, and R. That is, we place lines of length 2 and 3, respectively, at right angles; then R is the diagonal of the rectangle they determine, and θ is the angle between $A (= 2)$ and R:

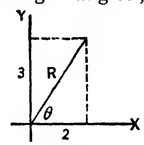


FIG. 64

$$R = \sqrt{A^2 + B^2} = \sqrt{2^2 + 3^2} = \sqrt{13};$$

and $\quad \tan \theta = B/A = 3/2 = 1.5;$

hence $\quad \theta = 56° \, 18'.6,$

and $\quad 2 \cos \alpha - 3 \sin \alpha = \sqrt{13} \cos(\alpha + 56° \, 18'.6).$

Example 2. Reduce the combination of two simple harmonic motions $3 \sin t + 4 \cos t$ to the form $R \sin(t + \theta)$. To find R and θ, draw a figure as in Ex. 1, with $A = 3$ and $B = 4$; then $R = 5$, and $\tan \theta = 4/3$ or $\theta = 53° \, 7'.8$. Hence, $3 \sin t + 4 \cos t = 5 \sin(t + 53° \, 7'.8)$.

The latter form in itself represents a simple harmonic motion, for if a point P moves on a circular path of radius 5 with constant angular speed unity (*i.e.* 1 radian per second), and if the time t is calculated (in sec.) from the time P was at A, where $\angle XOA = \theta = 53° \, 7'.8$, we have

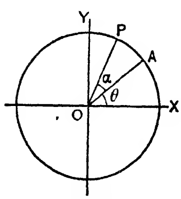


FIG. 65

$$\alpha = t \text{ and } x = 5 \cos(\alpha + \theta), \quad y = 5 \sin(\alpha + \theta).$$

Hence the motion of the projection of P on the y-axis is given by $\quad y = 5 \sin(t + \theta), \quad \theta = 53° \, 7'.8$

It follows that the sum of two simple harmonic motions $3 \sin t$ and $4 \cos t$ is a new simple harmonic motion. This fact is easily generalized to the case $A \sin kt + B \cos kt$:

$$y = A \sin kt + B \cos kt = R \sin(kt + \theta),$$

where $R = \sqrt{A^2 + B^2}$, and $\tan \theta = B/A$.

The angle θ is called the **phase angle**; in the cases considered in § 62, this angle was either 0 or $\pi/2$. To find the time when the moving point P was on the positive x-axis, we set $y = 0$; *i.e.* $\sin(kt + \theta) = 0$; this gives $t = -\theta/k$. This time $t = -\theta/k$ is called the **phase**.

* A consideration of one or two special cases shows that the sine of the sum of two angles is not equal to the sum of their sines: $\sin(\alpha + \beta) \neq \sin \alpha + \sin \beta$; *e.g.* $\sin 90° \neq \sin 60° + \sin 30°$, $\sin 180° \neq \sin 120° + \sin 60°$, $\sin 60° \neq 2 \sin 30°$. Similarly, show, by a trial, that $\cos(\alpha + \beta) \neq \cos \alpha + \cos \beta$.

65. Independent Proof of Addition Formulas. The formulas (6) and (7) of § 63 may be proved without reference to forces.

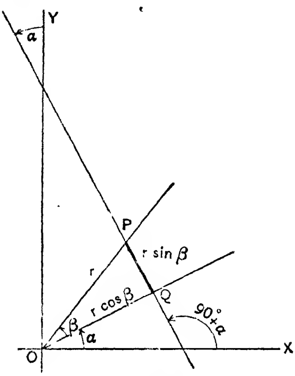


FIG. 66

Let α and β be any two acute angles; place their sum on coördinate axes. Let P be any point on the terminal side of $\alpha+\beta$ at a distance $r > 0$ from the origin. Draw PQ perpendicular to the terminal side of α. In Fig. 66, $\alpha+\beta$ is acute; in Fig. 67, $\alpha+\beta$ is obtuse.

In Fig. 66, the length of OP is r, and the line OP makes an angle $\alpha+\beta$ with the x-axis, while the angle between OP and the y-axis is $90° - (\alpha+\beta)$. In Fig. 67, OP makes an angle $(\alpha+\beta)$ with the x-axis and an angle of $(\alpha+\beta) - 90°$ with the y-axis. The length of OQ is $r \cos \beta$ and the line OQ makes an angle α with the x-axis while the angle between OQ and the y-axis is $90° - \alpha$. The length of QP is $r \sin \beta$, and the line QP makes an angle $90° + \alpha$ with the x-axis and an angle α with the y-axis. In either figure

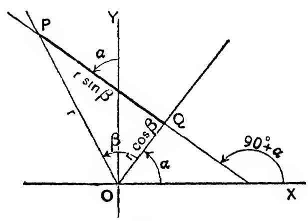


FIG. 67

(1) $$\text{Proj}_y\, OP = \text{Proj}_y\, OQ + \text{Proj}_y\, QP.$$

(2) $$\text{Proj}_x\, OP = \text{Proj}_x\, OQ + \text{Proj}_x\, QP,$$

Since $OQ = r \cos \beta$, $QP = r \sin \beta$, (1) becomes, by § 48,

(3) $$\begin{aligned} r \sin(\alpha+\beta) &= OQ \sin \alpha + QP \cos \alpha \\ &= (r \cos \beta) \sin \alpha + (r \sin \beta) \cos \alpha; \end{aligned}$$

and similarly, (2) becomes

(4) $$\begin{aligned} r \cos(\alpha+\beta) &= OQ \cos \alpha + QP(-\sin \alpha) \\ &= (r \cos \beta) \cos \alpha - (r \sin \beta) \sin \alpha. \end{aligned}$$

Dividing equations (3) and (4) through by r, we have, as in § 63:

I. $$\boldsymbol{\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta,}$$

II. $$\boldsymbol{\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta.}$$

66. Extension to Angles of Any Size. The formulas (I) and (II), § 65, have been proved to hold when α and β are any acute angles whatever; the student may show by direct substitution that they hold when either α or β, or both, are zero.

These formulas still hold when α and β are any angles whatever (zero, positive, or negative, of any magnitude). For any particular case, they may be proved by projection as in § 65. A general proof by mathematical induction follows.

Lemma. If there are any two angles for which formulas I and II hold, then they will hold also for the angles obtained by increasing one or both of these angles by 90°.

Proof of the lemma: Let $\alpha = a$, $\beta = b$, be two angles for which I and II hold. We have to show:

(1) $\sin(a + 90° + b) = \sin(a + 90°)\cos b + \cos(a + 90°)\sin b.$

(2) $\sin(a + b + 90°) = \sin a \cos(b + 90°) + \cos a \sin(b + 90°).$

(3) $\sin(a + 90° + b + 90°) = \sin(a + 90°)\cos(b + 90°) + \cos(a + 90°)\sin(b + 90°).$

(1′) $\cos(a + 90° + b) = \cos(a + 90°)\cos b - \sin(a + 90°)\sin b.$

(2′) $\cos(a + b + 90°) = \cos a \cos(b + 90°) - \sin a \sin(b + 90°).$

(3′) $\cos(a + 90° + b + 90°) = \cos(a + 90°)\cos(b + 90°) - \sin(a + 90°)\sin(b + 90°).$

These may all be verified by means of the relations of § 53, which have been proved to hold for all values of θ. Thus, to prove (1):

$$\sin(a + 90° + b) = \sin(90° + a + b) = \cos(a + b) = \cos a \cos b - \sin a \sin b.$$

$$\sin(a + 90°)\cos b + \cos(a + 90°)\sin b = \cos a \cos b - \sin a \sin b.$$

Similarly we may prove 2, 3, 1′, 2′, 3′; this establishes the lemma.

We know that the formulas I and II hold for $0° \leqq \alpha < 90°$, $0° \leqq \beta < 90°$; apply the lemma and we know that they hold for $0° \leqq \alpha < 180°$, $0° \leqq \beta < 180°$. Apply the lemma to this result and we know that they hold for $0° \leqq \alpha < 270°$, $0° \leqq \beta < 270°$, apply the lemma to this result and we know that they hold for $0° \leqq \alpha < 360°$, $0° \leqq \beta < 360°$. That is, we have now proved that the two formulas I and II hold for all angles between 0° and 360° (0° included).

Next let c and d be any two angles whatever, zero, positive, or negative; then by § 41 there is an angle $\alpha \cong c$, and an angle $\beta \cong d$, where α and β are both between 0° and 360° (0° included). Then $c + d \cong \alpha + \beta$. Hence, since I and II hold for α and β, and since the value of any trigonometric function is the same for any two congruent angles, it follows that I and II hold for c and d.

EXERCISES XXVI.—ADDITION FORMULAS

1. Given $\sin\alpha = 3/5$, $\sin\beta = 5/13$; find $\sin(\alpha+\beta)$.

(*a*) When α and β are both acute; (*b*) when α and β are both obtuse.

2. Find $\sin(45^\circ + x)$, $\cos(45^\circ + x)$, $\sin(30^\circ + x)$, $\cos(30^\circ + x)$ in terms of $\sin x$ and $\cos x$.

3. Given that x and y are both obtuse angles and that $\sin x = 1/2$, $\sin y = 1/3$; find $\sin(x+y)$ and $\cos(x+y)$.

4. Use the addition formulas to express $\sin(90^\circ+\alpha)$ and $\cos(90^\circ+\alpha)$ in terms of $\sin\alpha$ and $\cos\alpha$.

5. Prove that $\sin(60^\circ + x) - \cos(30^\circ + x) = \sin x$.

6. Express $\sin(\alpha+\beta+\theta)$ in terms of sines and cosines of α, β, and θ.

[Hint. Let $\phi = \alpha + \beta$ and obtain $\sin(\phi + \theta)$; then replace ϕ by its value, $\alpha + \beta$.]

7. Express $\cos(\alpha+\beta+\theta)$ in terms of sines and cosines of α, β, and θ.

8. Reduce the combination of two simple harmonic motions $5\cos t - 12\sin t$ to the form $r\cos(t+\theta)$.

9. Reduce $3\sin t + 4\cos t$ to the form $r\sin(t+\theta)$.

10. Reduce each of the following to the product of a number and the sine or the cosine of a single angle:

(*a*) $\sin x - 2\cos x$.
(*b*) $3\cos y - 4\sin y$.
(*c*) $5\cos\theta + 12\sin\theta$.
(*d*) $3\sin t - 3\cos t$.
(*e*) $\sqrt{3}\cos x - \sin x$.
(*f*) $\sin y + .5\cos y$.
(*g*) $.7\cos\theta - \sin\theta$.
(*h*) $.55667\sin c + .5\cos c$.

11. Given two forces of intensities 2 and 3 that make angles of 30° and 120°, respectively, with the positive x-axis; find the horizontal and the vertical components of their resultant without finding the resultant itself; find the same quantities by using the resultant.

12. Given the two simple harmonic motions $x = 2\cos t$ and $x = 5\sin t$, find a single S. H. M. which represents their sum. Find its amplitude; its phase-angle; its phase.

13. Express the S. H. M. $x = 6\sin(t + 60^\circ)$ as the sum of two S. H. M.'s whose phase-angle is 0° or 90°.

14. Given $.56\sin c + .5\cos c = -.34$, find an angle θ, and a number r, such that $.56\sin c + .5\cos c = r\sin(c+\theta)$, by means of § 63. Then, from $r\sin(c+\theta) = -.34$, find $\sin(c+\theta)$, and therefore (from the Tables) find $c+\theta$. Hence find c.

67. Functions of the Difference of two Angles. If ϕ and ψ are any two angles whatever, zero, positive, or negative, then formulas I, II, § 65 hold for $\alpha = \phi$, $\beta = -\psi$; hence

$$\begin{aligned}\sin(\phi - \psi) &= \sin\phi\cos(-\psi) + \cos\phi\sin(-\psi)\\ &= \sin\phi\cos\psi - \cos\phi\sin\psi.\end{aligned}$$

Likewise,
$$\begin{aligned}\cos(\phi - \psi) &= \cos\phi\cos(-\psi) - \sin\phi\sin(-\psi)\\ &= \cos\phi\cos\psi + \sin\phi\sin\psi.\end{aligned}$$

Since ϕ and ψ are any angles whatever, we have proved that

III. $\boldsymbol{\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta,}$

IV. $\boldsymbol{\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta,}$

hold for any two angles whatever.

68. Double Angles. Since formulas I and II, § 65, are true for all angles, they hold when $\alpha = a$, any angle whatever, and $\beta = a$, the same angle; hence,

$$\sin(a + a) = \sin a\cos a + \cos a\sin a,$$

and
$$\cos(a + a) = \cos a\cos a - \sin a\sin a.$$

Therefore the following formulas hold for any angle whatever:

V. $\boldsymbol{\sin 2\alpha = 2\sin\alpha\cos\alpha;}$

VI. $\boldsymbol{\cos 2\alpha = \cos^2\alpha - \sin^2\alpha;}$

or, since $\sin^2\alpha + \cos^2\alpha = 1,$

VI a. $\boldsymbol{\cos 2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1.}$

69. Tangent of a Sum or of a Difference. Since formulas I and II hold for all values of α and β, the formula

$$\frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} = \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta - \sin\alpha\sin\beta}$$

holds good for all values of α and β except those which make $\cos(\alpha + \beta) = 0$, *i.e.* except when $\alpha + \beta \cong 90°$, or $270°$. For example, it does not hold for $\alpha = 47°$, $\beta = 43°$.

Dividing both numerator and denominator by $\cos\alpha\cos\beta$, we obtain the formula

VII. $$\boldsymbol{\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta},}$$

which holds for all angles α and β such that α, β, and $\alpha + \beta$ have tangents.

Similarly from formulas III and IV, we obtain

$$\text{VIII.} \qquad \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta},$$

which holds for all angles α and β such that α, β, and $\alpha - \beta$ have tangents.

From formulas V and VI, we find

$$\text{IX.} \qquad \tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha},$$

which holds for every angle α such that α and 2α have tangents. The same formula may be obtained directly from VII by putting α in place of β.

70. Applications. The formulas of this chapter are frequently used for reducing expressions whose values are to be calculated, to a form in which logarithms may be used conveniently.

Example. Suppose the height of an object CD is to be determined and that it is not convenient to measure a base line bearing directly toward the base C. The following method is then sometimes employed. The angle of elevation α is measured from some convenient point A; a line $AB = d$ is then measured at right angles to the line AC; finally the angle of elevation, β, is observed from B. The height h can then be determined by solving a succession of triangles. With the aid of the formulas of this chapter it is frequently possible in such cases to reduce the calculation to a single logarithmic computation. In the case just mentioned we have

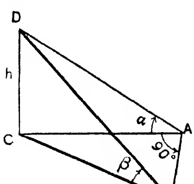


FIG. 68

$$BC = h\operatorname{ctn}\beta, \qquad AC = h\operatorname{ctn}\alpha,$$

$$\begin{aligned} d^2 = \overline{BC}^2 - \overline{AC}^2 &= h^2(\operatorname{ctn}^2\beta - \operatorname{ctn}^2\alpha) \\ &= h^2(\operatorname{ctn}\beta - \operatorname{ctn}\alpha)(\operatorname{ctn}\beta + \operatorname{ctn}\alpha) \\ &= h^2\frac{(\sin\alpha\cos\beta - \cos\alpha\sin\beta)(\sin\alpha\cos\beta + \cos\alpha\sin\beta)}{\sin^2\alpha\sin^2\beta}; \end{aligned}$$

hence, using formulas I and III, we have

$$h = \frac{d\sin\alpha\sin\beta}{\sqrt{\sin(\alpha-\beta)\sin(\alpha+\beta)}}.$$

Let the student show, by opening a book and studying the dihedral angle formed by two leaves, that $\alpha > \beta$.

EXERCISES XXVII. — SECONDARY FORMULAS — APPLICATIONS

1. Find $\sin 15°$, $\cos 15°$, $\tan 15°$ from the known values of $\sin 30°$, $\cos 30°$, $\tan 30°$, and $\sin 45°$, $\cos 45°$, $\tan 45°$. [HINT. $15° = 45° - 30°$.]

2. Find $\tan 75°$, $\tan 105°$, $\sin 165°$, $\cos 255°$. [HINT. $75° = 45° + 30°$, etc.]

3. Given $\sin 36° 52' = .6$; find the sine, cosine, and tangent of $66° 52'$; find $\sin 73° 44'$.

4. Given $\tan 26° 34' = .5$; find sine, cosine, tangent of $71° 34'$; find $\tan 53° 8'$.

5. Given $\sin \alpha = 5/13$ and $90° < \alpha < 180°$; $\cos \beta = 8/17$ and $0° < \beta < 90°$; find $\sin(\alpha - \beta)$, $\cos(\alpha - \beta)$, $\tan(\alpha + \beta)$, $\sin 2\alpha$, $\cos 2\beta$.

6. Given $\tan \alpha = 15/8$ and $0° < \alpha < 90°$; $\cos \beta = 4/5$ and $270° < \beta < 360°$; find $\sin(\alpha - \beta)$, $\cos(\beta - \alpha)$, $\tan 2\alpha$, $\cos 2\beta$.

7. Given $\sin \alpha = 1/3$ and $0° < \alpha < 180°$; find $\sin(135° - \alpha)$ and $\tan 2\alpha$.

8. The angular elevation of an object from an upper window is observed to be α. The angular elevation from a point on the ground h feet directly beneath the window is β. Show that the height of the object is $h \sin \beta \cos \alpha \div \sin(\beta - \alpha)$.

9. To determine the difference in elevation of two stations, a flagstaff of known height h is held at the upper of the two stations and the angles of elevation of its top and bottom are observed to be α and β, respectively. Show that the difference in elevation of the two stations is

$$h \tan \beta \div (\tan \alpha - \tan \beta);$$

reduce this expression to a form convenient for logarithmic computation.

10. A tree leans directly toward two points of observation distant a and b, respectively, from its foot. The angles of elevation of the top of the tree from these two points are α and β. Show that the perpendicular height of the tree is $(b - a) \div (\operatorname{ctn} \beta - \operatorname{ctn} \alpha)$; reduce this expression to a form suitable for logarithmic computation.

11. Prove that $\sin 3\alpha = \sin \alpha(3 - 4\sin^2 \alpha) = \sin \alpha(4\cos^2 \alpha - 1)$, and state for what values of α it holds. Use formulas I and II.

12. Prove that $\cos 3\alpha = \cos \alpha(4\cos^2 \alpha - 3) = \cos \alpha(1 - 4\sin^2 \alpha)$, and state for what values of α it holds. Use formulas I and II.

13. Prove that $\tan 3\alpha = (3\tan \alpha - \tan^3 \alpha) \div (1 - 3\tan^2 \alpha)$; show that it holds for all values of α such that α and 3α have tangents.

14. Prove that $\sin(45° + \alpha)\sin(45° - \alpha) = (1/2)\cos 2\alpha$ for all values of α.

15. Prove that $\sin(\alpha + \beta)\sin(\alpha - \beta) = \sin^2 \alpha - \sin^2 \beta$ for all values of α and β.

16. Prove that $\cos(\alpha + \beta)\cos \beta + \sin(\alpha + \beta)\sin \beta = \cos \alpha$.

71. Functions of Half-angles. The formulas

$$\cos^2 \alpha + \sin^2 \alpha = 1 \qquad \text{and} \qquad \cos^2 \alpha - \sin^2 \alpha = \cos 2\alpha$$

are true for all values of α. If we subtract one of these from the other, and if we also add them, we obtain the formulas:

(1) $2 \sin^2 \alpha = 1 - \cos 2\alpha$, (2) $2 \cos^2 \alpha = 1 + \cos 2\alpha$.

These formulas are true for all values of α; for $\alpha = \alpha'/2$ they become $2 \sin^2 (\alpha'/2) = 1 - \cos \alpha'$ and $2 \cos^2 (\alpha'/2) = 1 + \cos \alpha'$, or since these are true for all values of α', we may write

(3) $\sin (\alpha/2) = \pm \sqrt{\dfrac{1 - \cos \alpha}{2}}$, (4) $\cos (\alpha/2) = \pm \sqrt{\dfrac{1 + \cos \alpha}{2}}$;

which hold good for all values of α. The same formulas may be obtained from VI a by solving for $\sin (\alpha'/2)$, or for $\cos (\alpha'/2)$, after putting $\alpha'/2$ for α.

From (3) and (4) we get by division

(5) $\tan \alpha/2 = \pm \sqrt{\dfrac{1 - \cos \alpha}{1 + \cos \alpha}} = \dfrac{\sin \alpha}{1 + \cos \alpha} = \dfrac{1 - \cos \alpha}{\sin \alpha}$,

which hold for all values of α except when a denominator vanishes. The ambiguity of sign of the radical is determined in a given case by the fact that $\tan (\alpha/2)$ is positive or negative according as $\alpha/2$ is or is not in the 1st or 2d quadrant.

The relations between an angle and its half are frequently useful in problems that relate to a chord of a circle and the angle which it subtends at the center; this occurs, for example, in laying out railroad curves where it is convenient to make measurements along chords of the curve. This is illustrated in some of the exercises below. The relations are also useful in simplifying trigonometric expressions and in adapting formulas to logarithmic computation.

EXERCISES XXVIII. — HALF-ANGLE FORMULAS

1. Find the sine, the cosine, and the tangent of $22° 30'$ from the known values of $\sin 45°$, $\cos 45°$, $\tan 45°$.

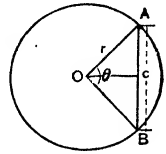
2. Find the sine, cosine, and tangent of $15°$.

3. Given that $\sin \alpha = 4/5$, and that α is an acute angle; find $\sin(\alpha/2)$ and $\tan(\alpha/2)$.

4. Given $\tan 26° 34' = 1/2$; find $\tan 13° 17'$.

5. Given $\tan 36° 52' = 3/4$; find sine, cosine, and tangent of $18° 26'$

6. If r denotes the radius of the circle in the accompanying figure, c a chord, and θ the angle which c subtends at the center; show that $\sin(\theta/2) = c/(2r)$.



7. In the figure, draw the line BD tangent to the circle, and AD perpendicular to BD from the opposite end of the chord BA. Show that (a) $\angle ABD = \theta/2$; (b) $BD = AB \cos(\theta/2) = 2r\sin(\theta/2)\cos(\theta/2) = r\sin\theta$.

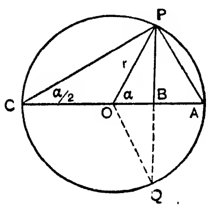
8. Prove that $\tan(45° + \alpha/2) = \sec\alpha + \tan\alpha$, if $\tan\alpha$ exists.

9. Prove that $\tan(45° + \alpha/2)\tan(45° - \alpha/2) = \tan 45°$ if $\tan\alpha$ exists.

10. Prove that $\tan(\alpha/2) + 2\sin^2(\alpha/2)\operatorname{ctn}\alpha = \sin\alpha$, if $\sin\alpha \neq 0$.

11. Prove that $\tan(\alpha/2) + \operatorname{ctn}(\alpha/2) = 2\csc\alpha$, if $\sin\alpha \neq 0$.

12. Prove that $[\sin(\alpha/2) + \cos(\alpha/2)]^2 = 1 + \sin\alpha$ for all values of α.



13. Prove that $[\sin(\alpha/2) - \cos(\alpha/2)]^2 = 1 - \sin\alpha$ for all values of α.

14. In the figure, COA is a diameter of a circle of radius r; $AOP = \alpha$ is any acute angle; $OCP = \alpha/2$, by geometry; and PB is perpendicular to OA. Show that

$$OB = r\cos\alpha,\quad BP = r\sin\alpha,\quad BA = r \operatorname{vers}\alpha,$$
$$CB = r(1+\cos\alpha),$$
$$CP = \sqrt{\overline{PB}^2 + \overline{CB}^2} = r\sqrt{2(1+\cos\alpha)}.$$

15. From Ex. 14, show that the functions of $\alpha/2$ can be read directly from the figure in the form:

$$\sin(\alpha/2) = \frac{r\sin\alpha}{r\sqrt{2(1+\cos\alpha)}} = \sqrt{\frac{1-\cos\alpha}{2}};$$

$$\cos(\alpha/2) = \frac{1+\cos\alpha}{\sqrt{2(1+\cos\alpha)}} = \sqrt{\frac{1+\cos\alpha}{2}};$$

$$\tan(\alpha/2) = \frac{\sin\alpha}{1+\cos\alpha} = \frac{\sqrt{1-\cos^2\alpha}}{1+\cos\alpha} = \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{1-\cos\alpha}{\sin\alpha}.$$

16. If a numerical value of any function of α is given, all the other functions of α and of $\alpha/2$ can be found geometrically from Ex. 14. Thus, if $\sin\alpha = 4/5$ is given, lay off $OP = 5$, $BP = 4$; then $OB = \sqrt{5^2 - 4^2} = 3$. Hence, $CB = 8$, $BA = 2$; and $CP = \sqrt{\overline{CB}^2 + \overline{BP}^2} = \sqrt{8^2 + 4^2} = \sqrt{80}$. It follows that

$$\sin\alpha = 4/5,\quad \cos\alpha = 3/5,\quad \tan\alpha = 4/3,$$
$$\sin(\alpha/2) = 4/\sqrt{80} = 1/\sqrt{5} = \sqrt{5}/5,$$
$$\cos(\alpha/2) = 8/\sqrt{80} = 2/\sqrt{5} = 2\sqrt{5}/5,$$
$$\tan(\alpha/2) = 4/8 = 1/2.$$

17. Find the remaining functions of α and those of $\alpha/2$ by means of Ex. 16, if $\cos\alpha = 5/13$; if $\tan\alpha = 1/3$.

18. The remaining functions of $(\alpha/2)$ and those of α can be found when any function of $\alpha/2$ is given from the figure of Ex. 14, by dropping a perpendicular from O to CP. Do this if $\tan(\alpha/2) = 3/4$.

19. Show that the results of Exs. 14–15 hold also if α is obtuse.

20. Since, in the figure of Ex. 14, by geometry $\overline{BP}^2 = CB \cdot BA$, show that
$$(1 + \cos\alpha)\operatorname{vers}\alpha = \sin^2\alpha.$$

21. Derive trigonometric formulas from the geometric identities (Ex. 14):
$$CB \cdot BA = \overline{BP}^2, \quad CA \cdot BA = \overline{AP}^2, \quad CA \cdot CB = \overline{CP}^2.$$

72. Factor Formulas. In adapting trigonometric formulas to logarithmic computation it is often desirable to express the sum (or difference) of two sines (or cosines) as the product of other functions.

Example 1. Reduce $\sin 35° + \sin 15°$ to the form $2\sin 25°\cos 10°$.

To do this, set $\quad x + y = 35°, \quad x - y = 15°,$

and solve for x and y: $\quad x = 25°, \; y = 10°.$

Then
$$\sin(x+y) = \sin x\cos y + \cos x\sin y,$$
$$\sin(x-y) = \sin x\cos y - \cos x\sin y;$$
whence, adding, $\sin(x+y) + \sin(x-y) = 2\sin x\cos y$;
substituting $x = 25°$, $y = 10°$, we get $\sin 35° + \sin 15° = 2\sin 25°\cos 10°$.
This method is general.

Example 2. Reduce $\sin s - \sin(s - c)$ to a product, where
$$s = (a + b + c)/2.$$

Let $x + y = s$, $x - y = s - c$; then $x = (a+b)/2$, $y = c/2$,
and
$$\sin(x+y) = \sin x\cos y + \cos x\sin y,$$
$$\sin(x-y) = \sin x\cos y - \cos x\sin y;$$
subtracting $\quad \sin(x+y) - \sin(x-y) = 2\cos x\sin y,$
whence $\quad \sin s - \sin(s-c) = 2\cos[(a+b)/2]\sin(c/2).$

EXERCISES XXIX. — FACTORING

1. Reduce each of the following forms to products:

(a) $\sin 70° - \sin 10°$. (b) $\sin 70° + \sin 50°$.

(c) $\sin 13° + \sin 41°$. (d) $\sin 34° - \sin 19°$.

(e) $\cos 26° - \cos 35°$. (f) $\sin 43° + \sin 28°$.

(g) $\cos 20° + \cos 10°$. (h) $\cos 51° - \sin 11°$.

(i) $\dfrac{\sin 15° + \cos 45°}{\cos 45° - \sin 15°}$. (j) $\dfrac{\sin 28° + \sin 12°}{\cos 28° + \cos 12°}$.

(k) $\dfrac{\sin 64° + \sin 16°}{\sin 64° - \sin 16°}$. (l) $\dfrac{\sin 80° - \sin 40°}{\cos 40° - \cos 80°}$.

2. Prove that $\cos(x+y) + \cos(x-y) = 2\cos x \cos y$.

3. Prove that $\cos(x+y) - \cos(x-y) = -2\sin x \sin y$.

4. Prove that

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

by substituting $A = x+y \quad B = x-y$ in Ex. 2.

5. Prove by means of Ex. 3 that

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}.$$

6. By the method of Example 1, § 72, show that

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}.$$

7. By the method of Example 2, § 72, show that

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}.$$

8. Prove $\dfrac{\sin x + \sin y}{\sin x - \sin y} = \tan\{(x+y)/2\}\operatorname{ctn}\{(x-y)/2\}$.

9. Prove $\dfrac{\cos x + \cos y}{\cos x - \cos y} = -\operatorname{ctn}\{(x+y)/2\}\operatorname{ctn}\{(x-y)/2\}$.

10. Prove $\dfrac{\sin\theta + \sin 2\theta}{\cos\theta - \cos 2\theta} = \operatorname{ctn}(\theta/2)$.

11. Prove $\dfrac{\sin(2x-3y) + \sin 3y}{\cos(2x-3y) + \cos 3y} = \tan x$.

12. $\sin(45° + x) + \sin(45° - x) = \sqrt{2}\cos x$.

13. $\sin 3x + \sin 5x = 2\sin 4x \cos x$.

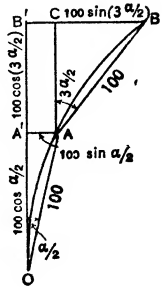
14. If $a + b + c = 2s$, show that

(a) $\cos(b-c) - \cos a = 2\sin(s-b)\sin(s-c)$;

(b) $\cos a - \cos(b+c) = 2\sin s \sin(s-a)$;

(c) $\dfrac{\sin s - \sin(s-c)}{\sin s + \sin(s-c)} = \dfrac{\tan(c/2)}{\tan\{(a+b)/2\}}$.

15. The so-called "method of offsets" for laying out a circular track is illustrated in the adjoining figure. The track OAB is tangent at O to OB', and the distances OA', $A'B'$, $A'A$, CB, are easily shown to be as marked in the figure, where $\alpha/2 = \angle AOA'$ is half the angle at the center subtended by a 100 ft. chord.



In practice, the line $OA'B'$ is run, and A' and B' marked. Show that $B'B$, the distance actually to be laid off from B', is

$$B'B = A'A + CB = 200\sin\alpha\cos(\alpha/2).$$

CHAPTER VII

TRIGONOMETRIC EQUATIONS INVERSE FUNCTIONS

PART I. IDENTITIES AND EQUATIONS

73. Identities in One Variable. The equation,

$$x^2 - 1 = (x - 1)(x + 1)$$

is satisfied when any number whatever is substituted for x; we say it is satisfied by *all values of* x. The equation

$$(x^2 - 1)/(x - 1) = x + 1$$

is satisfied by all values of x except $x = 1$. The student may verify both of these statements for $x = -2, -1, 1/2, 3/2, 2$, etc. Similarly, the equation $(1 + x)(1 - 1/x) = (x - 1)(1 + 1/x)$ is satisfied by every value of x except $x = 0$. The equation $\sin^2 x + \cos^2 x = 1$ is satisfied by every value of x; and the equation $\tan x \cos x = \sin x$ is satisfied by every value of x except $x =$ an odd multiple of $\pm 90°$. These are examples of identities: *Two expressions involving an unknown letter are said to be* **identically equal,** *or simply,* **identical,** *if they have the same value for every value of the unknown for which both are defined.** An equation whose sides are identically equal is called an **identity.**

74. Elementary Identities. An identity is to be regarded as a declaration to be proved: thus

$$\cos 2x \equiv (\cos x + \sin x)(\cos x - \sin x)$$

declares that for every angle x the cosine of twice that angle is equal to the product of the sum and difference of its cosine and its sine; this was proved in § 68. Among other identities

* The trigonometric functions $\sin x$ and $\cos x$ are defined for every value of x; $\tan x$ and $\sec x$, however, are not defined for $x =$ any odd multiple of 90°, while $\operatorname{ctn} x$ and $\csc x$ are not defined when $x =$ any even multiple of 90°. See p. 61. It is assumed that values of the unknown exist for which both sides are defined. A similar definition holds for identities in several variables.

that have been established are the Pythagorean relations, § 10, p. 16, the reciprocal relations, etc., § 6, p. 9.

75. Identities in Two Variables. In Chapter VI we found:

$$\sin(x+y) = \sin x \cos y + \cos x \sin y,$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y,$$

for all values of x and y. These are identities in two variables.

76. Illustrative Example. The truth of an identity is usually established by reducing both sides, either to the same expression, or to two expressions which are known to be identical.

Example 1. Prove that $1 - \sin\theta = \cos^2\theta/(1+\sin\theta)$ is an identity.

The right-hand side is not defined when $1 + \sin\theta = 0$. The left-hand side has a value for every value of θ. We are to show, then, that the two sides of the equation have the same value for every value of θ except those that make $1 + \sin\theta = 0$; *i.e.* except when $\theta \cong 270°$. To prove this we reduce the right-hand side to the left-hand side. Replace $\cos^2\theta$ by $1 - \sin^2\theta$, then $\cos^2\theta/(1+\sin\theta) = (1-\sin^2\theta)/(1+\sin\theta)$. Dividing the numerator by the denominator we obtain $1 - \sin\theta$, which is the left-hand side of the given equation. This division is permissible if $1 + \sin\theta \neq 0$.

EXERCISES XXX. — TRIGONOMETRIC IDENTITIES

Prove the truth of the following identities and state in each case the exceptional values of the variables, if any, for which one or both of the two sides are undefined.

1. $\cos^4 x - \sin^4 x + 1 = 2\cos^2 x.$
2. $\cos^3 x + \sin^3 x = (\sin x + \cos x)(1 - \sin x \cos x).$
3. $\tan(45° + x) - \tan(45° - x) = 2\tan 2x.$
4. $\sin 2x + \sin 2y = 2\sin(x+y)\cos(x-y).$
5. $\tan x + \operatorname{ctn} x = \sec x \csc x.$
6. $\sin 2x - \sin 2y = 2\cos(x+y)\sin(x-y).$
7. $(\sin x - \cos x)(\cos x - \sin x) = \sin 2x - 1.$
8. $2\cos x \sin y = \sin(x+y) - \sin(x-y).$
9. $2\sin x \cos y = \sin(x+y) + \sin(x-y).$
10. $(\sec x - \tan x)(1 + \sin x) = \cos x.$
11. $\sin^2 x \sec^2 x = \sec^2 x - 1.$
12. $(\sqrt{1+\sin x} - \sqrt{1-\sin x})^2 = 4\sin^2(x/2).$
13. $(\sqrt{1+\sin x} + \sqrt{1-\sin x})^2 = 4\cos^2(x/2).$
14. $2\cos x \cos y = \cos(x-y) + \cos(x+y).$
15. $2\sin x \sin y = \cos(x-y) - \cos(x+y).$

16. $\sin 3x = \sin x(3 - 4\sin^2 x) = \sin x\,(2\cos x - 1)(2\cos x + 1)$.

17. $\cos 3x = \cos x\,(4\cos^2 x - 3) = \cos x(1 - 2\sin x)(1 + 2\sin x)$.

18. $1 + \sin x - \cos 2x = \tan x(\cos x + \sin 2x)$.

19. $(1 + \cos 2x)\tan x = \sin 2x$.

20. $[\sin(x/2) - \cos(x/2)]^2 = 1 - \sin x = 2\cos^2(45° + x/2)$.

21. $[\sin(x/2) + \cos(x/2)]^2 = 1 + \sin x = 2\cos^2(45° - x/2)$.

22. $\sec(45° - x/2)\sec(45° + x/2) = 2\sec x$.

Prove that the following expressions are reciprocals:

23. $\sec x + \tan x$ and $\sec x - \tan x$.

24. $1 - \sin x$ and $\sec^2 x + \sec x \tan x$.

25. $1 + \cos x$ and $\csc^2 x - \csc x \operatorname{ctn} x$.

[NOTE. Two numbers are reciprocals if and only if their product is $+1$; in Ex. 23 we must prove that $(\sec x + \tan x)(\sec x - \tan x) = 1$. Just as in any other identity, values of x for which either side is meaningless are excluded. Values of x that make either of the given expressions vanish must be excluded; thus, in Ex. 24, $1 - \sin x = 0$ when $x \cong 90°$; and in Ex. 25, $1 + \cos x = 0$ when $x \cong 270°$.]

77. Conditional Equations. In the exercises of the preceding list it was frequently necessary to determine the values of x which would make a certain expression vanish. Thus in Exs. 24–25, the equations $1 - \sin x = 0$, $1 + \cos x = 0$, etc., were considered. These are not identities, since it is clear that there are values of x in each case for which the left-hand side is defined and for which that side is *different from zero.*

An equation in x which is not satisfied by all values of x for which each side is defined is called a **conditional equation,** or, when no ambiguity can arise, simply an equation.

Examples of conditional equations that are quite familiar are:

$$(a)\quad x^2 - 5x + 6 = 0,$$

which is satisfied by two and only two values of x: $x = 2$ and $x = 3$;

$$(b)\quad 8x^3 - 12x^2 + 6x = 1,$$

which is satisfied by one and only one value of x: $x = 1/2$;

$$(c)\quad 4\cos^4 x + \sin^2 2x = 2,$$

which is satisfied by $x = 45°$, $135°$, $225°$ $315°$, and all angles congruent to any one of these. This last equation, therefore, has an infinite number* of solutions, but nevertheless it is not an identity, since there exist values of x for which the two sides have two definite values that are different.

* There are an infinite number of things in a class of things, if, when you have counted out as many as you please, still others remain.

The purpose of what follows is to show how to find all* the solutions of certain simple forms of equations containing trigonometric functions of an unknown angle.

Any equation that is not an identity is to be regarded, not as a declaration to be proved, but rather as a question to be investigated and answered. Thus the equation,

$$8x^3 - 12x^2 + 6x = 1$$

implies the question, "Are there any values of x which make $8x^3 - 12x^2 + 6x$ equal to 1?" and the direction, "If so, find all of them." This is the meaning of the direction, "Solve the equation." This point of view is very important.

78. Illustrative Examples. The simplest trigonometric equations are of the form

$$\sin x = 1/2; \qquad \cos x = -4/5; \qquad \tan x = 5.3;$$
$$\text{ctn}\, x = -2; \qquad \sec x = \sqrt{2}; \qquad \csc x = -4/3.$$

The method of solving such equations is illustrated in the examples that follow:

Example 1. Solve the equation $\sin x = 1/2$.

We know that $x = 30°$ is a solution, and that any angle congruent to 30° must be a solution: $x = 30°, 390°, -330°$, etc. All these angles are included in the statement $x \cong 30°$; but there are still other solutions, since we know that the sine of an angle is also the sine of its supplement; the supplement of 30°, or 150°, must therefore be a solution, and hence all angles x such that $x \cong 150°$ are solutions. We shall show presently that there are no others.

Example 2. Solve the equation $\cos x = -4/5$.

The value $x = 143° 8'$ (approximately) is a solution, as may be verified by a table of cosines; hence other solutions are $x \cong 143° 8'$. Are there still others?

Example 3. Solve the equation $\tan x = 1/3$.

From the tables an approximate solution is found to be $x = 18° 26'$. Hence other solutions are $x \cong 18° 26'$. Are there still others?

79. General Principles. A general method of solving such equations depends upon the following theorems:

* We shall consider only real values of x, since in elementary work the *trigonometric* functions are not defined for imaginary values of the angle.

THEOREM I. *The equation $\sin x = s$, where $-1 < s < 1$, is satisfied by exactly two angles between $0°$ and $360°$.** Every angle congruent to either of these is a solution, and conversely, every solution of the equation is congruent to one or the other of these angles.

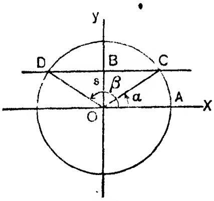


FIG. 69

The following construction will indicate the method of solution of such an equation for any particular value of s.

Draw a pair of coördinate axes and a unit circle whose center is at the origin. On the y-axis lay off $OB = s$ (above O if $s > 0$, below if $s < 0$); draw through B a line parallel to the x-axis. This line cuts the circle in two and only two points, C and D Draw the radii OC and OD. Then the positive angles

$$\alpha = AOC, \qquad \beta = AOD$$

are two angles (and the only two angles between $0°$ and $360°$) such that

$$\sin\alpha = \sin\beta = s.$$

Every angle congruent to either of these has the same sine. Therefore every such angle is a solution of the given equation.

Conversely, if any angle γ is a solution, when placed upon the axes, its terminal side must fall either upon OC or OD, since no other radius meets the circle at height s. Hence γ must be congruent to α or to β.

THEOREM II. *The equation $\cos x = c$, where $-1 < c < 1$, has exactly two solutions between $0°$ and $360°$.* Every angle congruent to either of these two is a solution, and conversely, every solution is congruent to one or the other of them.

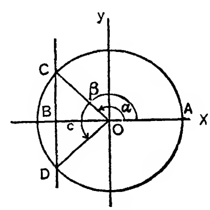


FIG. 70

To see this, draw a unit circle as for theorem I and lay off on the x-axis, $OB = c$ (to the right if $c > 0$, to the left if $c < 0$); through B draw a line parallel to the y-axis; this line meets the circle in two and only two points, C and D. The positive angles $\alpha = AOC$, $\beta = AOD$ are two angles (and the only angles between $0°$ and $360°$), which satisfy the equation $\cos x = c$. The student will easily see, as in theorem I, that all angles congruent to either of these are solutions, and that every solution is congruent to α or to β.

THEOREM III. *The equation $\tan x = t$, where t is any real number, has exactly two solutions between $0°$ and $360°$.* Every angle congruent to either of these two is a solution, and conversely every solution is congruent to one or the other of these two angles.

---

* In this chapter, by "between $0°$ and $360°$" we shall mean $0°$ included and $360°$ excluded; *i.e.* "x between $0°$ and $360°$" means $0° \leqq x < 360°$.

To see this, draw a unit circle as in the previous theorems and draw the tangent TAT'; on this tangent lay off $AB = t$ (upward if $t > 0$, downward if $t < 0$); through B draw a diameter; this diameter meets the circle in two and only two points C and D. The positive angles

$$\alpha = AOC, \qquad \beta = AOD$$

are the only solutions of the equation $\tan x = t$ between 0° and 360°. The student may complete the demonstration as in the previous theorems.

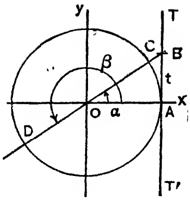


FIG. 71

THEOREM IV. *The equation* $\operatorname{ctn} x = 0$ *is equivalent* to* $\cos x = 0$. *The equations* $\operatorname{ctn} x = c$, $\sec x = c$, $\csc x = c$, *where* $c \neq 0$, *are equivalent, respectively, to* $\tan x = \frac{1}{c}$, $\cos x = \frac{1}{c}$, $\sin x = \frac{1}{c}$. The proof, which is almost obvious, is left to the student.

80. Use of the Graph. A second method for solving the equation $\sin x = s$ is as follows:

Plot the graph of $\sin x$, on the y-axis lay off $OA = s$ (above if $s > 0$, below if $s < 0$), and draw through O a line parallel to the x-axis. If $-1 < s < 1$, this line will cut the curve in points P_1, Q_1, P_2, Q_2, etc., and the projections of these points on the

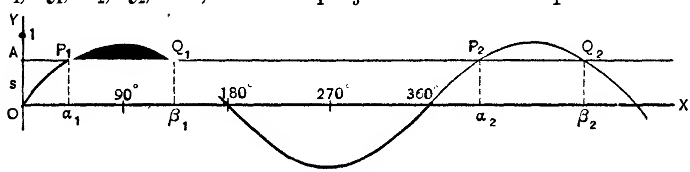


FIG. 72

x-axis determine the angles α_1, β_1, α_2, β_2, etc., which are solutions of the equation $\sin x = s$. It is obvious that $\alpha_1 \cong \alpha_2$, $\beta_1 \cong \beta_2$, etc. This method has an advantage over that shown on p. 94, in that it shows graphically more than the two solutions which lie between 0° and 360°; the former method has, however, the advantage that it requires very much less time to make the construction accurately.

This method can clearly be applied to any of the trigonometric functions.

* Two equations are equivalent when every solution of either is a solution of the other also.

EXERCISES XXXI. — SIMPLE TRIGONOMETRIC EQUATIONS

1. Solve the following equations by constructing a figure for each.

(*a*) $\sin x = 2/5$. (*b*) $\sin x = -1/2$. (*c*) $\sin x = -.8$.
(*d*) $\sin x = .866$. (*e*) $\sin x = .48$. (*f*) $\cos x = -1/2$.
(*g*) $\cos x = .63$. (*h*) $\cos x = \sqrt{3}/2$. (*i*) $\sin x = 0$.
(*j*) $\cos x = 0$. (*k*) $\sin x = 1$. (*l*) $\cos x = 1$.

[NOTE. The equations (*k*) and (*l*) are not included under theorems I and II, but the student can readily solve them by a similar method.]

2. $2\sin^2 x + \sin x = 1$.

[HINT. Solve this quadratic for $\sin x$ and apply theorem I.]

3. (*a*) $2\sin^2 x - 5\sin x + 2 = 0$. (*b*) $4\cos^2\theta + 8\cos\theta = 5$.

4. (*a*) $\tan x = 1$. (*b*) $\tan x = -1/2$. (*c*) $\tan x = 2$.
(*d*) $\tan x = -2.6$. (*e*) $\tan x = 5.3$. (*f*) $\tan x = 0$.

5. (*a*) $\tan^2 x = 3$. (*b*) $\tan^2\theta = 6\frac{1}{4}$ (*c*) $\tan^2\theta = 6 - 4\sqrt{2}$.

6. (*a*) $\tan^2\theta - 4\tan\theta + 1 = 0$. (*b*) $3\tan^2 x - 4\sqrt{3}\tan x + 3 = 0$.

7. (*a*) $\text{ctn}\, x = 1/2$. (*b*) $\text{ctn}\, x = .73$. (*c*) $\text{ctn}\, x = -1.31$.
(*d*) $\text{ctn}\, x = 0$. (*e*) $2\,\text{ctn}^2 x - 3\,\text{ctn}\, x + 1 = 0$.

8. (*a*) $\sec x = 2$. (*b*) $\sec x = 3.1$. (*c*) $\sec x = 10.57$.

9. (*a*) $\csc x = 5.3$. (*b*) $\csc x = 15$. (*c*) $\csc x = 7.4$.

10. (*a*) $\sec^2 x - 3\sec x + 2 = 0$. (*b*) $\sec^2 x = \sqrt{2}\sec x$.

11. (*a*) $2\csc^2 x - 5\csc x + 2 = 0$. (*b*) $2\csc^2 x = \sqrt{3}\csc x$.

81. Reduction of Equations to Standard Form. If a trigonometric equation contains more than one of the trigonometric functions, all but one can usually be eliminated; the resulting equation may then be solved algebraically for the function which remains, as in Ex. 2, above; the solutions may then be found by the methods of § 79.

Example 1. Solve the equation $\cos^2 t - \sin^2 t = \sin t$. In this equation $\cos^2 t$ may be replaced by its equal $1 - \sin^2 t$; the equation then becomes a quadratic in $\sin t$, viz.: $2\sin^2 t + \sin t - 1 = 0$. This equation is *equivalent* to the given one; *i.e.* every solution of either is a solution of the other. The solutions may now be found by factoring:

$$(2\sin t - 1)(\sin t + 1) = 0.$$

Hence we have either $\sin t + 1 = 0$, whence $\sin t = -1$, and $t \cong 270°$; or else, $2\sin t - 1 = 0$, whence $\sin t = 1/2$, and $t \cong 30°$ or $t \cong 150°$. There are no other solutions.

The process of solving equations consists chiefly in replacing one equation by another (or by a set of others) which is equiv-

alent, and which is more easily solved; in carrying out the necessary transformations, use may be made of any identities previously proved.* Certain operations (for example, squaring both sides of an equation) yield a new equation which, though not equivalent to the original, has all the solutions of the original equation, and perhaps other solutions. If such a transformation is employed, *every solution of the transformed equation must be tested by substitution in the given equation.*

Example 2. Solve the equation $\cos x - \sqrt{3}\sin x + 1 = 0$. Substituting $\sqrt{1 - \cos^2 x}$ for $\sin x$, transposing the radical, squaring both sides, and collecting terms, we obtain $4\cos^2 x + 2\cos x - 2 = 0$. The solutions of this equation are all included in the set $x \cong 60°$, $x \cong 180°$, $x \cong 300°$. By substitution in the given equation we see that it is not satisfied by any of the values $x \cong 300°$; hence these are not solutions of the given equation. Similarly, it is found that all the values $x \cong 60°$ and $x \cong 180°$, do satisfy the given equation and these are therefore the values required.

EXERCISES XXXII. — SOLUTION OF TRIGONOMETRIC EQUATIONS

Solve completely the following equations:

1. $2\sin^2 x - \cos x = 1$.
2. $\cos^2 x = \sin^2 x$.
3. $\cos 2x + 5\sin x = 3$.
4. $\cos 2x - \sin x = 1/2$.
5. $5\sin x + 2\cos^2 x = 5$.
6. $\sec^2 x + \tan x = 3$.
7. $4\sec^2 x + \tan x = 7$.
8. $\tan x + \text{ctn}\, x = 2$.
9. $\sin x + 3 = \csc x$.
10. $\sin 2x \cos x = \sin x$.
11. $\cos^2 x + 5\sin x = 1$.
12. $2\sin^2 x + \sin^2 2x = 2$.
13. $5\tan^2 x - 2\tan x = 1$.
14. $\tan^2 x - 6\tan x + 4 = 0$.
15. $2\sec^2\theta + (\tan\theta)/3 = 6$.
16. $4\sec^2\theta - 3 = 7\tan^2\theta$.
17. $\tan x + 3\,\text{ctn}\, x = 4$.
18. $\text{ctn}\,\theta - 2\tan\theta = 1$.
19. $\tan x + \tan(x + 45°) = 2$.
20. $\tan x + \tan(x - 60°) = 4$.

* Attention is called to the fact that we need for this process only the following rules of algebra:

(1) The following changes in an equation lead to an equivalent equation:

(*a*) transposition of terms,

(*b*) multiplication (or division) of all the terms by the same constant $\neq 0$. [Changing the signs of all the terms is the same as multiplying by -1.]

(*c*) substituting for one expression, another identically equal to it.

(2) If an equation is of the form $A = 0$ (*i.e.* has all of its terms on the left-hand side), and if A can be factored into $B \times C$ so that the equation can be written $BC = 0$, then the equation $A = 0$ is equivalent to the pair $B = 0$, $C = 0$.

In (1) (*c*), and (2) make sure that the expressions used are defined throughout the range of values of the unknown in which we are interested.

82. Special Methods of Solution. An equation of the form $a \sin x + b \cos x = c$, may be solved by the following device: Construct a right triangle whose sides are a and b and compute the hypotenuse $h = \sqrt{a^2 + b^2}$, as in § 64, p. 79. Divide the given equation through by h and replace a/h and b/h by the appropriate function of one of the acute angles, α or β, of the triangle. Then solve the equation as in § 79.

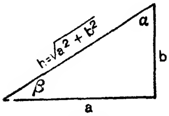


FIG. 73

Example 1. Solve the equation $3 \sin x - 4 \cos x = 2$.

Here the hypotenuse of the auxiliary triangle is 5 and the equation reduces to $(3/5) \sin x - (4/5) \cos x = 2/5$, or $\sin \alpha \sin x - \cos \alpha \cos x = 2/5$, whence $\cos (\alpha + x) = -2/5$, where $\alpha \cong 36° 52'$. By theorem II, the complete solution of this equation is $x + \alpha \cong 113° 35'$ or $x + \alpha \cong 246° 25'$; *i.e.* $x \cong 76° 43'$ or $x \cong 209° 33'$. If we had employed the angle $\beta = 53° 08'$ instead of α, we should have obtained $\sin x \cos \beta - \cos x \sin \beta = 2/5$. Show that this equation leads to the values of x found above.

Many equations can be solved by the following principles:
(1) If $\sin \alpha = \sin \beta$, then $\alpha \cong \beta$, or $\alpha \cong 180° - \beta$; and conversely.
(2) If $\cos \alpha = \cos \beta$, then $\alpha \cong \beta$, or $\alpha \cong -\beta$; and conversely.
(3) If $\tan \alpha = \tan \beta$, then $\alpha \cong \beta$, or $\alpha \cong 180° + \beta$; and conversely, provided either α or β has a tangent.

Example 2. Solve the equation $\sin 5\theta = \sin 2\theta$.

By (1) either $5\theta \cong 2\theta$; *i.e.* $3\theta = \pm n \cdot 360°$, and hence $\theta = \pm n \cdot 120°$; or, $5\theta \cong 180° - 2\theta$; *i.e.* $7\theta = 180° \pm n \cdot 360°$ and $\theta = (1 \pm 2n) 180°/7$.

Example 3. Solve the equation $\cos (5\theta/6) = \cos (\theta/3)$.

By (2) either $5\theta/6 \cong \theta/3$, whence $\theta = \pm 2n \cdot 360°$; or $5\theta/6 \cong -\theta/3$, whence $\theta = \pm (6/7)n \cdot 360° = \pm n \cdot (308° 34' 17'' 1/7)$.

Example 4. Solve the equation $\tan 7x = \tan 3x$.

By (3) either $7x \cong 3x$, whence $4x = n \cdot 360°$, that is, $x = n \cdot 90°$; or $7x \cong 3x + 180°$, whence $4x = 180° + n \cdot 360° = (2n + 1)180°$, that is, $x = (2n + 1)45°$. All these results may therefore be written $x = n \cdot 45°$. Of these values, those which make x an odd multiple of $90°$ must of course be excluded; and the only solutions are $x = n \cdot 45°$, provided n is any integer except twice an odd number.

Example 5. Solve the equation $\csc 2x = \csc 5x$.

Any solution of the equation $\sin 2x = \sin 5x$, except those that make $\sin 2x = \sin 5x = 0$ are solutions of the given equation. Hence $x = \pm n \cdot 120°$ are solutions, except when n is a multiple of 3; and $x = \pm (2m \pm 1) 180°/7$ are solutions, except when $2m + 1$ is a multiple of 7. See Example 1.

83. Graphical Methods. Any equation may be solved graphically by plotting the graphs of the two sides on the same pair of axes; then the points of intersection of the two curves, when projected on the x-axis, will determine the solutions.

Example 1. Solve graphically the equation $\sin x = \sin 2x$.

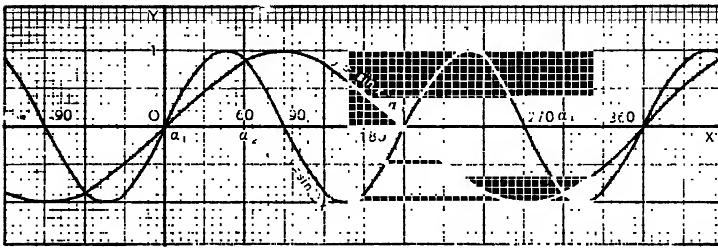


FIG. 74

The graphical solution is shown in Fig. 74. The solutions between 0° and 360° are $\alpha_1 = 0°$, $\alpha_2 = 60°$, $\alpha_3 = 180°$, $\alpha_4 = 300°$.

EXERCISES XXXIII. — SPECIAL METHODS — TRIGONOMETRIC EQUATIONS

Solve each of the following equations.

1. $2\sin^2 x + 3\cos x = 0.$
2. $\sin^2 x + \cos x = 1.$
3. $\sin 2x = 2\sin x.$
4. $2\cos 2x - 3\sin x \cos x = 0.$
5. $8\cos x - 5\sin 2x = 4\cos^3 x.$
6. $\sin^2 x \cos x - \sin 2x + \cos x = 0.$
7. $\sqrt{3}\cos x + \sin x = \sqrt{2}.$
8. $\sqrt{3}\sin x - \cos x = \sqrt{2}.$
9. $5\cos x - 2\sin x = 2.$
10. $6\cos\theta + 8\sin\theta = 9.$
11. $5(1 - \sin x) = 2\cos x.$
12. $\sin(60° + x) = \sin x.$
13. $\sec^2 x - 4\sin^2 x = 0.$
14. $4\sec^2 x = 9\tan^2 x.$
15. $\text{ctn}\, x + \csc^2 x = 3.$
16. $\tan^3 x = 3\tan x.$
17. $\csc 3x = \csc 2x.$
18. $\sec 4x = \sec 5x.$
19. $\tan 3x = 3\tan x.$
20. $\cos 3\theta = \cos\theta.$
21. $\sin x = \tan x - \tan 2x.$
22. $\text{ctn}\, x = 2\tan x + 3.$
23. $3\tan(x - 15°) = \tan(x + 15°).$
24. $\sin(28° 15' + x) = 1.11755 \sin x.$
25. $\cos(18° 30' - x) = .342 \sin x.$
26. $\sin mx + \sin nx = 0.$
27. $\cos mx + \sin nx = 0.$
28. $\csc\theta = 1 + \text{ctn}\,\theta.$

PART II. INVERSE FUNCTIONS — TRANSCENDENTAL EQUATIONS

84. Inverse Functions. We have seen that the equation

(1) $$\sin x = y$$

can be solved for x, if y is any number whatever between -1 and $+1$, and that there are an infinite number of solutions. Any one of these solutions is denoted by*

(2) $$x = \mathbf{arcsin}\ y.$$

Throughout this Chapter we shall suppose all angles measured in radians. Then (2) means that x *is the number of radians in an angle* (*or arc* †) *whose sine is* y; it is read "arc sine y" or "an angle whose sine is y."

The expressions $y = \sin x$, $x = \arcsin y$, are two aspects of one relation, just as are the two statements "A is the uncle of B" and "B is the nephew of A"; either one implies the other; both mean the same thing.

Likewise **arccos** y denotes *an angle whose cosine is* y; **arctan** y denotes *an angle whose tangent is* y; etc.

Whenever two quantities y and x are related in this dual manner, each is called the **inverse** of the other; thus, if $y = \sin x$, $\sin x$ is the *inverse* of $\arcsin y$; and conversely, $\arcsin y$ is the inverse of $\sin x$. Similarly, if $y = \cos x$, $\cos x$ and $\arccos y$ are inverse to each other. An analogous notation is usual for $\tan x$, and, indeed, for any function whatever.

85. Graphical Representation of Inverse Functions. Since the equations

(1) $$y = \sin x \quad \text{and} \quad x = \arcsin y$$

are equivalent, the same pairs of values of x and y which satisfy one of them satisfy the other. Hence either of these two equivalent equations is represented graphically by the curve drawn in §§ 55–56, p. 69.

If we wish to study the *arcsine function* for its own sake,

* The notation $\sin^{-1} y$ also is used very frequently to denote $\arcsin y$; it is necessary to notice carefully that $\sin^{-1} y$ does *not* mean $(\sin y)^{-1}$.

† If the unit angle is a radian, and the unit length be taken for the radius of a circle, the numerical measure of an angle at the center is equal to the numerical measure of the length of the intercepted arc. See § 44, p. 56.

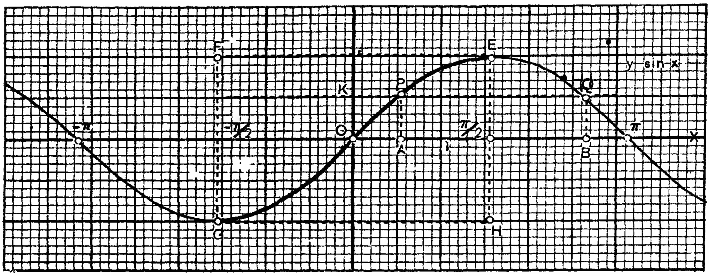


FIG. 75

it is convenient and customary to think of it as plotted in the ordinary manner, with the equation written in the form:

(2) $y = \arcsin x$, [*i.e.* $x = \sin y$]

which differs from the preceding form only in the interchange of the letters x and y. It follows that the equation (2) is represented by the curve formed by interchanging the two axes * in Fig. 75. This curve is shown in Fig. 76. It may also be plotted by points from the equation $x = \sin y$, precisely as in §§ 55–56.

It is usual to say that the curve of Fig. 75 represents the *sine* function, and that that of Fig. 76 represents the *arcsine* function.

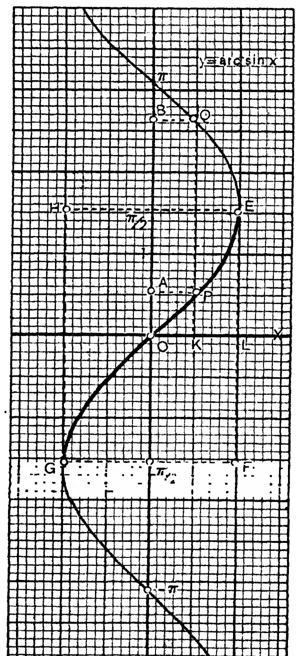


FIG. 76

* This interchange of the x and y axes on any curve is equivalent to leaving the axes fixed in space and rotating the curve through 180° about a line through the origin that makes an angle of 45° with the x-axis.

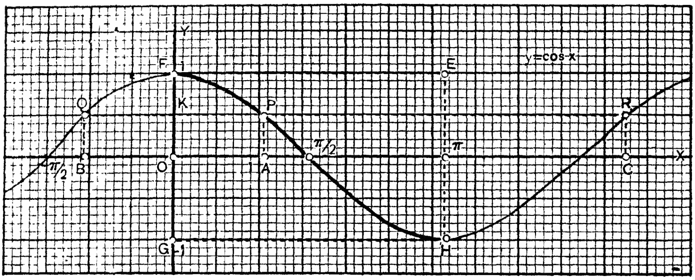


FIG. 77

Likewise, either of the equations,

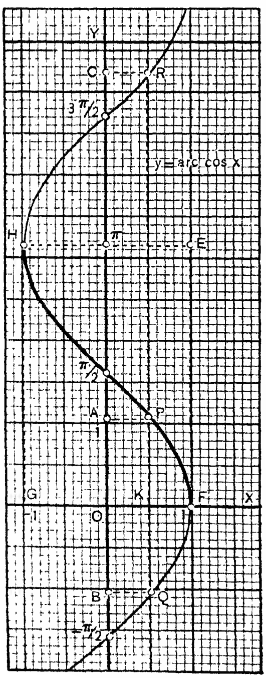


FIG. 78

(3) $y = \cos x$, $[i.e.\ x = \operatorname{arccos} y]$

is represented by a graph of the form shown in Fig. 77, but if x and y are interchanged, the equations become

(4) $y = \operatorname{arccos} x$, $[i.e.\ x = \cos y]$

and the graph appears as in Fig. 78, which is identical with Fig. 77, except for interchange of x and y. It is usual to say that Fig. 78 represents the arccosine function.

The similar figure for

(5) $y = \tan x$, $[i.e.\ x = \arctan y]$

and that for

(6) $y = \arctan x$, $[i.e.\ x = \tan y]$

are shown in Figs. 79 and 80, respectively.

The curve of Fig. 80 is usually thought of as representing the *arctangent* function.

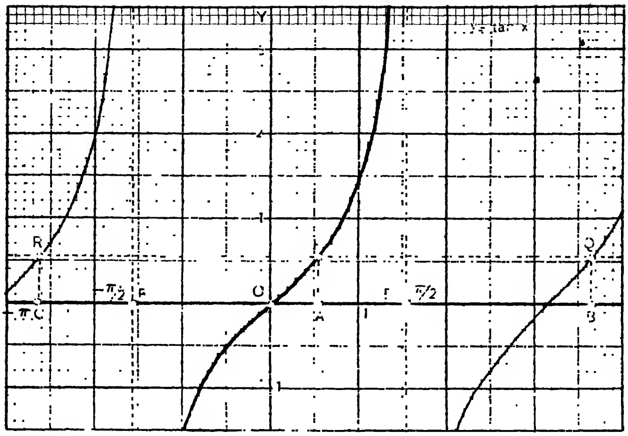
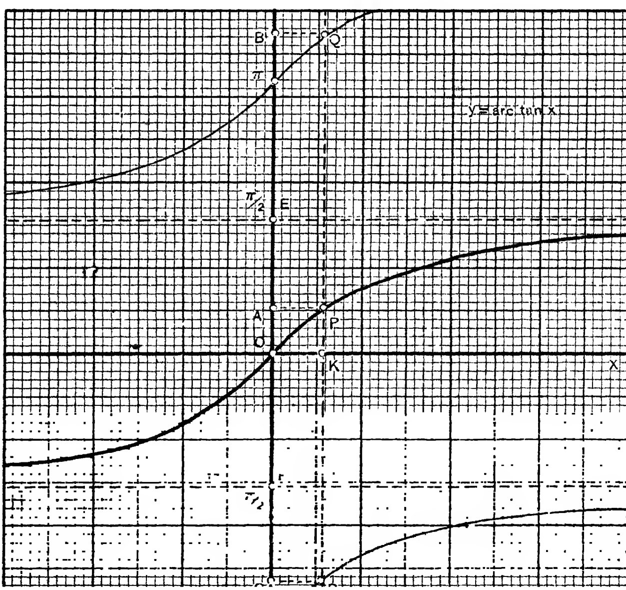


FIG. 79



86. Principal Values. We have seen (§ 79, p. 94), and it is also immediately evident from Fig. 76, that there is one and only one angle α between $-\pi/2$ and $+\pi/2$ * whose sine is x, if x lies between -1 and $+1$. We shall distinguish this one value from all other angles whose sine is x by capitalizing the initial letter A in the symbol arcsin; thus $\arcsin x$ has many values, but $\text{Arcsin}\, x$ has one and only one value α, if x is any given number between -1 and $+1$. We shall call this special value α ($= \text{Arcsin}\, x$) the **principal value** of $\arcsin x$.

Thus, if $x=1/2$, $\text{Arcsin}\, x = \pi/6$; if $x = -1/2$, $\text{Arcsin}\, x = -\pi/6$; etc. In Fig. 76, if x is represented by OK, a vertical line through K meets the curve in one and only one point P between $y = -\pi/2$ and $y = +\pi/2$.

Another value of $\arcsin x$ is (OB, Fig. 76) $\pi - \text{Arcsin}\, x$, which is obtained by subtracting the principal value from π, since $\sin(\pi - \alpha) = \sin \alpha = x$. It follows that all values of $\arcsin x$ are congruent either to $\text{Arcsin}\, x$ or to $\pi - \text{Arcsin}\, x$; that is, all values of $\arcsin x$ are contained in one or the other of the forms

(1) $$\text{Arcsin}\, x \pm 2\,n\pi, \quad \text{or} \quad (\pi - \text{Arcsin}\, x) \pm 2\,n\pi,$$

where n is a positive integer or zero.†

For example, if $x = 1/2$, $\text{Arcsin}\, x = \pi/6$; but another value of $\arcsin x$ is $\pi - \pi/6 = 5\,\pi/6$ (OB, Fig. 76). All other values are given by the forms $\pi/6 \pm 2\,n\pi$ or $5\,\pi/6 \pm 2\,n\pi$. Thus, $\pi/6 + 2\,\pi = 13\,\pi/6$ (OC, Fig. 76) and $5\,\pi/6 - 2\,\pi = -7\,\pi/6$ are other values of $\arcsin x$.

Similarly, $\arccos x$ has one and only one value α between ‡ 0 and π (see II, § 79, and OA, Fig. 78) if x lies between -1 and $+1$. This value will be denoted by $\text{Arccos}\, x$ and will be called the *principal value* of $\arccos x$.

Another value of $\arccos x$ is $-\text{Arccos}\, x$ (OB, Fig. 78) since $\cos(-\alpha) = \cos\alpha = x$. All values of $\arccos x$ are contained in one or the other of the forms:

* That is, $\pi/2$ *radians*, or a right angle. Here and throughout this Chapter, radian measure is to be understood.

† Or, in one formula, $\arcsin x = \text{Arcsin}\,[(-1)^n x] + n\pi$, $n = 0, \pm 1, \pm 2$, etc.

‡ Notice that values of y between $-\pi/2$ and $\pi/2$ would not be enough to include all values of $\cos y$.

(2) $\qquad \text{Arccos}\, x \pm 2\,n\pi, \quad \text{or} \quad -\text{Arccos}\, x \pm 2\,n\pi,$
where n is a positive integer or zero.

For example, $\text{Arccos}\,(1/2) = \pi/3$; another value of *arccos (1/2) is $-\pi/3$; all others are given by $\pi/3 \pm 2\,n\pi$ or $-\pi/3 \pm 2\,n\pi$. Thus, $-\pi/3 + 2\,\pi = 5\,\pi/3$ (see OC, Fig. 78) is another value of arccos (1/2).

Finally, arctan x has one and only one value α ($=$ Arctan x) between $-\pi/2$ and $+\pi/2$; and it will be called the *principal value* of arctan x. Another value of arctan x is $\pi +$ Arctan x, since $\tan(\pi + \alpha) = \tan\alpha = x$. All values of arctan x are contained in one or the other of the forms: Arctan $x \pm 2\,n\pi$, or (π + Arctan x) $\pm\, 2\,n\pi$, which are together equivalent to

(3) $\qquad \text{Arctan}\, x \pm n\pi,$
where n is a positive integer or zero.

Thus, $\text{Arctan}\,\sqrt{3} = \pi/3$; but other values of $\arctan\sqrt{3}$ are $\pi + \pi/3 = 4\,\pi/3$ and $\pi/3 - \pi = -2\,\pi/3$.

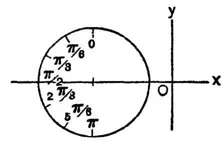
The principal values of the other inverse trigonometric functions are denoted by Arcctn x, Arcsec x, Arccsc x, etc., and are given in the following table:

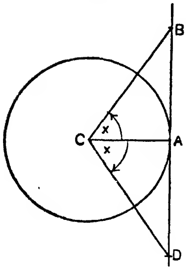
$y =$	**Arcsin x**	**Arccos x**	**Arctan x**
Range of x	$-1 \leqq x \leqq +1$	$-1 \leqq x \leqq +1$	All values
Range of y	$-\pi/2$ to $\pi/2$	0 to π	$-\pi/2$ to $\pi/2$
x positive	1st Quad.	1st Quad.	1st Quad.
x negative	4th Quad.	2d Quad.	4th Quad.
$y =$	**Arcctn x**	**Arcsec x**	**Arccsc x**
Range of x	All values	$x \geqq 1$ or $x \leqq -1$	$x \geqq 1$ or $x \leqq -1$
Range of y	0 to π	0 to π	$-\pi/2$ to $\pi/2$
x positive	1st Quad.	1st Quad.	1st Quad.
x negative	2d Quad.	2d Quad.	4th Quad.

* Or, in one formula, $\arccos x = \text{Arccos}\,[(-1)^n x] + n\pi$, $n = 0, \pm 1, \pm 2$, etc

EXERCISES XXXIV. — INVERSE FUNCTIONS. GRAPHS

1. Plot the curve $y = \cos x$ from $x = 0$ to $x = \pi$ by a construction similar to that of § 56, for $y = \sin x$, except in this case lay off the equal arcs from the highest point on the unit circle.





2. Plot the curve $y = \tan x$ for the range $-\pi/2 < x < \pi/2$, making use of the construction shown in the annexed figure: CA is one scale unit; ACB is the angle x radians; the ordinate of point B is $\tan x$.

3. On the graph of each of the following curves, mark in heavier ink the portion which corresponds to the principal value (see table above).
(*a*) $y = \text{arcctn}\, x$; (*b*) $y = \text{arcsec}\, x$; (*c*) $y = \text{arccsc}\, x$.

4. Prove each of the following relations:

(*a*) $\text{Arcsin}\,(-3/4) = -\text{Arcsin}\,(3/4)$.

[This can be seen from the graph of Arcsin x; proved as follows:
Let $\alpha = \text{Arcsin}\,(-3/4)$, then $\sin \alpha = -3/4$; let $\beta = \text{Arcsin}\, 3/4$, then $\sin \beta = 3/4$, where both α and β lie between $-\pi/2$ and $+\pi/2$. Now $\sin(-\beta) = -\sin \beta = -3/4$. Hence $\sin \alpha = \sin(-\beta)$; whence $\alpha = -\beta$, *i.e.* $\text{Arcsin}\,(-3/4) = -\text{Arcsin}\, 3/4$.]

(*b*) $\text{Arcsin}\,(-x) = -\text{Arcsin}\, x$ for $-1 < x < 1$.

(*c*) $\text{Arccos}\,(-2/5) = \pi - \text{Arccos}\,(2/5)$.

(*d*) $\text{Arccos}\,(-x) = \pi - \text{Arccos}\, x$ for $-1 \leqq x \leqq 1$.

(*e*) $\text{Arcsin}\, 2/3 + \text{Arccos}\, 2/3 = \pi/2$.

(*f*) $\text{Arcsin}\, x + \text{Arccos}\, x = \pi/2$ for $-1 \leqq x \leqq 1$.

(*g*) $\text{Arctan}\,(1/2) + \text{Arcctn}\,(1/2) = \pi/2$.

(*h*) $\text{Arctan}\, x + \text{Arcctn}\, x = \pi/2$ for all values of x.

(*i*) $\text{Arcsin}\, x = \pm \text{Arccos}\, \sqrt{1 - x^2}$, according as $x > 0$ or $x < 0$.

5. Show that $\text{Arctan}\, x + \text{Arctan}\, y = \text{Arctan}\left(\dfrac{x + y}{1 - xy}\right)$, provided x and y are both between -1 and $+1$.

6. $\text{Arctan}\, x = \text{Arcsin}\, x/\sqrt{1 + x^2}$ for all values of x.

7. $\text{Arctan}\, 1/2 + \text{Arctan}\, 1/3 = \pi/4$.

8. $\text{Arctan}\, 1/4 + \text{Arctan}\, 1/13 = \text{Arctan}\, 1/3$.

9. $\text{Arctan}\, 1/2 + \text{Arctan}\, 1/5 + \text{Arctan}\, 1/8 = \pi/4$.

10. $\text{Arctan}\, x + \text{Arcctn}\,(x + 1) = \text{Arctan}\,(x^2 + x + 1)$ for all x.

11. $\text{Arccos}\, 1/\sqrt{2} - \text{Arcsin}\, 1/\sqrt{5} = \text{Arctan}\, 1/3$.

12. Find the numerical values of the following:

(*a*) $\cos(\text{Arcsin}\, 8/17)$. (*b*) $\tan(\text{Arcsin}\, 5/13)$.

(*c*) $\tan(\text{Arctan}\, 4/3 - \text{Arctan}\, 1/7)$.

87. Transcendental Equations. In §§ 77–83 equations occur which involve only trigonometric functions of the unknown. In equations which involve also algebraic or other functions, the unknown angle is generally measured in radians. The graphical method is usually the best. All such equations are called Transcendental Equations.

Example 1. Solve the equation $x = \cos x$. To solve this equation we must find a number x, such that the cosine of x radians is equal to the number x. Draw the graphs of the equations $y = x$ and $y = \cos x$; the solution of the equation is the value of x at the point of intersection of the two graphs. There is clearly only one such point, and at that point $x = .74$, approximately; a still more accurate value is $x = .73908$.

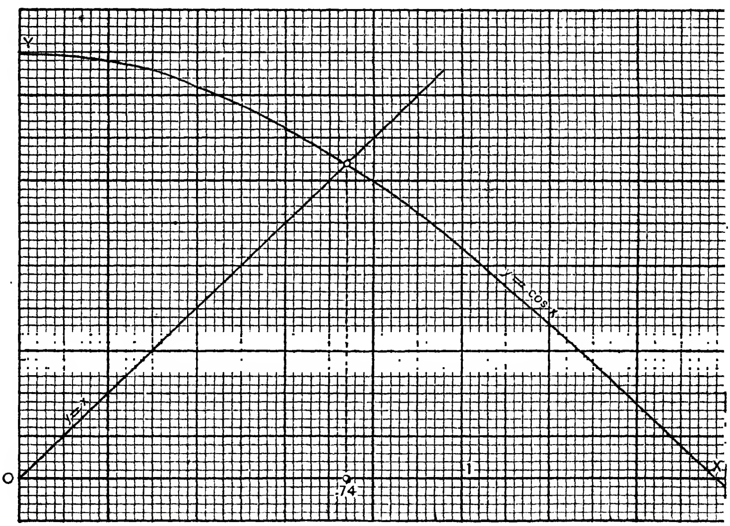


Fig. 81

EXERCISES XXXV. — TRANSCENDENTAL EQUATIONS

Solve the following equations:

1. $x - 1 = \sin x$.
2. $x + 1 = \tan x$.
3. $1/x = \sin x$.
4. $1/x = \cos x$.
5. $\log x = \cos x$.
6. $\log x = \sin x$.
7. $2^x = \sin x$.
8. $10^x = \cos x$.
9. $(2.7)^x = \sin x + \cos x$.

[Hint. Use logarithms to plot the curves $y = 2^x$, etc.]

CHAPTER VIII

SPHERICAL TRIGONOMETRY

PART I. SUMMARY OF GEOMETRIC THEOREMS

88. Purpose. Spherical Trigonometry has for its primary purpose the determination of certain parts of a spherical triangle when other parts are known. This is accomplished by the use of the formulas of plane trigonometry and certain elementary propositions of solid geometry. Several useful propositions are collected here without proof for convenience in reference.

89. Some Useful Propositions of Solid Geometry. (1) A **sphere** is the locus of points equidistant from a fixed point O called its center. The constant distance is called the **radius**, and will be denoted in all that follows by R.

(2) *If a sphere is cut by any plane, the intersection is a circle.* If the plane passes through the center O, the radius r of the circle is equal to R, and the circle is called a **great circle.** Otherwise we have $r < R$, and the circle is called a **small circle.**

(3) *A line perpendicular to a plane is perpendicular to every line of the plane through the foot of the perpendicular.*

[We shall use also several other theorems regarding lines and planes that are practically self-evident.]

(4) A line through the center O perpendicular to the plane of a given circle cuts the sphere in two points P and P', which are called the **poles** of that circle. The line PP' is called the **axis** of that circle.

For example, the north and south poles of the earth are the poles of the equator.

(5) *Through any two points A and B of a sphere there is at least one great circle, determined by the plane AOB; there is only one such great circle through A and B unless AOB is a straight line.*

(6) The shortest distance between the points A and B is the straight line which joins them, but this line does not lie on the sphere. The shortest line on the sphere joining A and B is the arc of a great circle; hence we make the definition:

The **distance** *between two points* **on a sphere** *is the length of the arc (not greater than a semicircle) of a great circle which joins them.* The unit distance on a sphere is the length of the arc of a great circle which subtends a unit angle at the center. When angles are measured in degrees, the unit distance is also called a degree. Hence the arc of a great circle and the angle which it subtends at the center have the same numerical measure.

(7) *The distance from the pole of any circle to a point on that circle is constant; for a great circle it is 90°, or one quadrant.*

(8) *The angle between two planes is the angle between two lines AP and AQ in those planes perpendicular to their line of intersection at the same point A.*

(9) *The angle between two great circles on a sphere is the angle between their planes;* hence it is equal to the angle between their tangents at the point of intersection, for the tangents are perpendicular to the diameter in which the planes intersect.

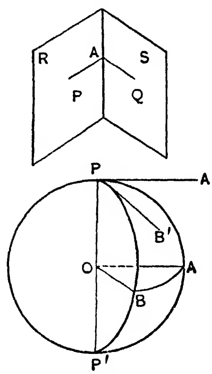


FIG. 82

(10) If two great circles meet at P and P', and if AB be another great circle whose pole is P, cutting the two given circles at A and B, respectively, then $AOP = BOP = 90°$ and angle AOB measures the angle between the two planes; hence *the angle between two great circles PA and PB is measured by the intercepted arc AB of a great circle whose pole is P.*

(11) *Any two great circles bisect each other at their points of intersection.*

(12) The portion of a sphere bounded by two great semicircles is called a **lune**.

The two angles of a lune are equal to each other; each of them is equal to the angle between the two great semicircles that bound the lune.

(13) The portion of a sphere bounded by arcs of three great circles is called a **spherical triangle.** The sides a, b, c, are the lengths of the arcs; the angles A, B, C, are the angles between the great circles of which the sides are arcs.

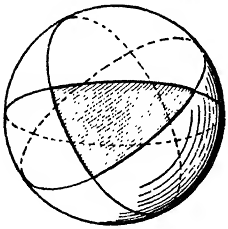


FIG. 83

(14) Any three distinct great circles that do not pass through a common point on the sphere divide the surface of the sphere into eight spherical triangles; none of these has an angle or a side greater than 180°. Hence we consider only such triangles.

(15) The two triangles into which a lune AA' is divided by an arc BC of any third great circle are called **co-lunar.** The two angles at C are supplementary; those at B are supplementary; those at A and A' are equal. Similarly $AC + CA' = 180°$, and $AB + BA' = 180°$.

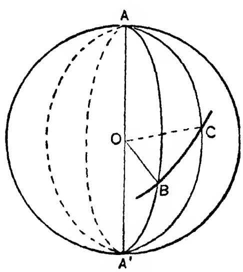


FIG. 84

(16) Two spherical triangles are **congruent** (sometimes called *equal*) if they can be superposed; *i.e.* when the sides and angles of one are equal to those of the other and are arranged in the same order. If the parts of two triangles are equal, but are arranged in reverse order, the triangles are called **symmetric.**

(17) A spherical triangle is *determined* when a sufficient number of parts are given so that one triangle on a sphere has these parts, and any other triangle on the same sphere that has the given parts is either congruent or symmetric to it. Thus a triangle is determined by (a) two sides and included angle; (b) two angles and included side; (c) three sides; (d) three angles; (e) (ambiguously) two sides and angle opposite one of them; (f) (ambiguously) two angles and the side opposite one of them.

(18) A right-angled spherical triangle is determined when any two parts besides the right angle are given, as in (17).

(19) *In any spherical triangle ABC:*

(*a*) The sum of the sides is less than 360° (or 2π radians).

(*b*) The sum of any two sides is greater than the third side. Hence any side is greater than the difference of the other two.

(*c*) Equal sides lie opposite equal angles; and conversely.

(*d*) The greater of two unequal sides lies opposite the greater angle; and conversely.

(*e*) The sum of the angles lies between 180° and 540° (or between π and 3π radians). The excess over 180° is called the **spherical excess** $\boldsymbol{E} = \angle A + \angle B + \angle C - 180°$ (or $\angle A + \angle B + \angle C - \pi$ radians). Hence $\boldsymbol{E} < 360°$.

(One or two or all the angles may be right or obtuse; if all are right angles, the triangle is called *tri-rectangular*.)

(*f*) If the sum of any two angles, A and B, is 180°, the sum of the two sides opposite them, a and b, is 180°.*

(*g*) If the sum of two angles, A and B, is less (greater) than 180°, the sum of the two sides opposite them, a and b, is less (greater) than 180°.†

(*h*) Of the three pairs of sides and their opposite angles a and A, b and B, c and C, at least two pairs are either both acute, or both obtuse, or both right angles.‡

(20) Areas. The area $\boldsymbol{A}$ § of the entire sphere is $4\pi R^2$.

(*a*) The area $\boldsymbol{A}$ of the entire sphere is four times the area of any great circle.

(*b*) The area $\boldsymbol{A}_l$ of a lune is proportional to its angle α: $\boldsymbol{A}_l = (\alpha/360)(4\pi R^2) = \pi R^2\alpha/90$ if α is in degrees; or $\boldsymbol{A}_l = 2R^2\alpha$ if α is in radians.

* For, the co-lunar triangle (see (15)) formed by extending a and b is congruent (16) to ABC.

† For, in the figure suggested in the preceding footnote, starting with $A + B = a + b = 180°$, if we let $\angle C$ and the point B remain fixed, but move A so as to increase b, the area of ABC increases. Hence $A + B$ increases. [See (20) (*d*).] It follows that $A + B > 180°$ if $a + b > 180°$; and conversely.

‡ For, suppose $a > 90°$ and $A < 90°$; and suppose b and B are also in opposite quadrants. Then $b > 90°$ and $B < 90°$, by (*d*); but this violates (*g*). Other cases of the theorem can be proved similarly.

§ The black-faced type $\boldsymbol{A}$ is used to denote areas, to distinguish emphatically from the angle A of the triangle ABC.

(c) The area of a tri-rectangular triangle is one eighth the area of the sphere, or $\pi R^2/2$.

(d) The area A_t of a triangle is proportional to its spherical excess E; by (c), since the excess for a tri-rectangular triangle is 90° or $\pi/2$ radians, $A_t = \pi R^2 E/180°$, if E is in degrees; or $A_t = R^2 E$, if E is in radians.*

(21) *The volume of a sphere is* $4\,\pi R^3/3$.

(22) In any triangle ABC, mark the pole of c which lies on the same side of c as does C, and call it C'; and let A' be the corresponding pole of a, and B' the corresponding pole of b. Then $A'B'C'$ is called the **polar triangle** to ABC. Conversely, ABC is itself polar to $A'B'C'$.

(23) The sides of any spherical triangle are the *supplements* of the opposite *angles* of the polar triangle:

$$A + a' = 180°,$$
$$a + A' = 180°,$$
$$B + b' = 180°,$$
$$b + B' = 180°,$$
$$C + c' = 180°,$$
$$c + C' = 180°.$$

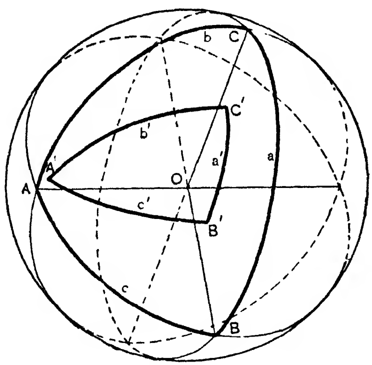


FIG. 85

Thus, if the parts of a triangle are known, those of the polar triangle can be found, and conversely.

(24) Any theorem that is true for all spherical triangles can be changed into a new theorem by applying it first to the polar triangle; thus from "The sum of the angles of a spherical triangle is greater than 180°" follows "The sum of the sides of a spherical triangle is less than 360°."

(25) It may be easier to replace a given triangle by its polar triangle before the missing parts are found: thus, if a triangle has one side equal to 90°, the polar triangle is right-angled. *If the principles of polar triangles be remembered, there are really only three cases:* viz., (a), (c), (e), of (17).

* The formulas in 20 (b) and 20 (d) are simplest in radian measure.

PART II. FUNDAMENTAL PRINCIPLES OF SOLUTION

90. Fundamental Laws. If a sufficient number of parts of a spherical triangle are given, the others can be found. We proceed to find methods for doing this.

I. The Law of Cosines. Let ABC be a spherical triangle on a sphere whose center is O, in which at least two sides (b and c, say) are each less than $90°$. The third side and the angles may have any magnitudes from $0°$ to $180°$.

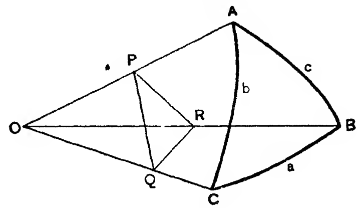


FIG. 86

At P, a point of OA, pass a plane perpendicular to OA cutting OC in Q and OB in R. Then, by the law of cosines in the plane triangles OQR and PQR, since $\angle QPR = \angle A$ and $\angle COB = a$,

$$(1) \qquad \overline{QR}^2 = \overline{OQ}^2 + \overline{OR}^2 - 2\,\overline{OQ} \cdot \overline{OR} \cos a,$$

$$\overline{QR}^2 = \overline{PQ}^2 + \overline{PR}^2 - 2\,\overline{PQ} \cdot \overline{PR} \cos A.$$

Equate these two values of QR^2, and notice that we also have

$\overline{OQ}^2 - \overline{PQ}^2 = \overline{OP}^2 = \overline{OR}^2 - \overline{PR}^2$, since $OPQ = OPR = 90°$; then

$$(2) \qquad 2\,\overline{OP}^2 - 2\,\overline{OQ} \cdot \overline{OR} \cos a = -2\,\overline{PQ} \cdot \overline{PR} \cos A.$$

Transposing, and dividing by $2\,\overline{OQ} \cdot \overline{OR}$, we have

$$(3) \qquad \cos a = \frac{OP}{OQ} \cdot \frac{OP}{OR} + \frac{PQ}{OQ} \cdot \frac{PR}{OR} \cdot \cos A,$$

or, since

$$OP/OQ = \cos b, \quad PQ/OQ = \sin b,$$

$$OP/OR = \cos c, \quad PR/OR = \sin c,$$

$$\text{I.} \qquad \mathbf{\cos \boldsymbol{a} = \cos \boldsymbol{b} \cos \boldsymbol{c} + \sin \boldsymbol{b} \sin \boldsymbol{c} \cos \boldsymbol{A}}.$$

The cosine of any side of a spherical triangle is equal to the product of the cosines of the other two sides plus the product of the sines of those two sides into the cosine of their included angle. Compare this with the law of cosines for plane triangles, § 25.

This result, called the **law of cosines**, is true in general for any side of any spherical triangle.*

II. The Law of Sines. Let ABC be a spherical triangle in which at least two sides b and c and the angles opposite are each less than 90°. From P, a point of OA, drop PQ perpendicular to the plane BOC, and through PQ pass a plane PQR perpendicular to OC and a plane PQS perpendicular to OB; then OR is perpendicular to PR and to QR, and OS is perpendicular to PS and QS.

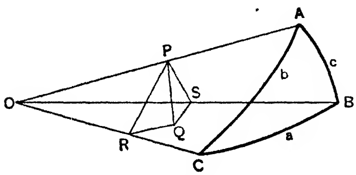


FIG. 87

In the right triangles ORP and OSP,

(1) $$PR = OP \sin b$$
and
$$PS = OP \sin c.$$

In the right triangles PQR and PQS,

(2) $$PQ = PR \sin C = PS \sin B.$$

Substituting these values from (1) in (2), we find:

$$OP \sin b \sin C = OP \sin c \sin B,$$

---

* (1) The formula [I] has been proved when $b < 90°$, $c < 90°$.

Logically, it remains to consider the following cases; it is shown below that each of them either reduces to this first case, or can be verified directly.

(2) Suppose $b < 90°$, $c = 90°$.

(a) If $A < 90°$, produce side b to D, making $AD = 90°$, and draw the arc BD. Apply [I] to the right triangle BCD, in which the sides BD $(= \angle A)$ and CD $(= 90° - b)$ are each less than 90° [Case (1)].

(b) If $A = 90°$, then $a = 90°$, and the formula may be verified by substitution.

(c) If $A > 90°$, produce sides a and c to E to form a lune; the formula applies to the triangle ACE since $CA < 90°$, $AE = 90°$; and the included angle is less than 90° [Case (2a)].

(3) Suppose $b = 90°$, $c = 90°$. Verify by substitution.

(4) Suppose $b < 90°$, $c > 90°$. Produce sides a and c to D to form a lune. Apply the formula to side CD of the triangle ACD in which AC and AD are each less than 90° [Case (1)].

(5) Suppose $b = 90°$, $c > 90°$. Produce sides b and c to D to form a lune. Apply the formula to the triangle BCD in which $BD < 90°$, $CD = 90°$ [Case (2)].

(6) Suppose $b > 90°$, $c > 90°$. Produce sides b and c to D to form a lune. Apply the formula to the triangle BCD in which BD and CD are each less than 90° [Case (1)].

whence

I. $$\sin b/\sin c = \sin B/\sin C.$$

This result, called the **law of sines**, is true in general for any two sides and the angles opposite them in any spherical triangle;* it may also be written symmetrically as follows:

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}.$$

The sines of any two sides of any spherical triangle are to each other as the sines of the angles opposite them. Compare this result with the law of sines for plane triangles, § 24.

91. Solution of Spherical Triangles. The law of cosines, the law of sines, and the properties of polar triangles are sufficient to solve all cases of spherical triangles.†

Example 1. *Two sides* of a spherical triangle are $a = 116°$ and $b = 102°$ *and their included angle* C is $129°$. Solve the triangle.

To find the third side, use the law of cosines

$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$
$$= (-.43837)(-.20791) + (.89879)(.97815)(-.62932) = -.46212$$

Therefore, $c = 117° \, 31'.4$

* The general proof may be made as in the footnote on page 114; but it may be deduced from the law of cosines, which has been proved in all cases, as follows:

$$\cos a = \cos b \cos c + \sin b \sin c \cos A.$$
$$\cos A = (\cos a - \cos b \cos c)/(\sin b \sin c).$$
$$\sin^2 A = 1 - \cos^2 A = [\sin^2 b \sin^2 c - (\cos a - \cos b \cos c)^2]/(\sin^2 b \sin^2 c).$$
$$\sin^2 b \sin^2 c \sin^2 A = (1 - \cos^2 b)(1 - \cos^2 c) - (\cos a - \cos b \cos c)^2.$$
$$\sin b \sin c \sin A = \sqrt{1 - \cos^2 a - \cos^2 b - \cos^2 c + 2 \cos a \cos b \cos c}.$$
$$\sin A/\sin a = \sqrt{1 - \cos^2 a - \cos^2 b - \cos^2 c + 2 \cos a \cos b \cos c}/(\sin a \sin b \sin c). \quad (1)$$

Starting from $\cos b = \cos c \cos a + \sin c \sin a \cos A$, we should obtain the same result for $\sin B/\sin b$, by replacing a by b, b by c, c by a, A by B, B by C, C by A. Likewise, by a second permutation of the letters, $\sin C/\sin c$ has the same value. Therefore $\sin A/\sin a = \sin B/\sin b = \sin C/\sin c$.

† The expression "Solve a triangle" tacitly assumes that a sufficient number of parts of an actual spherical triangle are given. In spherical trigonometry, as in plane, a proposed problem may violate this assumption and there will be no solution. Thus, there is no plane triangle whose sides are 14, 24, and 40. Likewise, there is no spherical triangle of which the sides are 14°, 24°, and 40°; nor one in which one angle is 72°, the side opposite it is 97°, and a second side is 84° (§ 92). Such impossible problems give rise to contradiction; thus, the sine of some angle may be found to be greater than 1, which is absurd. *Any triangle which can be constructed can be solved.*

To find A use the law of sines, $\sin A/\sin C = \sin a/\sin c$,

$$\begin{aligned} \log \sin C &= 9.89050 - 10 \\ \log \sin a &= 9.95366 - 10 \\ \text{colog} \sin c &= \underline{0.05211} \\ \log \sin A &= 9.89627 - 10 \end{aligned}$$

Hence, $A = 51° 57'.3$ or else $A = 128° 2'.7$. In order to find which of these is correct, we may use the cosine law

$$\cos a = \cos b \cos c + \sin b \sin c \cos A,$$

and substitute crudely the approximate values:

$$\cos a = -.4, \quad \cos b = -.2, \quad \cos c = -.5, \quad \sin b = .98, \quad \sin c = .9,$$

which can be quickly found from the table of p. 15, or from Table C; this gives *

$$-.4 = (-.2)(-.5) + (.98)(.9) \cos A, \quad \text{or} \quad .88 \cos A = -.5.$$

Without any further calculation whatever, we see that $\cos A$ is certainly *negative;* hence A lies in the second quadrant and the correct one of the above values is $A = 128° 2'.7$

In a similar manner, show that $B = 121° 0'.8$

Example 2. Given two angles and their included side:

$$A = 64°, \quad B = 78°, \quad c = 51°.$$

To solve this triangle, first compute the corresponding parts of the *polar* triangle [see (23), p. 112]; they are

$$a' = 116°, \quad b' = 102°, \quad C' = 129°.$$

This is, in fact, the triangle solved in Example 1; hence

$$c' = 117° 31'.4, \quad A' = 128° 2'.7, \quad B' = 121° 0'.8.$$

Hence, the required parts of the original triangle are:

$$C = 62° 29'.6, \quad a = 51° 57'.3, \quad b = 58° 59'.2.$$

Example 3. Given three sides: $a = 116°$, $b = 102°$, $c = 117° 31'.4$.

Find one angle, say A, by the cosine law:

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c} = \frac{(-.43837) - (-.20791)(-.46212)}{(.97815)(.88682)} = -.61613,$$

whence $A = 128° 2'.7$. The remainder of the work may be done precisely as in Example 1; or B and C may be found as A was found.

If the three angles are given, solve the polar triangle. There will be one and only one solution if the sum of the given angles is between 180° and 540°; otherwise, no solution.

* It is better to use the cosine law, as here, only to determine in which quadrant A lies, since the sine law is adapted to logarithmic computation. The same fact may be derived from 19(c) and 19(g), p. 111; for, since $b < a$, it follows that $B < A$, by 19(c); and since $a + b > 180°$, $A + B > 180°$ by 19 (g); hence $A > 90°$. It is usually a saving of time to use the cosine law, however, since that process is *always* successful.

92. Temporary Ambiguity. In Example 1, above, we found it convenient to use the cosine law to determine the quadrant in which A lies. A similar temporary ambiguity arises whenever an angle (or side) is found by means of its sine; it does not occur when it is found by its cosine or its tangent because the cosine and tangent are positive when the angle is in the first quadrant and negative when it is in the second quadrant. For this reason careful attention must be given to the signs of the various factors entering into the calculation of an unknown angle or side. In the case of an angle, such an ambiguity can be determined by means of the cosine law, as in Example 1 above, when the three sides are known. Inspection of that formula, or at most a rough mental or slide rule calculation will be sufficient to determine the sign of the cosine. Other cases also may be decided by the cosine law, but it is often easier to apply (19), p. 111.

Another rule which is often useful is the following: *A side (angle) which differs more from 90° than another side (angle) is in the same quadrant as its opposite angle (side).**

93. The Ambiguous Cases. *If two sides and an angle opposite one of them are given,* the third side can be found by applying the law of cosines to the side opposite the given angle. This will determine one value, two values, or no value for the third side between 0° and 180°; and accordingly, there will be one triangle, two triangles, or no triangle satisfying the given data. The other two angles can then be found by the law of sines and the temporary ambiguity determined as in § 92.

Example 1. One angle of a spherical triangle is 130°, the side opposite is 70°, and a second side is 120°. Solve the triangle.

To find c, apply the law of cosines in the form:

$$\cos a = \cos b \cos c + \sin b \sin c \cos A,$$

* These differences are taken without regard to sign. Suppose a differs more from 90° than does b or c; then $\cos a$ is (numerically) greater than $\cos b \cos c$. But, by the cosine law, $\cos a - \cos b \cos c = \sin b \sin c \cos A$, whence $\cos a$ and $\cos A$ must have like signs. The angles follow the same rule as the sides, since $\sin A > \sin B > \sin C$ if $\sin a > \sin b > \sin c$.

or $.34202 = (-.5)\cos c + (.86603)\sin c\,(-.64279),$

whence $.55667 \sin c + .5 \cos c = -.34202$

The left-hand side of this equation can be reduced, by the method of § 82, to the form

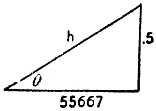


FIG. 88

$$h\,(\sin c \cos \theta + \cos c \sin \theta) = -.34202,$$

where

$$h = .74826 \text{ and } \theta = 41° \, 55'.9$$

Hence

$$\sin(c + \theta) = \sin c \cos \theta + \cos c \sin \theta = -.4571$$

$$c + \theta = 207° \, 12', \quad \text{or} \quad c + \theta = 332° \, 48';$$

$$c = 165° \, 16'.1, \quad \text{or} \quad c = 290° \, 52';$$

the value $c = 290° \, 52'$ is impossible, by (14), § 89; hence **only one solution** exists in this example. By the law of sines determine $B \neq 45° \, 54'.5$, $B = 134° \, 5'.5$ since it must be in the same quadrant as b. Similarly $C = 168° \, 2'.2$.

Example 2. Given $c = 40° \, 16'$, $C = 52° \, 30'$, $a = 47° \, 44'$.

To find b apply the law of cosines to side c.

$$\cos c = \cos a \cos b + \sin a \sin b \cos C.$$

$$.76304 = .67258 \cos b + .74002 \sin b\,(.60876)$$

$$.45047 \sin b + .67258 \cos b = .76304$$

Placing $\qquad h(\sin b \cos \theta + \cos b \sin \theta) = .76304,$

we find $h = .80950$ and $\theta = 56° \, 11'.6$; hence $\sin(b + \theta) = .94260$,

whence $\qquad b + \theta = 70° \, 29'.6 \quad$ or $\quad b + \theta = 109° \, 30'.4,$

and $\qquad b_1 = 14° \, 18' \qquad\qquad b_2 = 53° \, 18'.8.$ **(Two solutions.)**

By the law of sines find $A_1 = 65° \, 16'.5$, $B_1 = 79° \, 52'.8$, $A_2 = 114° \, 43'.5$, $B_2 = 17° \, 37'.3$.

Example 3. Given $b = 40°$, $B = 50°$, $c = 60°$.

By the cosine law, $.55567 \sin a + .5 \cos a = .76604$; hence,

$\sin(a + \theta) = 1.2+$, which is impossible. **(No solution.)**

Example 4. Given $a = 97°$, $A = 72°$, $b = 84°$.

By the cosine law, $.30723 \sin c + .10453 \cos c = -.12187$;

hence $\qquad \sin(c + \theta) = -.37539, \quad \theta = 18° \, 47'.4,$

and $\qquad c + \theta = 202° \, 2'.9$ or $c + \theta = 337° \, 57'.1,$

but this gives no value for c between 0° and 180°. **(No solution.)**

If two angles and the side opposite one of them are given, solve the polar triangle. There will be the same number of solutions of the given triangle as of the polar triangle.

94. Résumé of Cases. To summarize, the cases are:

Case I. *Given two sides and the included angle*; Use the cosine law to find the third side; then use the sine law to find the other angles, determining the correct quadrant by (19), p. 111, or by means of the cosine law. See § 92.

Case $\mathrm{I_p}$. **(Polar of Case I.)** *Given two angles and the included side.* Solve the polar triangle.

Case II. *Given three sides.* Find one angle by the cosine law; then proceed as in Case I.

Case $\mathrm{II_p}$. **(Polar of Case II.)** *Given three angles.** Solve the polar triangle.

Case III. **(Ambiguous Case.)** *Given two sides and an angle opposite one of them:* Determine the third side by the cosine law, as in Example 1, p. 117, deciding the ambiguity as in Examples 1–4, § 93. Then proceed as in Case I.

Case $\mathrm{III_p}$. **(Polar of Case III.)** *Given two angles and a side opposite one of them.*† Solve the polar triangle.

The student should note that we cannot construct a spherical triangle nor compute the lengths of its sides in ordinary linear units unless the radius of the sphere is given.

EXERCISES XXXVI. — SPHERICAL TRIANGLES

1. Solve each of the following triangles:

(*a*) $a = 100°$, $b = 65°$, $A = 96°$.	(*f*) $A = 110°$, $B = 58°$, $c = 55°$.
(*b*) $a = 100°$, $b = 70°$, $A = 80°$.	(*g*) $a = 75°$, $b = 54°$, $c = 36°$.
(*c*) $A = 75°$, $B = 85°$, $b = 94°$.	(*h*) $a = 31°$, $b = 23°$, $c = 19°$.
(*d*) $a = 104°$, $b = 64°$, $C = 99°$.	(*i*) $A = 110°$, $B = 58°$, $C = 75°$.
(*e*) $A = 140°$, $B = 99°$, $c = 60°$.	(*j*) $A = 100°$, $B = 80°$, $C = 75°$.

2. Find the distance, in degrees, from St. Petersburg (lat. 60° N., lon. 30° E.) to Hopedale (lat. 55° 30′ N., lon. 60° W.) and the angle which the arc joining the two places makes with the meridian through each.

[HINT. Use the north pole for the third vertex of a triangle.]

* This case does not arise in plane triangles, since a plane triangle is not determined by its three angles.

† The corresponding case in the plane is not ambiguous because the sum of the angles of a plane triangle is precisely 180°.

3. Reduce the distance computed in Ex. 2 to miles, assuming that the radius R of the earth is 3956 mi.

4. Find the distance, in degrees, from San Francisco (lat. 37° 48′ N., lon. 122° 28′ W.) to Manila (lat. 14° 35′.5 N., 120° 58′.1 E.). Express the same distance in miles.

5. By means of the cosine law and the *polar* triangle, show that, for any triangle,

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a. \quad \text{[Polar Cosine Law.]}$$

PART III. SPECIAL METHODS FOR RIGHT TRIANGLES

95. Right Spherical Triangles. The methods of § 94 will solve all cases, including right triangles; but on account of their frequent occurrence and the simplicity of the formulas in case one angle is 90°, a separate treatment seems desirable.

A right spherical triangle can be solved when any two of its parts (in addition to the right angle) are given; in fact, if two parts are given, the other three parts can be computed in succession from the given data. All possible cases are provided for in ten formulas which, for a reason to be explained presently, are collected into two groups.

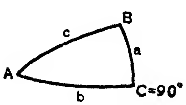


Fig. 89

Let ABC be a spherical triangle, right-angled at C; we shall call a and b its sides, c its hypotenuse, A and B its angles. The ten formulas are:

Group I	Group II
1. $\sin a = \sin c \sin A$.	6. $\sin a = \tan b \operatorname{ctn} B$.
2. $\sin b = \sin c \sin B$.	7. $\sin b = \tan a \operatorname{ctn} A$.
3. $\cos c = \cos a \cos b$.	8. $\cos c = \operatorname{ctn} A \operatorname{ctn} B$.
4. $\cos A = \cos a \sin B$.	9. $\cos A = \operatorname{ctn} c \tan b$.
5. $\cos B = \cos b \sin A$.	10. $\cos B = \operatorname{ctn} c \tan a$.

To prove 1, 2, use law of sines; to prove 3, use law of cosines; to prove 4, 5, apply the law of cosines to the polar triangle, remembering that $c' = 90°$. To prove 6, substitute in 1 the values of $\sin b$ and $\sin A$ from 2 and 5; similarly prove 7, 8, 9, 10, by substituting in 2, 3, 4, 5.

96. Napier's Rules. The preceding formulas are simple, are adapted to the use of logarithms, and are easy to apply; but there are so many of them that they become a burden to the memory. To avoid this difficulty an ingenious device was given by Napier, a Scotch mathematician (1550–1617) who invented logarithms and made important contributions to trigonometry. He states two rules which enable one to write down the particular formula needed in any given computation without attempting to remember the whole set of formulas.

These two rules of Napier employ what are called "*the five circular parts*" of a right spherical triangle. These are as indicated in the diagram, Fig. 90, *the two sides, the complement of the hypotenuse, and the complements of the two angles.* The right angle is not counted and is not indicated on the diagram.

If five objects are placed on a closed contour, *e.g.* five persons sitting at a round table, each has two neighbors which are *adjacent,* one on the right, the other on the left; and the two remaining are nonadjacent or *opposite.* So on the diagram of the five circular parts of a right spherical triangle, if we select any one, there are two others which are adjacent and the remaining two are opposite; moreover, if any three are selected, one of these may always be chosen and called the *middle part* so that the other two are either both adjacent or both opposite. Napier's rules refer to these circular parts and are as follows:

RULE 1. *The sine of the middle part is equal to the product of the cosines of the opposite parts.*

RULE 2. *The sine of the middle part is equal to the product of the tangents of the adjacent parts.*

These rules may be remembered by the alliteration of the first vowel in the words cosine and opposite, tangent and adjacent. When applied in succession to the five circular parts, rule 1 gives the formulas of group I; and rule 2, those of group II.

To apply these rules to the solution of a right spherical triangle, use the diagram of the five circular parts; mark the given parts and the part to be found; of these three choose the middle part (which may or may not be the unknown) and note

whether the other two are opposite or adjacent; then apply the appropriate rule to write down the formula, remembering that any function of the complement of an angle is the corresponding cofunction of the angle itself; *e.g.* $\sin \text{co } c = \cos c$, $\tan \text{co } B = \text{ctn } B$, $\cos \text{co } A = \sin A$, etc.

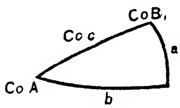


FIG. 90

Example 1. The hypotenuse c of a right spherical triangle is 124° 50′ and the angle A is 37° 25′. Solve the triangle.

To find a, mark co A, co c, and a; a is the middle part and the others are opposite; then as you say to yourself, "The sine of a is equal to the product of the cosines of co A and co c;" write $\sin a = \sin c \sin A$.

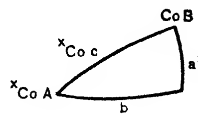


FIG. 91

Before beginning a computation by logarithms, attention must first be paid to the signs of the factors. They are here both positive, so we proceed to compute:

$$\begin{aligned} \log \sin 124° 50' &= 9.91425 - 10 \\ \log \sin \ \ 37° 25' &= \underline{9.78362 - 10} \\ \log \sin a \qquad &= 9.69787 - 10 \end{aligned}$$

Since side a is in the same quadrant as its opposite angle, $a = 29° 55'$.

To find b, mark co A, co c, and b; co A is the middle part, the others are adjacent. Say, "Sin co A is equal to the product of tan co c and tan b;" write $\cos A = \text{ctn } c \tan b$, whence $\tan b = \overset{+}{\cos} A \overset{-}{\tan} c$. Cos A is positive, tan c is negative; therefore, tan b is negative and b is in the second quadrant.

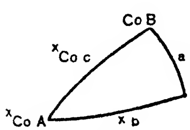


FIG. 92

To make all the factors positive we change the signs of the negative factors and write the formula

$$\tan (180° - b) = \cos A \tan (180° - c),$$

and compute as follows:

$$\begin{aligned} \log \cos 37° 25' &= 9.89995 - 10 \\ \log \tan 55° 10' &= \underline{0.15746 \qquad\ \ } \\ \log \tan (180° - b) &= 0.05741 \\ 180° - b = 48° 46'.6, &\quad b = 131° 13'.4 \end{aligned}$$

To find B, mark co A, co c, and co B; co c is the middle part, the others are adjacent:

$$\cos c = \text{ctn } A \text{ ctn } B, \text{ whence } \text{ctn } B = \cos c \tan A.$$

Signs of the factors indicate that B is in the 2d quadrant; changing signs of negative factors, the formula becomes

$$\text{ctn}\,(180^\circ - B) = \cos\,(180^\circ - c)\tan A$$

$$\begin{aligned}
\log\cos 55^\circ\,10' &= 9.75678 - 10\\
\log\tan 37^\circ\,25' &= \underline{9.88367 - 10}\\
\log\text{ctn}\,(180^\circ - B) &= 9.64045 - 10
\end{aligned}$$

$$180^\circ - B = 66^\circ\,23'.8, \quad B = 113^\circ\,36'.2$$

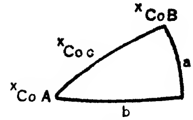


FIG. 93

97. Ambiguity. When an unknown part of a right spherical triangle is to be determined by its sine, the following statements are useful to determine whether the angle is in the first or second quadrant.

1. An angle and the opposite side are always in the same quadrant, by (4), § 95, since sin B is surely positive.

2. When the two sides are in the same quadrant, the hypotenuse is in the first quadrant; when the two sides are in different quadrants, the hypotenuse is in the second quadrant; and conversely. This follows from (3), § 95.

3. *The Ambiguous Case.* In case there are given one of the angles of a right spherical triangle and the side opposite, there are, always, two solutions which are colunar, except that if the given side is also 90°, the triangle is birectangular and the two solutions coincide. These facts are obvious geometrically, since the lune whose angle is the given angle is divided by the given side into two colunar triangles, either of which is a solution.

In no other case, when two parts besides the right angle are given, is there more than one solution.

98. Quadrantal and Isosceles Spherical Triangles. A quadrantal spherical triangle is one that has one side equal to a quadrant or 90°. Such triangles are readily solved by means of their polar triangles, which are right angled.

An isosceles spherical triangle may be divided into two symmetric right triangles by an arc perpendicular to the base through the vertex of the angle included between the equal sides.

EXERCISES XXXVII. — RIGHT SPHERICAL TRIANGLES

1. Solve the following right spherical triangles:

No.	Hypotenuse	First Side	Second Side	Angle Opposite First Side	Angle Opposite Second Side
(a)		48°	72°		
(b)		108°			38°
(c)	78°			46°	
(d)				32°	66°
(e)		70°	108°		
(f)	101° 16′.3	115° 42′.6			
(g)		160°		150°	
(h)		155° 46′.7			80° 10′.5
(i)				60° 47′.4	57° 16′.3
(j)		116°	16°		
(k)	110° 46′.3			80° 10′.5	

2. Solve the following spherical triangles:

(a) One side is 90°, and the angles adjacent are 72° 15′ and 104° 26′.
(b) The three sides are 125° 40′, 90°, 115° 10′.
(c) Two sides are 54° 20′ and 90°, and the included angle is 57° 55′.
(d) Two sides are each 80° 28′, and the angles opposite are each 33° 20′.

3. Quito is on the equator in longitude 78° 50′ W.; New York is in lat. 40° 43′, lon. 74° W. Find the distance between them and the angle that the arc connecting the two places makes with the meridian at each.

4. The arc of the great circle connecting St. Petersburg and Hopedale (lat. 55° 30′ N., lon. 60° W.) is 44° 28′ long and makes an angle of 45° 33′ with the meridian through Hopedale. Find the distance from St. Petersburg to St. Nicholas (lat. 34° 30′ S., lon. 60° W.) and the angle between the two arcs at St. Petersburg.

5. Prove that in any quadrantal spherical triangle, if c denotes the quadrantal side, *i.e.* if $c = 90°$, then:

(a) $\cos C + \cos A \cos B = \cos c$.
(b) $\cos C + \operatorname{ctn} a \operatorname{ctn} b = \cos c$.
(c) $\sin a \cos B = \cos b, \quad \sin b \cos A = \cos a$.

6. Prove that in any right spherical triangle, if C denotes the right angle, then:

$$\sin a = \cos c \tan b \tan A.$$

PART IV. SPECIAL LOGARITHMIC METHODS

99. Tangents of the Half-angles. When adapted to logarithmic computation, the law of cosines of spherical trigonometry yields two sets of formulas which are useful in the solution of triangles and are analogous to those of plane trigonometry given in §§ 26–27, pp. 34–36.

Let ABC be a spherical triangle; a small circle can be inscribed in it as follows: draw arcs of great circles bisecting its angles; these bisectors meet in some point P. This is the pole of the inscribed circle; for, from P draw arcs of great circles PL, PM, PN, perpendicular respectively to the sides $BC=a$, $CA=b$, $AB=c$. The right triangles PMA, PNA are symmetric, having the hypotenuse and an angle of the one equal respectively to the hypotenuse and an angle of the other; therefore $PM=PN$, and similarly each of these is equal to PL. Let $r=PL=PM=PN$, the polar distance of the inscribed circle. Also from the same symmetric triangles, $AM=AN$, $BN=BL$, and $CL=CM$; hence if we set $2s=a+b+c$, $AN+BL+CL=s$, whence $AN=s-(BL+CL)=s-a$, and similarly, $BL=s-b$ and $CM=s-c$.

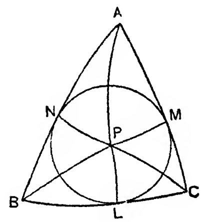


FIG. 94

In the right triangle ANP, we have, by (6) § 95, or by Napier's Rule 2,

$$\sin AN = \tan PN \cdot \operatorname{ctn} PAN,$$

$$\sin(s-a) = \tan r \operatorname{ctn}(A/2),$$

and therefore

$$\textbf{(1)} \qquad \boldsymbol{\tan(A/2) = \tan r/\sin(s-a).}$$

Similarly from triangles BLP and CMP,

$$(2) \qquad \tan(B/2) = \tan r/\sin(s-b).$$

$$(3) \qquad \tan(C/2) = \tan r/\sin(s-c).$$

Compare these with (1) § 27, p. 36.

Eliminate $\tan r$ from (1) and (2) by division:

$$(4) \qquad \frac{\tan(A/2)}{\tan(B/2)} = \frac{\sin(s-b)}{\sin(s-a)}.$$

Produce the sides AB and AC to form a lune, and apply (4) to the colunar triangle $A'B'C'$:

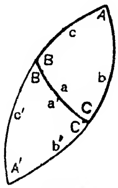


FIG. 95

$$\frac{\tan(A'/2)}{\tan(B'/2)} = \frac{\sin(s'-b')}{\sin(s'-a')},$$

or, since $s' = 180° - (s-a)$, $s'-b' = s-c$, $s'-a' = 180° - s$,

$$(5) \qquad \frac{\tan(A/2)}{\operatorname{ctn}(B/2)} = \frac{\sin(s-c)}{\sin s}.$$

Multiplying (4) and (5), we find

$$\tan^2(A/2) = \frac{\sin(s-b)\sin(s-c)}{\sin s \sin(s-a)}$$
$$= \frac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin^2(s-a)\sin s};$$

whence

$$(6) \qquad \tan(A/2) = \frac{1}{\sin(s-a)}\sqrt{\frac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s}}.$$

Comparing (1) and (6), we find

$$(7) \qquad \boldsymbol{\tan r = \sqrt{\frac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s}}}.$$

Compare this result with that of § 29, p. 39.

Formulas (1), (2), (3), and (7) hold for all spherical triangles which have no side or angle greater than 180°.*

100. The Law of Tangents, or Napier's Analogies. Let ABC be a spherical triangle; suppose $A > B$. Then at B, the smaller of the two angles, lay off angle $ABA' = A$. Apply formula (4) of § 99 to angles B' and C' of the triangle $A'B'C'$.

$$\frac{\tan(B'/2)}{\tan(C'/2)} = \frac{\sin(s'-c')}{\sin(s'-b')},$$

$B' = A - B$, $C' = 180° - C$, $s' = (a + b' + c')/2$; but since triangle $A'AB$ is isosceles, $c' = b + b'$; hence $s' = (a+b)/2 + b'$,

* All these formulas can be deduced from the law of cosines by an algebraic process without reference to any particular figure. See, for example, Chauvenet's *Treatise on Spherical Trigonometry*.

$s'-b'=(a+b)/2$, $s'-c'=(a-b)/2$. Therefore

$$\frac{\tan[(A-B)/2]}{\operatorname{ctn}(C/2)}=\frac{\sin[(a-b)/2]}{\sin[(a+b)/2]},$$

or

$$(1)\qquad \tan\frac{A-B}{2}=\frac{\sin[(a-b)/2]}{\sin[(a+b)/2]}\operatorname{ctn}\frac{C}{2}.$$

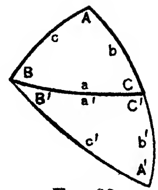


FIG. 96

If $A<B$, the result is obviously

$$\tan\frac{B-A}{2}=\frac{\sin[(b-a)/2]}{\sin[(b+a)/2]}\operatorname{ctn}\frac{C}{2},$$

which comes to the same thing as (1).

If $A=B$, then $a=b$ and (1) is satisfied. Therefore, the formula holds for all cases, since (4) of § 99 does.

Produce the sides AB and BC to form a lune, and apply formula (1) to the triangle $A'B'C'$, colunar to ABC.

$$\tan\frac{A'-B'}{2}=\frac{\sin[(a'-b')/2]}{\sin[(a'+b')/2]}\operatorname{ctn}\frac{C'}{2}.$$

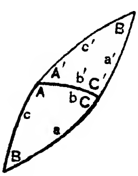


FIG. 97

Substitute $180°-A$ for A', B for B', and so on:

$$\operatorname{ctn}\frac{A+B}{2}=\frac{\cos[(a+b)/2]}{\cos[(a-b)/2]}\tan\frac{C}{2},$$

or

$$(2)\qquad \tan\frac{A+B}{2}=\frac{\cos[(a-b)/2]}{\cos[(a+b)/2]}\operatorname{ctn}\frac{C}{2}.$$

If we divide (1) by (2) we obtain a formula

$$\frac{\tan[(A-B)/2]}{\tan[(A+B)/2]}=\frac{\tan[(a-b)/2]}{\tan[(a+b)/2]},$$

which is analogous to the law of tangents for plane triangles.

Formulas (1) and (2) are called **Napier's Analogies.** They furnish another proof of (19) (f) and (g), p. 111.

101. Solutions of Triangles by Logarithms. *When two sides and the included angle are given* (Case I of § 94), the other two angles can be found by Napier's Analogies, (1) and (2) of § 100, and then the third side can be found by the law of sines.

Example 1. Assuming the radius of the earth to be 3956 miles, find the shortest distance from Seattle (lat. 47° 36′ N., lon. 122° 20′ W.) to Manila (lat. 14° 35′.5 N., lon. 120° 58′.1 E.), and the direction of the course at each place.

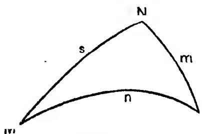


FIG. 98

Take the north pole for the third vertex; then

$$m = 42° 24', \quad s = 75° 24'.5; \quad N = 116° 41'.9$$

To find the angles at M and S, we write, since $s > m$,

$$\tan\frac{S-M}{2} = \frac{\sin[(s-m)/2]}{\sin[(s+m)/2]}\operatorname{ctn}\frac{N}{2}, \qquad [(1), \S\ 100]$$

$$\tan\frac{S+M}{2} = \frac{\cos[(s-m)/2]}{\cos[(s+m)/2]}\operatorname{ctn}\frac{N}{2}, \qquad [(2), \S\ 100]$$

$(s-m)/2 = 16° 30'.2 \qquad (s+m)/2 = 58° 54'.2 \qquad N/2 = 58° 20'.9$

Arrange the computation so as to look up both sine and cosine of $(s-m)/2$ and $(s+m)/2$ at one reading:

$$\begin{aligned}
\log\sin 16° 30'.2 &= 9.45343 - 10\\
\log\operatorname{ctn} 58° 20'.9 &= 9.78990 - 10\\
\operatorname{colog}\sin 58° 54'.2 &= \underline{0.06737}\\
\log\tan[(S-M)/2] &= 9.31070 - 10\\
(S-M)/2 &= 11° 33'.5
\end{aligned}$$

$$\begin{aligned}
\log\cos 16° 30'.2 &= 9.98173 - 10\\
\log\operatorname{ctn} 58° 20'.9 &= 9.78990 - 10\\
\operatorname{colog}\cos 58° 54'.2 &= \underline{0.28694}\\
\log\tan[(S+M)/2] &= 0.05857\\
(S+M)/2 &= 48° 51'.1
\end{aligned}$$

By addition and subtraction, $\quad S = 60° 24'.6 \qquad M = 37° 17'.6$

The course leaves Seattle going N. 60° 24′.6 W. and arrives at Manila going S. 37° 17′.6 W. To find n, use $\sin n/\sin s = \sin N/\sin S$:

$$\begin{aligned}
\log\sin 75° 24'.5 &= 9.98576 - 10\\
\log\sin 116° 41'.9 &= 9.95104 - 10\\
\operatorname{colog}\sin 60° 24'.6 &= \underline{0.06069}\\
\log\sin n &= 9.99749 - 10
\end{aligned}$$

Hence either $\quad n = 83° 51' \quad$ or $\quad n = 96° 9'$.

By the law of cosines, $\cos n = (.74)(.25) - (.67)(.97)(.45)$ approximately; hence $\cos n$ is negative; therefore $n = 96° 9'$.

Hence the length of the course in miles (to the nearest mile) is $n\pi R/180 = 96.15 \times 3.1416 \times 3956/180 = 6639.$ (Compute by logarithms.)

CASE II. *When the three sides are given* (Case II of § 94), the angles can be found by the half-angle formulas of § 99.

Illustrative Example. The face angles of a trihedral angle are 50° 12′.1, 116° 44′.8, 129° 11′.7; find the dihedral angles.

$$\begin{array}{rl} a = & 50^\circ\ 12'.1 \\ b = & 116^\circ\ 44'.8 \\ c = & \underline{129^\circ\ 11'.7} \\ 2) & \underline{296^\circ\ \ 8'.6} \\ s = & 148^\circ\ \ 4'.3 \\ s-a = & 97^\circ\ 52'.2 \\ s-b = & 31^\circ\ 19'.5 \\ s-c = & 18^\circ\ 52'.6 \end{array}
\qquad
\begin{array}{rl} \log\sin(s-a) = & 9.99589-10 \\ \log\sin(s-b) = & 9.71591-10 \\ \log\sin(s-c) = & 9.50992-10 \\ \text{colog}\sin s = & \underline{0.27666\qquad\quad} \\ 2) & 19.49838-20 \\ \log\tan r = & 9.74919-10 \\ \log\tan(A/2) = & 9.75330-10 \\ \log\tan(B/2) = & 0.03328 \\ \log\tan(C/2) = & 0.23927 \end{array}$$

$$\begin{array}{lll} A/2 = 29^\circ\ 32'.2 & B/2 = 47^\circ\ 11'.6 & C/2 = 60^\circ\ 2'.4 \\ A = 59^\circ\ \ 4'.4 & B = 94^\circ\ 23'.2 & C = 120^\circ\ 4'.8 \end{array}$$

CASE III. *When two sides and the angle opposite one of them are given* (Case III of § 94), first find the angle opposite the other given side by the law of sines, and determine the number of solutions (see § 93). Use one of Napier's Analogies to find the third angle, and the law of sines to find the third side.

Example 1. One angle of a spherical triangle is 103° 40′, the side opposite is 56° 40′, and another side is 30° 50′. Solve the triangle.

Let $C = 103^\circ\ 40'$, $c = 56^\circ\ 40'$, $b = 30^\circ\ 50'$; to find B, use law of sines:

$$\sin B/\sin C = \sin b/\sin c$$

$$\begin{array}{rl} \log\sin 103^\circ\ 40' = & 9.98753-10 \\ \log\sin 30^\circ\ 50' = & 9.70973-10 \\ \text{colog}\sin 56^\circ\ 40' = & \underline{0.07806\qquad\quad} \\ \log\sin B = & 9.77532-10 \end{array}$$

Therefore, since by § 92, b and B must be in the same quadrant, we have $B = 36^\circ\ 35'.5$, and there is only one solution.

To find A use one of Napier's Analogies, say (1) of § 100:

$$\text{ctn}\,\frac{A}{2} = \frac{\sin[(c+b)/2]}{\sin[(c-b)/2]}\tan\frac{C-B}{2}$$

$$(c+b)/2 = 43^\circ\ 45',\ (c-b)/2 = 12^\circ\ 55',\ (C-B)/2 = 33^\circ\ 32'.2$$

$$\begin{array}{rl} \log\sin 43^\circ\ 45' = & 9.83980-10 \\ \log\tan 33^\circ\ 32'.2 = & 9.82139-10 \\ \text{colog}\sin 12^\circ\ 55' = & \underline{0.65066\qquad\quad} \\ \log\text{ctn}\,(A/2) = & 0.31185 \end{array}
\qquad
\begin{array}{rl} A/2 = & 25^\circ\ 59'.9 \\ A = & 51^\circ\ 59'.9 \end{array}$$

Finally, from $\sin a/\sin c = \sin A/\sin C$, we find

$$\log\sin a = 9.83093 - 10;\ \text{hence,}$$

since a and A must be in the same quadrant, $a = 42^\circ\ 39'.1$.

EXERCISES XXXVIII. — SOLUTION BY LOGARITHMIC METHODS

Solve the following spherical triangles :

No.	a	b	c	A	B	C
1	30° 20′	46° 30′		36° 40′		
2		147° 6′			110° 10′	100°
3	130°				66°	
4	115° 10′	125° 20′	70° 30′			
5	70°	80°	170°			
6	109° 20′		82°	107° 40′		
7				56° 32′	69° 7′	78° 58′
8	93° 20′		56° 30′		74° 40′	
9			74° 20′	67° 30′	45° 50′	
10		40° 35′		51° 58′		83° 54′
11	28° 48′	98° 10′	80° 12′			
12			127°		132°	140°
13	75°	120°				145°
14	62° 20′	54° 10′	97° 50′			
15		100°	65°	96°		
16		40° 40′		31° 40′		122° 20′
17			42°	51° 58′	83° 54′	
18	40°	118° 20′		29° 40′		
19	130°				55° 25′	44° 58′
20	78°				55° 25′	44° 58′
21		99° 40′	64° 20′		95° 40′	
22		43° 18′	19° 24′	74° 22′		
23	47° 15′		42° 45′			56° 30′

24. Find one of the dihedral angles of a regular tetrahedron.

[Hint. Each face of a regular tetrahedron is an equilateral triangle.]

25. Find one of the dihedral angles of a regular dodecahedron.

26. The dihedrals of a trihedral angle are 60° 53′, 60° 53′, 66° 50′. Find the face angles.

27. Two face angles of a trihedral angle are 96° 24′ and 68° 27′; the dihedral between these faces is 84° 46′. Find the other face angle.

28. Find the shortest distance from Sandy Hook (lat. 40° N., lon. 74° W.) to Gibraltar (lat. 36° 6′.4 N., lon. 5° 20′.9 E.) and the direction of the course of each place.

29. Find the distance from San Francisco (lat. 37° 48′ N., lon. 122° 28′ W.) to Hong Kong (lat. 22° 17′ N., lon. 114° 10′ E.) and the direction of the course at each place.

102. Area of Spherical Triangles. A **spherical degree** is 1/720 of the surface of the sphere. Thus, the area of a lune whose angle is 1° is two spherical degrees. The area of a birectangular triangle whose third angle is 1° is one spherical degree.

The **spherical excess**, E, of a triangle is the excess of the sum of its angles over two right angles. (See (19) and (20), § 89.) It is proved in geometry that the area of the entire surface of a sphere is $4\pi r^2$, and that the number of spherical degrees in the area A_t of a triangle is equal to its spherical excess. Therefore if E denotes the spherical excess of a triangle and R the radius of the sphere, then the area A_t of the triangle is

$$A_t = \pi R^2 E/180, \tag{1}$$

if E denotes the number of *degrees* in the spherical excess; or

$$A_t = R^2 E, \tag{2}$$

if E denotes the number of *radians* in the spherical excess. (See (20), § 89.) We can therefore compute the area of a spherical triangle as soon as we know its angles.

Example. Find the area of a triangle whose angles are 94° 30′, 116° 25′, 72° 20′, on a sphere of radius 50 ft.

$$E = 103°.25\ ;\ A_t = \frac{3.1416 \times \overline{50}^2 \times 103.25}{180}.$$

$$\begin{aligned} \log 3.1416 &= 0.49715 \\ 2 \log 50 &= 3.39794 \\ \log 103.25 &= 2.01389 \\ \text{colog } 180 &= \underline{7.74473 - 10} \\ \log A_t &= 3.65371 \qquad A_t = 4505.2. \end{aligned}$$

If a triangle is given by means of other parts than the angles, the angles may be computed by the preceding methods and the spherical excess E can be found as above. Special formulas are sometimes given for direct computation of E without first finding the angles.*

* Thus, if two sides and the included angle are given, say a, b, and C, then

$$\tan \tfrac{1}{2} E = \frac{\tan \frac{1}{2} a \tan \frac{1}{2} b \sin C}{1 + \tan \frac{1}{2} a \tan \frac{1}{2} b \cos C}.$$

If the three sides are given,

$$\tan \tfrac{1}{4} E = \sqrt{\tan \tfrac{1}{2} s \tan \tfrac{1}{2}(s-a) \tan \tfrac{1}{2}(s-b) \tan \tfrac{1}{2}(s-c)}.$$

For proof of these formulas, consult, *e.g.*, Chauvenet, *Treatise on Spherical Trigonometry*.

EXERCISES XXXIX. — AREAS OF SPHERICAL TRIANGLES

1. Find the areas of the following spherical triangles:

No.	a	b	c	A	B	C	R: Radius of Sphere
(a)				64° 48′	40° 24′	120° 46′	1000 ft.
(b)				86° 30′	54° 46′	63° 12′	750 m.
(c)	43° 30′	72° 24′	87° 50′				100 m.
(d)		73° 58′	38° 45′	46° 33′			20 mi.
(e)			120° 10′	65° 13′	49° 27′		36 yd.
(f)	18° 12′		90°		74° 45′		40 rd.
(g)			90°	110° 48′	135° 35′		200 ft.
(h)	108° 14	75° 29′	56° 37′				600 ft.
(i)	132° 14′.2		97° 13′.1		81° 58′.9		2.3 mi.
(j)	88° 12′		59° 4′		132° 18′		30 mi.

2. Find the area of a trirectangular triangle on the earth's surface. (Radius = 3956 mi.)

3. Find the area on the earth's surface of a birectangular triangle whose third angle is 21°.

4. Find the third angle of a birectangular triangle on the earth's surface whose area is 10,000 sq. mi.

5. Find the area of a triangle whose vertices are San Francisco (lat. 37° 48′ N., lon. 122° 28′ W.), Seattle (lat. 47° 36′ N., lon. 122° 20′ W.), and Manila (lat. 14° 35′.5 N., lon. 120° 58′.1 E.).

6. If a survey of a triangle is made on the earth's surface, a map drawn from it on plane paper will not check precisely. If the area surveyed is less than one thousand square miles, show that the discrepancy in the sum of the angles of any triangle in it is less than 14″.

7. Show that in general the discrepancy, in seconds, on a plane map of the earth of any triangle whose area in square miles is A_t is less than $.01315 \times A_t$ (seconds).

8. If the angles must check on a map to within one second in any triangle, how large an area can be safely surveyed on the assumption that the earth is a plane?

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PREPARED UNDER THE DIRECTION OF

EARLE RAYMOND HEDRICK

TO ACCOMPANY THE

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ALFRED MONROE KENYON

AND

LOUIS INGOLD

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EXPLANATION OF THE TABLES*

TABLE I. FIVE-PLACE COMMON LOGARITHMS OF NUMBERS FROM 1 TO 10 000

1. Powers of 10. Consider the following table of values of powers of 10:

Column A		Column B	Column A		Column B
10^1	=	10	10^0	=	1.
10^2	=	100	10^{-1}	=	.1
10^3	=	1000	10^{-2}	=	.01
10^4	=	10000	10^{-3}	=	.001
10^5	=	100000	10^{-4}	=	.0001
10^6	=	1000000	10^{-5}	=	.00001
10^7	=	10000000	10^{-6}	=	.000001
10^8	=	100000000	10^{-7}	=	.0000001
10^9	=	1000000000	10^{-8}	=	.00000001
10^{10}	=	10000000000	10^{-9}	=	.000000001

This table may be used for multiplying or dividing powers of 10, by means of the rules $10^a \cdot 10^b = 10^{a+b}$, $10^a \div 10^b = 10^{a-b}$. Thus, to multiply 1000 by 100,000, add the exponent of 10 in column A opposite 1000 to the exponent of 10 opposite 100,000: $3 + 5 = 8$; and look for the number in column B opposite 10^8, *i.e.* 100,000,000. Similarly $1{,}000{,}000 \times .0001 = 100$, since $6 + (-4) = 2$.

To divide 1,000,000 by 100, from the exponent of 10 opposite 1,000,000 subtract the exponent of 10 opposite 100; $6 - 2 = 4$; and look for the number opposite 10^4, *i.e.* 10,000. Similarly $.001 \div 1{,}000{,}000 = .000000001$, since $-3 - 6 = -9$. To find the 4th power of 100, multiply the exponent of 10 opposite 100 by 4: $4 \times 2 = 8$, and look for the number opposite 10^8, *i.e.* 100,000,000. Likewise $(.001)^3 = .000000001$, since $3 \times (-3) = -9$. To find the cube root of 1,000,000,000, divide the exponent of 10 opposite 1,000,000,000 by 3, $9 \div 3 = 3$, and look for the number opposite 10^3.

* This Explanation, written to accompany the five-place tables, may be used also for the four-place tables by omitting the last figure in each example in a manner obvious to the teacher.

2. Common Logarithms. The exponent of 10 in any row of column A is called the common logarithm* of the number opposite in column B; thus $\log 10 = 1$, $\log 100 = 2$, $\log 1000 = 3$, etc.; $\log 1 = 0$, $\log .1 = -1$; $\log .01 = -2$, $\log .001 = -3$, etc. In general, if $10^l = n$, l is called the *common logarithm of* n, and is denoted by $\log n$.

3. Fundamental Principles. Logarithms are useful in reducing the labor of performing a series of operations of multiplication, division, raising to powers, extracting roots, as above; they have no necessary connection with trigonometry, since all the operations could be performed without them; but they are a great labor-saving device in arithmetical computations. They do not apply to addition and subtraction.

The principles of their application are stated as follows:

I. *The logarithm of a product is equal to the sum of the logarithms of the factors:* $\log ab = \log a + \log b$. This follows from the fact that if $10^l = a$ and $10^L = b$, $10^{l+L} = a \cdot b$. In brief: *to multiply, add logarithms.*

II. *The logarithm of a fraction is equal to the difference obtained by subtracting the logarithm of the denominator from the logarithm of the numerator:* $\log (a/b) = \log a - \log b$. For, if $10^l = a$ and $10^L = b$, then $10^{l-L} = a \div b$. In brief: *to divide, subtract logarithms.*

III. *The logarithm of a power is equal to the logarithm of the base multiplied by the exponent of the power:* $\log a^b = b \log a$. This follows from the fact that if $10^l = a$, then $10^{lb} = a^b$.

IV. *The logarithm of a root of a number is found by dividing the logarithm of the number by the index of the root:* $\log \sqrt[b]{a} = (\log a)/b$. This follows from the fact that if $10^l = a$, then $10^{l/b} = a^{1/b} = \sqrt[b]{a}$.

Corollary of II. *The logarithm of the reciprocal of a number is the negative of the logarithm of the number:* $\log (1/a) = -\log a$, since $\log 1 = 0$.

4. Characteristic and Mantissa. It is shown in algebra that every real positive number has a real common logarithm, and that if a and b are any two real positive numbers such that $a < b$, then $\log a < \log b$. Neither zero nor any negative number has a real logarithm.

An inspection of the following table, which is a restatement of a part

a	1	10	100	1000	10000	100000	1000000	10000000
$\log a$	0	1	2	3	4	5	6	7

* Common logarithms are exponents of the base 10; other systems of logarithms have bases different from 10; Napierian logarithms (see Table VII, p. 112) have a base denoted by e, an irrational number whose value is approximately 2.71828. When it is necessary to call attention to the base, the expression $\log_{10} n$ will mean common logarithm of n; $\log_e n$ will mean the Napierian logarithm, etc.; but in this book $\log n$ denotes $\log_{10} n$ unless otherwise explicitly stated.

of the table of § 1, p. v, shows that

the logarithm of every number between 1 and 10 is a proper fraction,
the logarithm of every number between 10 and 100 is 1 + a fraction,
the logarithm of every number between 100 and 1000 is 2 + a fraction;
and so on. It is evident that the logarithm of every number (not an exact power of 10) consists of a whole number + a fraction (usually written as a decimal). The whole number is called the **characteristic**; the decimal is called the **mantissa**. The characteristic of the logarithm of any number greater than 1 may be determined as follows:

RULE I. *The characteristic of any number greater than 1 is one less than the number of digits before the decimal point.*

The following table, which is taken from § 1, p. v, shows that

a	.0000001	.000001	.00001	.0001	.001	.01	.1	1
$\log a$	-7	-6	-5	-4	-3	-2	-1	0

the logarithm of every number between .1 and 1 is -1 + a fraction,
the logarithm of every number between .01 and .1 is -2 + a fraction,
the logarithm of every number between .001 and .01 is -3 + a fraction;
and so on.

Thus the characteristic of every number between 0 and 1 is a negative whole number; there is a great practical advantage, however, in computing, to write these characteristics as follows: $-1 = 9 - 10$, $-2 = 8 - 10$, $-3 = 7 - 10$, etc. *E.g.* the logarithm of .562 is $-1 + .74974$, but this should be written $9.74974 - 10$; and similarly for all numbers less than 1.

RULE II. *The characteristic of a number less than 1 is found by subtracting from 9 the number of ciphers between the decimal point and the first significant digit, and writing -10 after the result.*

Thus, the characteristic of $\log 845$ is 2 by Rule I; the characteristic of $\log 84.5$ is 1 by (I); of $\log 8.45$ is 0 by (I); of $\log .845$ is $9 - 10$ by (II); of $\log .0845$ is $8 - 10$ by (II).

An important consequence of what precedes is the following:

To move the decimal point in a given number one place to the right is equivalent to adding one unit to its logarithm, because this is equivalent to multiplying the given number by 10. Likewise, to move the decimal point one place to the left is equivalent to subtracting one unit from the logarithm. Hence, moving the decimal point any number of places to the right or left does not change the mantissa but only the characteristic.*

Thus, 5345, 5.345, 534.5, .05345, 534500 all have the same mantissa.

* Another rule for finding the characteristic, based on this property, is often useful: if the decimal point were just after the first significant figure, the characteristic would be zero; start at this point and count the digits passed over to the left or right to the actual decimal point; the number obtained is the characteristic, except for sign; the sign is negative if the movement was to the left, positive if the movement was to the right.

5. Use of the Table. To use logarithms in computation we need a table arranged so as to enable us to find, with as little effort and time as possible, the logarithms of given numbers and, vice versa, to find numbers when their logarithms are known. Since the characteristics may be found by means of Rules I and II, p. vii, only mantissas are given. This is done in Table I. Most of the numbers in this table are irrational, and must be represented in the decimal system by approximations. A five-place table is one which gives the values correct to five places of decimals.

PROBLEM 1. *To find the logarithm of a given number.* First, determine the characteristic, then look in the table for the mantissa.

To find the mantissa in the table when the given number (neglecting the decimal point) consists of four, or less, digits (exclusive of ciphers at the beginning or end), look in the column marked N for the first three digits and select the column headed by the fourth digit: the mantissa will be found at the intersection of this row and this column. Thus to find the logarithm of 72050, observe first (Rule I) that the characteristic is 4. To find the mantissa, fix attention on the digits 7205; find 720 in column N, and opposite it in column 5 is the desired mantissa, .85763; hence $\log 72050 = 4.85763$. The mantissa of .007826 is found opposite 782 in column 6 and is .89354; hence $\log .007826 = 7.89354 - 10$.

6. Interpolation. If there are more than four significant figures in the given number, its mantissa is not printed in the table; but it can be found approximately by assuming that the mantissa varies as the number varies in the small interval not tabulated; while this assumption is not strictly correct, it is sufficiently accurate for use with this table.

Thus, to find the logarithm of 72054 we observe that $\log 72050 = 4.85763$ and that $\log 72060 = 4.85769$. Hence a change of 10 in the number causes a change of .00006 in the mantissa; we assume therefore that a change of 4 in the number will cause, approximately, a change of $.4 \times .00006 = .00002$ (dropping the sixth place) in the mantissa; and we write $\log 72054 = 4.85763 + .00002 = 4.85765$.

The difference between two successive values printed in the table is called a **tabular difference** (.00006, above). The proportional part of this difference to be added to one of the tabular values is called the **correction** (.000002, above), and is found by multiplying the tabular difference by the appropriate fraction (.4, above). These proportional parts are usually written *without the zeros*, and are printed at the right-hand side of each page, to be used when mental multiplications seem uncertain.

Example 1. Find the logarithm of .0012647. Opposite 126 in column 4 find .10175; the tabular difference is 34 (zeros dropped); $.7 \times 34$ is given in the margin as 24; this correction added gives .10199 as the mantissa of .0012647; hence $\log .0012647 = 7.10199 - 10$.

Example 2. Find the logarithm of 1.85643. Opposite 185 in column 6 find .26858; tabular difference 23; $.43 \times 23$ is given in the margin as 10; this correction added gives .26868 as the mantissa of 1.85643; hence $\log 1.85643 = 0.26868$.

7. Reverse Reading of the Table. Problem 2. *To find the number when its logarithm is known.* First, fixing attention on the mantissa only, find from the table the number having this mantissa, then place the decimal point by means of the two following rules:*

Rule III. *If the characteristic of the logarithm is positive (in which case the mantissa is not followed by -10), begin at the left, count digits one more than the characteristic, and place the decimal point to the right of the last digit counted.*

Rule IV. *If the characteristic is negative (in which case the mantissa will be preceded by a number n and followed by -10†), prefix $9-n$ ciphers, and place the decimal point to the left of these ciphers.*

Example 1. Given $\log x = 1.22737$, to find x.

Since the mantissa is 22737, we look for 22 in the first column and to the right and below for 737, which we find in column 8 opposite 168. The number is therefore 1688. Since the characteristic is $+1$, we begin at the left, count 2 places, and place the point; hence $x = 16.88$.

Example 2. Given $\log x = 2.24912$, to find x.

This mantissa is not found in the table; in such cases we interpolate as follows: select the mantissa in the table next less than the given mantissa, and write down the corresponding number; here, 1774; the tabular difference is 25; the actual difference (found by subtracting the mantissa of 1774 from the given mantissa) is 17; hence the proportionality factor is $17/25 = .68$ or $.7$ (to the nearest tenth). Since moving the decimal point does not affect the mantissa, it follows that the digits in the required number are 17747 (to five places). The characteristic 2 directs to count 3 places from the left; hence $x = 177.47$.

Rule. *In general, when the given mantissa is not found in the table, write down four digits of the number corresponding to the mantissa in the table next less than the given mantissa, determine a fifth figure by dividing the actual difference by the tabular difference, and locate the decimal point by means of the characteristic.*

8. Illustrations of the Use of Logarithms in Computation.

Example 1. To find $832.43 \times 302.43 \times 16.725 \times .000178$.

$$\begin{aligned}
\log 832.43 &= 2.92034 \\
\log 302.43 &= 2.48062 \\
\log 16.725 &= 1.22337 \\
\log .000178 &= \underline{6.25042 - 10} \text{ (add)} \\
\log x &= 2.87475 \qquad \text{whence } x = 749.47.
\end{aligned}$$

Example 2. To find $461.29 \div 21.4$.

$$\begin{aligned}
\log 461.29 &= 2.66397 \\
\log 21.4 &= \underline{1.33041} \qquad \text{(subtract)} \\
\log x &= 1.33356 \qquad \text{whence } x = 21.556.
\end{aligned}$$

* Another convenient form of these rules is as follows: if the characteristic were zero, the decimal point would fall just after the first significant figure; move the decimal point one place to the right for each positive unit in the characteristic, one place to the left for each negative unit in the characteristic.

† In rare cases -20, -30, etc.

Illustration of Cologarithms

Example 3. To find $\frac{48.25 \times 132.76 \times .1745}{1415.3}$.

We might add the logarithms of the factors in the numerator and from this sum subtract the logarithm of the denominator; but we can shorten the operation by *adding* the negative of the logarithm of the denominator instead of subtracting the logarithm itself. The negative of the logarithm of a number (when written in convenient form for computation) is called the **cologarithm** of the number. We may find the negative of any number by subtracting it from zero, and it is convenient in logarithmic computation to write zero in the form 10.00000 − 10. Thus the negative of 2.17 is 7.83 − 10; the negative of 1.1432 − 10 is 8.8568. Remembering that the cologarithm of a number is its negative we have the following rule:

To find the cologarithm of a number begin at the left of its logarithm (including the characteristic) and subtract each digit from 9, except the last, which subtract from 10; if the logarithm has not − 10 after the mantissa, write − 10 after the result; if the logarithm has − 10 after the mantissa, do not write − 10 after the result.*

By this rule the cologarithm of a number can be read directly out of the table without taking the trouble to write down the logarithm. Attention must be given not to forget the characteristic. The use of the cologarithm is governed by the principle:

Adding the cologarithm is equivalent to subtracting the logarithm.

Returning to the computation of the given problem we should write:

$$
\begin{aligned}
\log 48.25 &= 1.68350 \\
\log 132.76 &= 2.12307 \\
\log .1745 &= 9.24180 - 10 \\
\operatorname{colog} 1415.3 &= \underline{6.84915 - 10} \quad \text{(add)} \\
\log x &= 9.89752 - 10 \quad \text{whence } x = .7898
\end{aligned}
$$

Example 4. Find the 5th power of 7.26842

$$
\begin{aligned}
\log 7.26842 &= 0.86144 \\
&\quad\ \underline{\quad 5} \quad \text{(multiply)} \\
\log x &= 4.30720 \quad \text{whence } x = 20286.
\end{aligned}
$$

Example 5. Find the 4th root of .007564

$$\log .007564 = 7.87875 - 10.$$

(It is convenient to have, after the division by 4, − 10 after the mantissa; hence before the division we add 30.00000 − 30.)

$$
\begin{aligned}
\log .007564 &= 37.87875 - 40 \quad \text{(divide by 4)}, \\
\log x &= 9.46969 - 10 \quad \text{whence } x = .2949
\end{aligned}
$$

Example 6. Find the value of $\sqrt[3]{\frac{(34.55)(-856.7)(-43.5)}{(98.75)(-186.3)}}$.

We have no logarithms of negative numbers, but an inspection of this problem shows that the result will be negative and numerically the same as though all the factors were positive; hence we proceed as follows:

$$
\begin{aligned}
\log 34.55 &= 1.53845 \\
\log 856.7 &= 2.93283 \\
\log 43.5 &= 1.63849 \\
\operatorname{colog} 98.75 &= 8.00546 - 10 \\
\operatorname{colog} 186.3 &= \underline{7.72979} - 10 \quad \text{(add)} \\
&\quad 1.84502 \quad \text{(divide by 3)} \\
\log(-x) &= 0.61501 \quad \text{whence } x = -4.121
\end{aligned}
$$

* If the logarithm ends in one or more ciphers, the last significant digit is to be understood here.

9. The Slide Rule. A slide rule consists of two pieces of the shape of a ruler, one of which slides in grooves in the other; each is marked

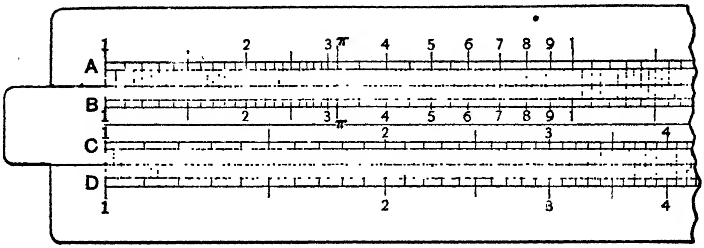

FIG. 1

(Fig. 1) in divisions (scale A and scale B) whose distances from one end are proportional to the logarithms of the numbers marked on them.

It follows that the sum of two logarithms can be obtained by simply

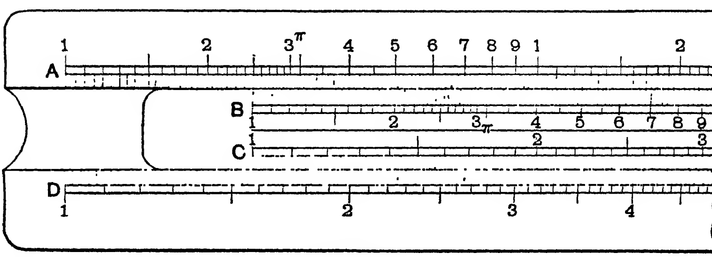

FIG. 2

sliding one rule along the other; thus if (see Fig. 2) the point marked 1 on scale B is set opposite the point marked 2.5 on scale A, the point on scale B marked 2 will be opposite the point on scale A marked 5, since $\log 2.5 + \log 2 = \log 5$. Likewise, opposite 3 (scale B) read 7.5 (scale A); opposite 2.5 (B) read 6.25 (A), *i.e.* $2.5 \times 2.5 = 6.25$.

Other multiplications can be performed in an analogous manner. Divisions can be performed by reversing the operation. Thus, if 4.5 (B) be set on 11.25 (A), then 1 (B) will be opposite 2.5 (A), as in Fig. 2.

Scales C and D are made just twice as large as scales A and B. It follows that the numbers marked on C and D are the square roots of the numbers marked opposite them on scales A and B.

For a description of more elaborate slide rules, and full directions for use, see the catalogues of instrument makers.

The student should use a slide rule in **checking results**; practice may be had by checking many of the results of the following list of exercises.

10. Graphical Representation of Interpolation. In the process of interpolation, values are inserted as if the logarithm varied directly as the number, between the two nearest values given in the table. Graphically, this means that the interpolation is made as if the curve $y = \log x$ consisted of a straight line segment.

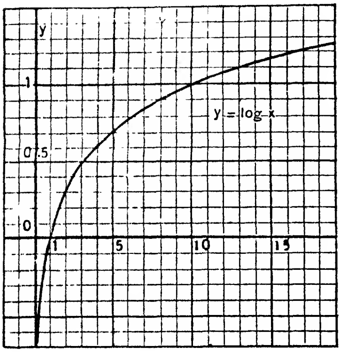

FIG. 3

If the values of x and $y = \log x$ are plotted in the usual manner, the curve obtained is that shown in Fig. 3. The values of x and y given in the table fall so close to each other on this figure that the interpolating line cannot be shown. But if the portion of the figure near $x = 2$, $y = .30103$ be enlarged in the ratio 1 to 10000 on the x-axis and 1 to 1000 on the y-axis, the resulting figure is as shown in Fig. 4. The point A shows $x = 2.001$, $y = .30125$; the point B shows $x = 2.002$, $y = .30146$; if we draw the straight line ANB, it is clear that the straight line differs from the true curve AMB, but the difference is very slight.

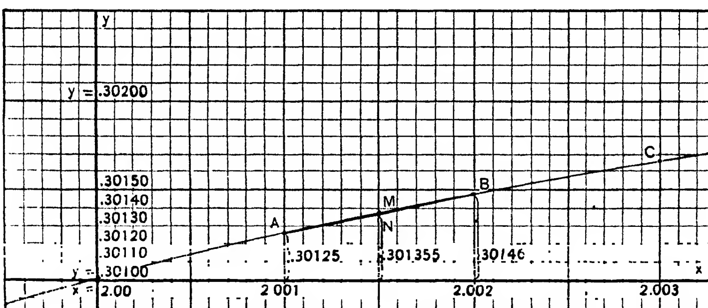

FIG. 4

Thus, the value of y given by interpolation for $x = 2.0015$ is shown at N; it is $y = .301355$. The true value of $\log 2.0015$, found from a higher place table is really .3013556; but either of these results would be written .30136, so that the error made in using the straight line ANB in place of the curve AMB does not affect the fifth place of decimals.

EXERCISES

1. Find the values of each of the following products by logarithms; check each computation by a multiplication of round numbers.

(*a*) 3.1416×205.6. (*b*) 64.32×2780.5. (*c*) $32.16 \times (-44.52)$.
(*d*) $231.6 \times .0024$. (*e*) $(-.003714) \times (1206.5)$. (*f*) $.963752 \times .0010746$.

2. Substitute $\div$ for $\times$ in each of the parts of Ex. 1, and then find the indicated quotient in each case by logarithms.

3. Find the value of each of the following expressions by logarithms; check each computation.

(*a*) $\dfrac{3.1416 \times 2109.4}{732.56 \times 23.5}$. (*b*) $\dfrac{725 \times (-3.472)}{6805.4 \times .0126}$. (*c*) $(3.1416)^2$. (*d*) $\sqrt{3.1416}$. (*e*) $(1.728)^5$. (*f*) $(2.469)^{3/2}$.
(*g*) $(-27.845)^3$. (*h*) $(.000165)^{1/7}$. (*i*) $(3.1416)(2.34)^3 \div (.006)^{1/3}$.

4. Find the area of a circle whose radius is 47.5 ft.

5. Find the area of a rectangle whose base is 231.75 and whose height is 514.25.

6. Find the area and the volume of a sphere whose radius is 4.6152.

7. Given 1 cm. = .3937 in., reduce 4752.6 cm. to inches.

8. Reduce 675 sq. cm. to square inches.

9. Given 365.242 mean solar days = 366.242 sidereal days, express 1 mean solar day in terms of sidereal days; express 1 sidereal day in terms of mean solar days.

10. The amount a of a principal p at compound interest of rate r for n years is given by the formula: $a = p(1+r)^n$. Find a if $p = 12{,}753$, $r = .06$, and $n = 5$.

11. Evaluate each of the following expressions:

(*a*) $\sqrt{3}$. $\sqrt[3]{5}$. $\sqrt[5]{7}$. (*c*) $\dfrac{5.62 \times (4.8)^{1.5}}{(.684)^{2.3}}$. (*d*) $\dfrac{\sqrt[3]{10000}}{(49.52)^{4.6}}$.
(*b*) $5^{2/3} \div (12.7)^{3/2}$.

II. FIVE-PLACE TABLE OF THE ACTUAL VALUES OF THE TRIGONOMETRIC FUNCTIONS OF ANGLES

11. Direct Readings. This table gives the sines, cosines, tangents, and cotangents of the angles from 0° to 45°; and by a simple device, indicated by the printing, the values of these functions for angles from 45° to 90° may be read directly from the same table. For angles less than 45° read down the page, the degrees being found at the top and the minutes on the left; for angles greater than 45° read up the page, the degrees being found at the bottom and the minutes on the right.

To find a function of an angle (such as 15° 27′.6, for example) which does not reduce to an integral number of minutes, we employ the process of interpolation. To illustrate, let us find tan 15° 27′.6. In the table we find tan 15° 27′ = .27638 and tan 15° 28′ = .27670; we know that tan 15° 27′.6 lies between these two numbers. The process of interpolation depends on the assumption that between 15° 27′ and 15° 28′ the tangent of the angle varies directly as the angle; while this assumption is not strictly true, it gives an approximation sufficiently accurate for a five-place table. Thus we should assume that tan 15° 27′.5 is halfway between .27638 and .27670. We may state the problem as follows: An increase of 1′ in the angle increases the tangent .00032; assuming that the tangent

varies as the angle, an increase of $0'.6$ in the angle will increase the tangent by $.6 \times .00032 = .00019$ (retaining only five places); hence

$$\tan 15^\circ 27'.6 = .27638 + .00019 = .27657.$$

The difference between two successive values in the table is called, as in Table I, the *tabular difference* (.00032 above). The proportional part of the tabular difference which is used is called the *correction* (.00019 above), and is found by multiplying the tabular difference by the appropriate fraction of the smallest unit given in the table.

Example 1. Find $\sin 63^\circ 52'.8$.

We find $\sin 63^\circ 52' = .89777$;
tabular difference $= .00013$ (subtracted mentally from the table),
correction $= .8 \times .00013 = .00010$ (to be added).

Hence $\sin 63^\circ 52'.8 = .89787$.

Example 2. Find $\tan 37^\circ 45'.4$.

$\tan 37^\circ 45' = .77428$;
dropping useless zeros, tabular difference $= 47$; $.4 \times 47 = 19$ (to be added).

Hence $\tan 37^\circ 45'.4 = .77447$.

Example 3. Find $\cos 65^\circ 24'.8$.

$\cos 65^\circ 24' = .41628$;
tabular difference $= 26$; $.8 \times 26 = 21$
(to be subtracted because the cosine decreases as the angle increases).

Hence $\cos 65^\circ 24'.8 = .41607$.

Example 4. Find $\text{ctn}\, 32^\circ 18'.5$.

$\text{ctn}\, 32^\circ 18' = 1.5818$;
tabular difference $= 10$; $.5 \times 10 = 5$ (to be subtracted).

Hence $\text{ctn}\, 32^\circ 18'.5 = 1.5813$.

RULE. *To find a trigonometric function of an angle by interpolation: select the angle in the table which is next smaller than the given angle, and read its sine (cosine or tangent or cotangent as the case may be) and the tabular difference. Compute the correction as the proper proportional part of the tabular difference. In case of sines or tangents* **add** *the correction; in case of cosines or cotangents,* **subtract** *it.*

12. Reverse Readings. Interpolation is also used in finding the angle when one of its functions is given.

Example 1. Given $\sin x = .32845$, to find x.

Looking in the table we find the sine which is next less than the given sine to be .32832, and this belongs to $19^\circ 10'$. Subtract the value of the sine selected from the given sine to obtain the actual difference $= .00013$; note that the tabular difference $= .00027$. The actual difference divided by the tabular difference gives the correction $= 13/27 = .5$ as the decimal of a minute (to be added). Hence $x = 19^\circ 10'.5$.

Example 2. Given $\cos x = .28432$, to find x.

The cosine in the table next less than this is .28429 and belongs to $73^\circ 29'$; the tabular difference is 28; the actual difference is 3; correction $= 3/28 = .1$ (to be subtracted). Hence $x = 73^\circ 28'.9$.

Example 3. Given $\tan x = 2.8573$, to find x.

The tangent in the table next less than this is 2.8556 and belongs to $70^\circ 42'$; the tabular difference is 26; the actual difference is 17; correction $17/26 = .7$ (to be added). Hence $x = 70^\circ 42'.7$

Rule. *To find an angle when one of its trigonometric functions is given: select from the table the same named function which is next less than the given function, noting the corresponding angle and the tabular difference; compute the actual difference (between the selected value of the function and the given value) and divide it by the tabular difference; this gives the correction which is to be added if the given function is sine or tangent, and to be subtracted if the given function is cosine or cotangent.*

III. FIVE-PLACE COMMON LOGARITHMS OF THE TRIGONOMETRIC FUNCTIONS

13. Use of the Table. If it is required to find the numerical value of $x = 27.85 \times \sin 51° 27'$, we may apply logarithms as follows:

$$\log 27.85 = 1.44483.$$
$$\log \sin 51° 27' = \underline{9.89324 - 10} \text{ (add).}$$
$$\log x = 1.33807 \qquad x = 21.78$$

The only new idea here is the method of finding $\log \sin 51° 27'$, which means the logarithm of the sine of $51° 27'$. The most obvious way is to find in Table I, $\sin 51° 27' = .78206$, and then to find in Table II, $\log .78206 = 9.89324 - 10$, but this involves consulting two tables. To avoid the necessity of doing this, Table III gives the logarithms of the sines, cosines, tangents, and cotangents. The arrangement and the principles of interpolation are similar to those given on p. viii for Table I. The student should note carefully that Table III does not give the sines, cosines, etc., of angles, but rather their logarithms; also that the sines and cosines of all acute angles, the tangents of all acute angles less than 45° and the cotangents of all acute angles greater than 45° are proper fractions, and their logarithms end with -10, which is not printed in the table, but which should be written down whenever such a logarithm is used.

Example 1. Find $\log \sin 68° 25'.4$.

On the page having 68° at the bottom, and in the row having 25′ on the right find $\log \sin 68° 25' = 9.96843 - 10$; the tabular difference is 5; $.4 \times 5$ is given in the margin as 2; this is the correction to be added, giving $\log \sin 68° 25'.4 = 9.96845 - 10$.

(In case of sine and tangent *add* the correction.)

Example 2. Find $\log \cos 48° 39'.4$.

$\log \cos 48° 39' = 9.81998 - 10$, tabular difference 15.

$.4 \times 15 = 6$ (subtract) therefore $\log \cos 48° 39'.4 = 9.81992 - 10$.

(In case of cosine and cotangent, subtract the correction.)

Example 3. Given $\log \tan x = 0.77663$, to find x.

The logarithmic tangent in Table III next less than the given one is 0.77639 and belongs to 80° 30′; the actual difference is 24; the tabular difference is 78; hence the correction is $24/78 = .3$ (add); hence $x = 80° 30'.3$.

Example 4. Given $\log \cos x = 9.72581 - 10$, to find x.

The logarithmic cosine next less than the given one is $9.72562 - 10$ and belongs to 57° 58′; the actual difference is 19; the tabular difference is 20; hence the correction is $19/20 = 1.0$ (to the nearest tenth); (subtract); hence $x = 57° 52'.0$.

In finding $\log \operatorname{ctn} \alpha$ for any angle α, note that $\log \operatorname{ctn} \alpha = -\log \tan \alpha$, since $\operatorname{ctn} \alpha = 1/\tan \alpha$. Hence *the tabular differences for* log ctn *are precisely the same as those for* log tan *throughout the table, but taken in reversed order.* Likewise, $\log \sec \alpha = -\log \cos \alpha$, $\log \csc \alpha = -\log \sin \alpha$; hence $\log \sec \alpha$ and $\log \csc \alpha$ are omitted.

For angles near 0° or near 90°, the interpolations are not very accurate if the differences are large. A special process, called *logarithmic interpolation*, is given on p. 45, for angles below 3° or above 87°.

IV–V. RADIAN MEASURE

14. Computations in Radian Measure. The reduction of degrees to radians is facilitated by Table IV — *Conversion of Degrees to Radians.*

The values of $\sin x$, $\cos x$, $\tan x$, are stated for every angle x from 0.00 radians to 1.60 radians at intervals of .01 radian in Table V — *Trigonometric Functions in Radian Measure.*

The reduction of radians to degrees can be performed directly by Table V; or, for greater accuracy, by the supplementary Table V*a*.

VI. POWERS — ROOTS — RECIPROCALS

15. Arrangement. This table is arranged so that the square, cube, square root, cube root, or reciprocal can be read directly to five decimal places for any number n of three significant figures. To attain this, not only n^2, n^3, $\sqrt{n}$, $\sqrt[3]{n}$, $1/n$, but also $\sqrt{10\,n}$, $\sqrt[3]{10\,n}$, $\sqrt[3]{100\,n}$ are printed on every page. All values have been carefully recomputed and checked.

Thus to find $\sqrt{1.17}$, read in $\sqrt{n}$ column the result: 1.08167. To find $\sqrt{11.7}$, read *in the same line*, in $\sqrt{10\,n}$ column the result: 3.42053. To find $\sqrt{117}$, read 10 times the entry in $\sqrt{n}$ column, since $\sqrt{117} = 10\sqrt{1.17}$.

Similarly, $\sqrt[3]{1.17} = 1.05373$ from $\sqrt[3]{n}$ column; $\sqrt[3]{11.7} = 2.27019$ from *the same line* in $\sqrt[3]{10\,n}$ column; $\sqrt[3]{117} = 4.89097$ from *the same line* in $\sqrt[3]{100\,n}$ column.

The effect of a change in the decimal point in n^2, n^3, and $1/n$ is only to shift the decimal point in the result, without altering the digits printed.

16. Uses. One principal use of this table in Trigonometry is to make the *Pythagorean Theorem* and the *Law of Cosines* practicable as formulas for actual computation, in an obvious manner.

For mensuration formulas, etc., all the entries are very convenient.

VII. NAPIERIAN OR NATURAL LOGARITHMS

17. The Base *e*. — Natural Logarithms. The number $e = 2.7182818 \cdots$ is called the **natural base** of logarithms. The logarithms of numbers to this base are given in Table VII at intervals of .01 from 0.01 to 10.09, and at unit intervals from 10 to 409. The fundamental relation $\log_e n = \log_e 10 \times \log_{10} n$ enables us to transfer from the base 10 to the base e, or conversely; where $\log_e 10 = 2.30258509$.

A — B — C. FOUR-PLACE TABLES

18. Four-place Tables. These are duplicates of the preceding five-place tables, reduced to four places, and with larger intervals between the tabulations. The value of such four-place tables consists in the greater speed with which they can be used, in case the degree of accuracy they afford is sufficient for the purpose in hand.

***A.* Logarithms of Numbers.** The only special feature of this table is that *the proportional parts are printed for every tenth in every row;* hence the logarithm of any number of *four* significant figures can be read directly, by a mental addition of the proportional part corresponding to the last figure. There may be an error of 1 in the last place in the result.

***B.* Antilogarithms.** Attention is called to the table of antilogarithms, in which the *numbers* corresponding to *given logarithms* are tabulated. This table, together with the accompanying four-place logarithm table, will be found to facilitate approximate calculations to a marked degree, especially when great accuracy is not necessary. Thus these tables are convenient in *checking* results found otherwise. The proportional parts are stated in the right-hand margin for each row separately; hence the antilogarithm of a number of four significant figures can be read almost immediately, the addition of the proper correction being performed mentally. This arrangement, with the corresponding one in Table A, makes the tables *effectively* four-place each way.

***C.* Values and Logarithms of Trigonometric Functions.** In this table, the values of sin α, cos α, tan α, ctn α, and their common logarithms, are stated for each 10 minute interval in α. The characteristics of the logarithms are omitted, since they can be supplied readily from the value, as in the case of Table A.

19. Sources and Checks used. In arranging all of these tables, several extant tables have been used as sources; and the proofs have been read against the standard seven-place tables of Vega, and at least one other table, or against at least two independent sources when the figures are not given by Vega. In all cases, the stereotyped plates have been proof-read five times, by three different persons.

In case of apparent doubt, especially in the last place of decimals, the values have been recomputed, either by series or by the condensed fifteen-place tables of Hoüel.

While errors may occur, it is believed that they must be purely typographical; in most cases such an error is revealed by the unreasonable differences it creates.

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Greek Alphabet

LETTERS	NAMES	LETTERS	NAMES	LETTERS	NAMES	LETTERS	NAMES
Α α	Alpha	Η η	Eta	Ν ν	Nu	Τ τ	Tau
Β β	Beta	Θ θ	Theta	Ξ ξ	Xi	Υ υ	Upsilon
Γ γ	Gamma	Ι ι	Iota	Ο ο	Omicron	Φ φ	Phi
Δ δ	Delta	Κ κ	Kappa	Π π	Pi	Χ χ	Chi
Ε ε	Epsilon	Λ λ	Lambda	Ρ ρ	Rho	Ψ ψ	Psi
Ζ ζ	Zeta	Μ μ	Mu	Σ σ ς	Sigma	Ω ω	Omega

LOGARITHMIC AND TRIGONOMETRIC TABLES

TABLE I

COMMON LOGARITHMS OF NUMBERS

FROM

1 TO 10 000

TO

FIVE DECIMAL PLACES

1—100

N	Log	N	Log	N	Log	N	Log	N	Log
0	——	**20**	1.30 103	**40**	1.60 206	**60**	1.77 815	**80**	1.90 309
1	0.00 000	21	1.32 222	41	1.61 278	61	1.78 533	81	1.90 849
2	0.30 103	22	1.34 242	42	1.62 325	62	1.79 239	82	1.91 381
3	0.47 712	23	1.36 173	43	1.63 347	63	1.79 934	83	1.91 908
4	0.60 206	24	1.38 021	44	1.64 345	64	1.80 618	84	1.92 428
5	0.69 897	25	1.39 794	45	1.65 321	65	1.81 291	85	1.92 942
6	0.77 815	26	1.41 497	46	1.66 276	66	1.81 954	86	1.93 450
7	0.84 510	27	1.43 136	47	1.67 210	67	1.82 607	87	1.93 952
8	0.90 309	28	1.44 716	48	1.68 124	68	1.83 251	88	1.94 448
9	0.95 424	29	1.46 240	49	1.69 020	69	1.83 885	89	1.94 939
10	1.00 000	**30**	1.47 712	**50**	1.69 897	**70**	1.84 510	**90**	1.95 424
11	1.04 139	31	1.49 136	51	1.70 757	71	1.85 126	91	1.95 904
12	1.07 918	32	1.50 515	52	1.71 600	72	1.85 733	92	1.96 379
13	1.11 394	33	1.51 851	53	1.72 428	73	1.86 332	93	1.96 848
14	1.14 613	34	1.53 148	54	1.73 239	74	1.86 923	94	1.97 313
15	1.17 609	35	1.54 407	55	1.74 036	75	1.87 506	95	1.97 772
16	1.20 412	36	1.55 630	56	1.74 819	76	1.88 081	96	1.98 227
17	1.23 045	37	1.56 820	57	1.75 587	77	1.88 649	97	1.98 677
18	1.25 527	38	1.57 978	58	1.76 343	78	1.89 209	98	1.99 123
19	1.27 875	39	1.59 106	59	1.77 085	79	1.89 763	99	1.99 564
N	Log	N	Log	N	Log	N	Log	N	Log

N.	0	1	2	3	4	5	6	7	8	9
100	00 000	043	087	130	173	217	260	303	346	389
01	432	475	518	561	604	647	689	732	775	817
02	860	903	945	988	*030	*072	*115	*157	*199	*242
03	01 284	326	368	410	452	494	536	578	620	662
04	703	745	787	828	870	912	953	995	*036	*078
05	02 119	160	202	243	284	325	366	407	449	490
06	531	572	612	653	694	735	776	816	857	898
07	938	979	*019	*060	*100	*141	*181	*222	*262	*302
08	03 342	383	423	463	503	543	583	623	663	703
09	743	782	822	862	902	941	981	*021	*060	*100
110	04 139	179	218	258	297	336	376	415	454	493
11	532	571	610	650	689	727	766	805	844	883
12	922	961	999	*038	*077	*115	*154	*192	*231	*269
13	05 308	346	385	423	461	500	538	576	614	652
14	690	729	767	805	843	881	918	956	994	*032
15	06 070	108	145	183	221	258	296	333	371	408
16	446	483	521	558	595	633	670	707	744	781
17	819	856	893	930	967	*004	*041	*078	*115	*151
18	07 188	225	262	298	335	372	408	445	482	518
19	555	591	628	664	700	737	773	809	846	882
120	918	954	990	*027	*063	*099	*135	*171	*207	*243
21	08 279	314	350	386	422	458	493	529	565	600
22	636	672	707	743	778	814	849	884	920	955
23	991	*026	*061	*096	*132	*167	*202	*237	*272	*307
24	09 342	377	412	447	482	517	552	587	621	656
25	691	726	760	795	830	864	899	934	968	*003
26	10 037	072	106	140	175	209	243	278	312	346
27	380	415	449	483	517	551	585	619	653	687
28	721	755	789	823	857	890	924	958	992	*025
29	11 059	093	126	160	193	227	261	294	327	361
130	394	428	461	494	528	561	594	628	661	694
31	727	760	793	826	860	893	926	959	992	*024
32	12 057	090	123	156	189	222	254	287	320	352
33	385	418	450	483	516	548	581	613	646	678
34	710	743	775	808	840	872	905	937	969	*001
35	13 033	066	098	130	162	194	226	258	290	322
36	354	386	418	450	481	513	545	577	609	640
37	672	704	735	767	799	830	862	893	925	956
38	988	*019	*051	*082	*114	*145	*176	*208	*239	*270
39	14 301	333	364	395	426	457	489	520	551	582
140	613	644	675	706	737	768	799	829	860	891
41	922	953	983	*014	*045	*076	*106	*137	*168	*198
42	15 229	259	290	320	351	381	412	442	473	503
43	534	564	594	625	655	685	715	746	776	806
44	836	866	897	927	957	987	*017	*047	*077	*107
45	16 137	167	197	227	256	286	316	346	376	406
46	435	465	495	524	554	584	613	643	673	702
47	732	761	791	820	850	879	909	938	967	997
48	17 026	056	085	114	143	173	202	231	260	289
49	319	348	377	406	435	464	493	522	551	580
150	609	638	667	696	725	754	782	811	840	869
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	44	**43**	**42**
1	4.4	4.3	4.2
2	8.8	8.6	8.4
3	13.2	12.9	12.6
4	17.6	17.2	16.8
5	22.0	21.5	21.0
6	26.4	25.8	25.2
7	30.8	30.1	29.4
8	35.2	34.4	33.6
9	39.6	38.7	37.8

	41	**40**	**39**
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1

	38	**37**	**36**
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4

	35	**34**	**33**
1	3.5	3.4	3.3
2	7.0	6.8	6.6
3	10.5	10.2	9.9
4	14.0	13.6	13.2
5	17.5	17.0	16.5
6	21.0	20.4	19.8
7	24.5	23.8	23 1
8	28.0	27.2	26.4
9	31.5	30.6	29.7

	32	**31**	**30**
1	3.2	3.1	3.0
2	6.4	6.2	6 0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
150	17 609	638	667	696	725	754	782	811	840	869
51	898	926	955	984	*013	*041	*070	*099	*127	*156
52	18 184	213	241	270	298	327	355	384	412	441
53	469	498	526	554	583	611	639	667	696	724
54	752	780	808	837	865	893	921	949	977	*005
55	19 033	061	089	117	145	173	201	229	257	285
56	312	340	368	396	424	451	479	507	535	562
57	590	618	645	673	700	728	756	783	811	838
58	866	893	921	948	976	*003	*030	*058	*085	*112
59	20 140	167	194	222	249	276	303	330	358	385
160	412	439	466	493	520	548	575	602	629	656
61	683	710	737	763	790	817	844	871	898	925
62	952	978	*005	*032	*059	*085	*112	*139	*165	*192
63	21 219	245	272	299	325	352	378	405	431	458
64	484	511	537	564	590	617	643	669	696	722
65	748	775	801	827	854	880	906	932	958	985
66	22 011	037	063	089	115	141	167	194	220	246
67	272	298	324	350	376	401	427	453	479	505
68	531	557	583	608	634	660	686	712	737	763
69	789	814	840	866	891	917	943	968	994	*019
170	23 045	070	096	121	147	172	198	223	249	274
71	300	325	350	376	401	426	452	477	502	528
72	553	578	603	629	654	679	704	729	754	779
73	805	830	855	880	905	930	955	980	*005	*030
74	24 055	080	105	130	155	180	204	229	254	279
75	304	329	353	378	403	428	452	477	502	527
76	551	576	601	625	650	674	699	724	748	773
77	797	822	846	871	895	920	944	969	993	*018
78	25 042	066	091	115	139	164	188	212	237	261
79	285	310	334	358	382	406	431	455	479	503
180	527	551	575	600	624	648	672	696	720	744
81	768	792	816	840	864	888	912	935	959	983
82	26 007	031	055	079	102	126	150	174	198	221
83	245	269	293	316	340	364	387	411	435	458
84	482	505	529	553	576	600	623	647	670	694
85	717	741	764	788	811	834	858	881	905	928
86	951	975	998	*021	*045	*068	*091	*114	*138	*161
87	27 184	207	231	254	277	300	323	346	370	393
88	416	439	462	485	508	531	554	577	600	623
89	646	669	692	715	738	761	784	807	830	852
190	875	898	921	944	967	989	*012	*035	*058	*081
91	28 103	126	149	171	194	217	240	262	285	307
92	330	353	375	398	421	443	466	488	511	533
93	556	578	601	623	646	668	691	713	735	758
94	780	803	825	847	870	892	914	937	959	981
95	29 003	026	048	070	092	115	137	159	181	203
96	226	248	270	292	314	336	358	380	403	425
97	447	469	491	513	535	557	579	601	623	645
98	667	688	710	732	754	776	798	820	842	863
99	885	907	929	951	973	994	*016	*038	*060	*081
200	30 103	125	146	168	190	211	233	255	276	298
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	29	28	27
1	2.9	2.8	2.7
2	5.8	5.6	5.4
3	8.7	8.4	8.1
4	11.6	11.2	10.8
5	14.5	14.0	13.5
6	17.4	16.8	16.2
7	20.3	19.6	18.9
8	23.2	22.4	21.6
9	26.1	25.2	24.3

	26	25	24
1	2.6	2.5	2.4
2	5.2	5.0	4.8
3	7.8	7.5	7.2
4	10.4	10.0	9.6
5	13.0	12.5	12.0
6	15.6	15.0	14.4
7	18.2	17.5	16.8
8	20.8	20.0	19.2
9	23.4	22.5	21.6

	23	22	21
1	2.3	2.2	2.1
2	4.6	4.4	4.2
3	6.9	6.6	6.3
4	9.2	8.8	8.4
5	11.5	11.0	10.5
6	13.8	13.2	12.6
7	16.1	15.4	14.7
8	18.4	17.6	16.8
9	20.7	19.8	18.9

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
200	30 103	125	146	168	190	211	233	255	276	298
01	320	341	363	384	406	428	449	471	492	514
02	535	557	578	600	621	643	664	685	707	728
03	750	771	792	814	835	856	878	899	920	942
04	963	984	*006	*027	*048	*069	*091	*112	*133	*154
05	31 175	197	218	239	260	281	302	323	345	366
06	387	408	429	450	471	492	513	534	555	576
07	597	618	639	660	681	702	723	744	765	785
08	806	827	848	869	890	911	931	952	973	994
09	32 015	035	056	077	098	118	139	160	181	201
210	222	243	263	284	305	325	346	366	387	408
11	428	449	469	490	510	531	552	572	593	613
12	634	654	675	695	715	736	756	777	797	818
13	838	858	879	899	919	940	960	980	*001	*021
14	33 041	062	082	102	122	143	163	183	203	224
15	244	264	284	304	325	345	365	385	405	425
16	445	465	486	506	526	546	566	586	606	626
17	646	666	686	706	726	746	766	786	806	826
18	846	866	885	905	925	945	965	985	*005	*025
19	34 044	064	084	104	124	143	163	183	203	223
220	242	262	282	301	321	341	361	380	400	420
21	439	459	479	498	518	537	557	577	596	616
22	635	655	674	694	713	733	753	772	792	811
23	830	850	869	889	908	928	947	967	986	*005
24	35 025	044	064	083	102	122	141	160	180	199
25	218	238	257	276	295	315	334	353	372	392
26	411	430	449	468	488	507	526	545	564	583
27	603	622	641	660	679	698	717	736	755	774
28	793	813	832	851	870	889	908	927	946	965
29	984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36 173	192	211	229	248	267	286	305	324	342
31	361	380	399	418	436	455	474	493	511	530
32	549	568	586	605	624	642	661	680	698	717
33	736	754	773	791	810	829	847	866	884	903
34	922	940	959	977	996	*014	*033	*051	*070	*088
35	37 107	125	144	162	181	199	218	236	254	273
36	291	310	328	346	365	383	401	420	438	457
37	475	493	511	530	548	566	585	603	621	639
38	658	676	694	712	731	749	767	785	803	822
39	840	858	876	894	912	931	949	967	985	*003
240	38 021	039	057	075	093	112	130	148	166	184
41	202	220	238	256	274	292	310	328	346	364
42	382	399	417	435	453	471	489	507	525	543
43	561	578	596	614	632	650	668	686	703	721
44	739	757	775	792	810	828	846	863	881	899
45	917	934	952	970	987	*005	*023	*041	*058	*076
46	39 094	111	129	146	164	182	199	217	235	252
47	270	287	305	322	340	358	375	393	410	428
48	445	463	480	498	515	533	550	568	585	602
49	620	637	655	672	690	707	724	742	759	777
250	794	811	829	846	863	881	898	915	933	950
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	22	21	20
1	2.2	2.1	2.0
2	4.4	4.2	4.0
3	6.6	6.3	6.0
4	8.8	8.4	8.0
5	11.0	10.5	10.0
6	13.2	12.6	12.0
7	15.4	14.7	14.0
8	17.6	16.8	16.0
9	19.8	18.9	18.0

	19	18	17
1	1.9	1.8	1.7
2	3.8	3.6	3.4
3	5.7	5.4	5.1
4	7.6	7.2	6.8
5	9.5	9.0	8.5
6	11.4	10.8	10.2
7	13.3	12.6	11.9
8	15.2	14.4	13.6
9	17.1	16.2	15.3

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
250	39 794	811	829	846	863	881	898	915	933	950
51	967	985	*002	*019	*037	*054	*071	*088	*106	*123
52	40 140	157	175	192	209	226	243	261	278	295
53	312	329	346	364	381	398	415	432	449	466
54	483	500	518	535	552	569	586	603	620	637
55	654	671	688	705	722	739	756	773	790	807
56	824	841	858	875	892	909	926	943	960	976
57	993	*010	*027	*044	*061	*078	*095	*111	*128	*145
58	41 162	179	196	212	229	246	263	280	296	313
59	330	347	363	380	397	414	430	447	464	481
260	497	514	531	547	564	581	597	614	631	647
61	664	681	697	714	731	747	764	780	797	814
62	830	847	863	880	896	913	929	946	963	979
63	996	*012	*029	*045	*062	*078	*095	*111	*127	*144
64	42 160	177	193	210	226	243	259	275	292	308
65	325	341	357	374	390	406	423	439	455	472
66	488	504	521	537	553	570	586	602	619	635
67	651	667	684	700	716	732	749	765	781	797
68	813	830	846	862	878	894	911	927	943	959
69	975	991	*008	*024	*040	*056	*072	*088	*104	*120
270	43 136	152	169	185	201	217	233	249	265	281
71	297	313	329	345	361	377	393	409	425	441
72	457	473	489	505	521	537	553	569	584	600
73	616	632	648	664	680	696	712	727	743	759
74	775	791	807	823	838	854	870	886	902	917
75	933	949	965	981	996	*012	*028	*044	*059	*075
76	44 091	107	122	138	154	170	185	201	217	232
77	248	264	279	295	311	326	342	358	373	389
78	404	420	436	451	467	483	498	514	529	545
79	560	576	592	607	623	638	654	669	685	700
280	716	731	747	762	778	793	809	824	840	855
81	871	886	902	917	932	948	963	979	994	*010
82	45 025	040	056	071	086	102	117	133	148	163
83	179	194	209	225	240	255	271	286	301	317
84	332	347	362	378	393	408	423	439	454	469
85	484	500	515	530	545	561	576	591	606	621
86	637	652	667	682	697	712	728	743	758	773
87	788	803	818	834	849	864	879	894	909	924
88	939	954	969	984	*000	*015	*030	*045	*060	*075
89	46 090	105	120	135	150	165	180	195	210	225
290	240	255	270	285	300	315	330	345	359	374
91	389	404	419	434	449	464	479	494	509	523
92	538	553	568	583	598	613	627	642	657	672
93	687	702	716	731	746	761	776	790	805	820
94	835	850	864	879	894	909	923	938	953	967
95	982	997	*012	*026	*041	*056	*070	*085	*100	*114
96	47 129	144	159	173	188	202	217	232	246	261
97	276	290	305	319	334	349	363	378	392	407
98	422	436	451	465	480	494	509	524	538	553
99	567	582	596	611	625	640	654	669	683	698
300	712	727	741	756	770	784	799	813	828	842
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	18	17	16
1	1.8	1.7	1.6
2	3.6	3.4	3.2
3	5.4	5.1	4.8
4	7.2	6.8	6.4
5	9.0	8.5	8.0
6	10.8	10.2	9.6
7	12.6	11.9	11.2
8	14.4	13.6	12.8
9	16.2	15.3	14.4

	15	14
1	1.5	1.4
2	3.0	2.8
3	4.5	4.2
4	6.0	5.6
5	7.5	7.0
6	9.0	8.4
7	10.5	9.8
8	12.0	11.2
9	13.5	12.6

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
300	47 712	727	741	756	770	784	799	813	828	842
01	857	871	885	900	914	929	943	958	972	986
02	48 001	015	029	044	058	073	087	101	116	130
03	144	159	173	187	202	216	230	244	259	273
04	287	302	316	330	344	359	373	387	401	416
05	430	444	458	473	487	501	515	530	544	558
06	572	586	601	615	629	643	657	671	686	700
07	714	728	742	756	770	785	799	813	827	841
08	855	869	883	897	911	926	940	954	968	982
09	996	*010	*024	*038	*052	*066	*080	*094	*108	*122
310	49 136	150	164	178	192	206	220	234	248	262
11	276	290	304	318	332	346	360	374	388	402
12	415	429	443	457	471	485	499	513	527	541
13	554	568	582	596	610	624	638	651	665	679
14	693	707	721	734	748	762	776	790	803	817
15	831	845	859	872	886	900	914	927	941	955
16	969	982	996	*010	*024	*037	*051	*065	*079	*092
17	50 106	120	133	147	161	174	188	202	215	229
18	243	256	270	284	297	311	325	338	352	365
19	379	393	406	420	433	447	461	474	488	501
320	515	529	542	556	569	583	596	610	623	637
21	651	664	678	691	705	718	732	745	759	772
22	786	799	813	826	840	853	866	880	893	907
23	920	934	947	961	974	987	*001	*014	*028	*041
24	51 055	068	081	095	108	121	135	148	162	175
25	188	202	215	228	242	255	268	282	295	308
26	322	335	348	362	375	388	402	415	428	441
27	455	468	481	495	508	521	534	548	561	574
28	587	601	614	627	640	654	667	680	693	706
29	720	733	746	759	772	786	799	812	825	838
330	851	865	878	891	904	917	930	943	957	970
31	983	996	*009	*022	*035	*048	*061	*075	*088	*101
32	52 114	127	140	153	166	179	192	205	218	231
33	244	257	270	284	297	310	323	336	349	362
34	375	388	401	414	427	440	453	466	479	492
35	504	517	530	543	556	569	582	595	608	621
36	634	647	660	673	686	699	711	724	737	750
37	763	776	789	802	815	827	840	853	866	879
38	892	905	917	930	943	956	969	982	994	*007
39	53 020	033	046	058	071	084	097	110	122	135
340	148	161	173	186	199	212	224	237	250	263
41	275	288	301	314	326	339	352	364	377	390
42	403	415	428	441	453	466	479	491	504	517
43	529	542	555	567	580	593	605	618	631	643
44	656	668	681	694	706	719	732	744	757	769
45	782	794	807	820	832	845	857	870	882	895
46	908	920	933	945	958	970	983	995	*008	*020
47	54 033	045	058	070	083	095	108	120	133	145
48	158	170	183	195	208	220	233	245	258	270
49	283	295	307	320	332	345	357	370	382	394
350	407	419	432	444	456	469	481	494	506	518
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	15	14
1	1.5	1.4
2	3.0	2.8
3	4.5	4.2
4	6.0	5.6
5	7.5	7.0
6	9.0	8.4
7	10.5	9.8
8	12.0	11.2
9	13.5	12.6

	13	12
1	1.3	1.2
2	2.6	2.4
3	3.9	3.6
4	5.2	4.8
5	6.5	6.0
6	7.8	7.2
7	9.1	8.4
8	10.4	9.6
9	11.7	10.8

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
350	54 407	419	432	444	456	469	481	494	506	518
51	531	543	555	568	580	593	605	617	630	642
52	654	667	679	691	704	716	728	741	753	765
53	777	790	802	814	827	839	851	864	876	888
54	900	913	925	937	949	962	974	986	998	*011
55	55 023	035	047	060	072	084	096	108	121	133
56	145	157	169	182	194	206	218	230	242	255
57	267	279	291	303	315	328	340	352	364	376
58	388	400	413	425	437	449	461	473	485	497
59	509	522	534	546	558	570	582	594	606	618
360	630	642	654	666	678	691	703	715	727	739
61	751	763	775	787	799	811	823	835	847	859
62	871	883	895	907	919	931	943	955	967	979
63	991	*003	*015	*027	*038	*050	*062	*074	*086	*098
64	56 110	122	134	146	158	170	182	194	205	217
65	229	241	253	265	277	289	301	312	324	336
66	348	360	372	384	396	407	419	431	443	455
67	467	478	490	502	514	526	538	549	561	573
68	585	597	608	620	632	644	656	667	679	691
69	703	714	726	738	750	761	773	785	797	808
370	820	832	844	855	867	879	891	902	914	926
71	937	949	961	972	984	996	*008	*019	*031	*043
72	57 054	066	078	089	101	113	124	136	148	159
73	171	183	194	206	217	229	241	252	264	276
74	287	299	310	322	334	345	357	368	380	392
75	403	415	426	438	449	461	473	484	496	507
76	519	530	542	553	565	576	588	600	611	623
77	634	646	657	669	680	692	703	715	726	738
78	749	761	772	784	795	807	818	830	841	852
79	864	875	887	898	910	921	933	944	955	967
380	978	990	*001	*013	*024	*035	*047	*058	*070	*081
81	58 092	104	115	127	138	149	161	172	184	195
82	206	218	229	240	252	263	274	286	297	309
83	320	331	343	354	365	377	388	399	410	422
84	433	444	456	467	478	490	501	512	524	535
85	546	557	569	580	591	602	614	625	636	647
86	659	670	681	692	704	715	726	737	749	760
87	771	782	794	805	816	827	838	850	861	872
88	883	894	906	917	928	939	950	961	973	984
89	995	*006	*017	*028	*040	*051	*062	*073	*084	*095
390	59 106	118	129	140	151	162	173	184	195	207
91	218	229	240	251	262	273	284	295	306	318
92	329	340	351	362	373	384	395	406	417	428
93	439	450	461	472	483	494	506	517	528	539
94	550	561	572	583	594	605	616	627	638	649
95	660	671	682	693	704	715	726	737	748	759
96	770	780	791	802	813	824	835	846	857	868
97	879	890	901	912	923	934	945	956	966	977
98	988	999	*010	*021	*032	*043	*054	*065	*076	*086
99	60 097	108	119	130	141	152	163	173	184	195
400	206	217	228	239	249	260	271	282	293	304
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	13	12
1	1.3	1.2
2	2.6	2.4
3	3.9	3.6
4	5.2	4.8
5	6.5	6.0
6	7.8	7.2
7	9.1	8.4
8	10.4	9.6
9	11.7	10.8

	11	10
1	1.1	1.0
2	2.2	2.0
3	3.3	3.0
4	4.4	4.0
5	5.5	5.0
6	6.6	6.0
7	7.7	7.0
8	8.8	8.0
9	9.9	9.0

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
400	60 206	217	228	239	249	260	271	282	293	304
01	314	325	336	347	358	369	379	390	401	412
02	423	433	444	455	466	477	487	498	509	520
03	531	541	552	563	574	584	595	606	617	627
04	638	649	660	670	681	692	703	713	724	735
05	746	756	767	778	788	799	810	821	831	842
06	853	863	874	885	895	906	917	927	938	949
07	959	970	981	991	*002	*013	*023	*034	*045	*055
08	61 066	077	087	098	109	119	130	140	151	162
09	172	183	194	204	215	225	236	247	257	268
410	278	289	300	310	321	331	342	352	363	374
11	384	395	405	416	426	437	448	458	469	479
12	490	500	511	521	532	542	553	563	574	584
13	595	606	616	627	637	648	658	669	679	690
14	700	711	721	731	742	752	763	773	784	794
15	805	815	826	836	847	857	868	878	888	899
16	909	920	930	941	951	962	972	982	993	*003
17	62 014	024	034	045	055	066	076	086	097	107
18	118	128	138	149	159	170	180	190	201	211
19	221	232	242	252	263	273	284	294	304	315
420	325	335	346	356	366	377	387	397	408	418
21	428	439	449	459	469	480	490	500	511	521
22	531	542	552	562	572	583	593	603	613	624
23	634	644	655	665	675	685	696	706	716	726
24	737	747	757	767	778	788	798	808	818	829
25	839	849	859	870	880	890	900	910	921	931
26	941	951	961	972	982	992	*002	*012	*022	*033
27	63 043	053	063	073	083	094	104	114	124	134
28	144	155	165	175	185	195	205	215	225	236
29	246	256	266	276	286	296	306	317	327	337
430	347	357	367	377	387	397	407	417	428	438
31	448	458	468	478	488	498	508	518	528	538
32	548	558	568	579	589	599	609	619	629	639
33	649	659	669	679	689	699	709	719	729	739
34	749	759	769	779	789	799	809	819	829	839
35	849	859	869	879	889	899	909	919	929	939
36	949	959	969	979	988	998	*008	*018	*028	*038
37	64 048	058	068	078	088	098	108	118	128	137
38	147	157	167	177	187	197	207	217	227	237
39	246	256	266	276	286	296	306	316	326	335
440	345	355	365	375	385	395	404	414	424	434
41	444	454	464	473	483	493	503	513	523	532
42	542	552	562	572	582	591	601	611	621	631
43	640	650	660	670	680	689	699	709	719	729
44	738	748	758	768	777	787	797	807	816	826
45	836	846	856	865	875	885	895	904	914	924
46	933	943	953	963	972	982	992	*002	*011	*021
47	65 031	040	050	060	070	079	089	099	108	118
48	128	137	147	157	167	176	186	196	205	215
49	225	234	244	254	263	273	283	292	302	312
450	321	331	341	350	360	369	379	389	398	408
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Pts.

	11	10	9
1	1.1	1.0	0.9
2	2.2	2.0	1.8
3	3.3	3.0	2.7
4	4.4	4.0	3.6
5	5.5	5.0	4.5
6	6.6	6.0	5.4
7	7.7	7.0	6.3
8	8.8	8.0	7.2
9	9.9	9.0	8.1

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
450	65 321	331	341	350	360	369	379	389	398	408
51	418	427	437	447	456	466	475	485	495	504
52	514	523	533	543	552	562	571	581	591	600
53	610	619	629	639	648	658	667	677	686	696
54	706	715	725	734	744	753	763	772	782	792
55	801	811	820	830	839	849	858	868	877	887
56	896	906	916	925	935	944	954	963	973	982
57	992	*001	*011	*020	*030	*039	*049	*058	*068	*077
58	66 087	096	106	115	124	134	143	153	162	172
59	181	191	200	210	219	229	238	247	257	266
460	276	285	295	304	314	323	332	342	351	361
61	370	380	389	398	408	417	427	436	445	455
62	464	474	483	492	502	511	521	530	539	549
63	558	567	577	586	596	605	614	624	633	642
64	652	661	671	680	689	699	708	717	727	736
65	745	755	764	773	783	792	801	811	820	829
66	839	848	857	867	876	885	894	904	913	922
67	932	941	950	960	969	978	987	997	*006	*015
68	67 025	034	043	052	062	071	080	089	099	108
69	117	127	136	145	154	164	173	182	191	201
470	210	219	228	237	247	256	265	274	284	293
71	302	311	321	330	339	348	357	367	376	385
72	394	403	413	422	431	440	449	459	468	477
73	486	495	504	514	523	532	541	550	560	569
74	578	587	596	605	614	624	633	642	651	660
75	669	679	688	697	706	715	724	733	742	752
76	761	770	779	788	797	806	815	825	834	843
77	852	861	870	879	888	897	906	916	925	934
78	943	952	961	970	979	988	997	*006	*015	*024
79	68 034	043	052	061	070	079	088	097	106	115
480	124	133	142	151	160	169	178	187	196	205
81	215	224	233	242	251	260	269	278	287	296
82	305	314	323	332	341	350	359	368	377	386
83	395	404	413	422	431	440	449	458	467	476
84	485	494	502	511	520	529	538	547	556	565
85	574	583	592	601	610	619	628	637	646	655
86	664	673	681	690	699	708	717	726	735	744
87	753	762	771	780	789	797	806	815	824	833
88	842	851	860	869	878	886	895	904	913	922
89	931	940	949	958	966	975	984	993	*002	*011
490	69 020	028	037	046	055	064	073	082	090	099
91	108	117	126	135	144	152	161	170	179	188
92	197	205	214	223	232	241	249	258	267	276
93	285	294	302	311	320	329	338	346	355	364
94	373	381	390	399	408	417	425	434	443	452
95	461	469	478	487	496	504	513	522	531	539
96	548	557	566	574	583	592	601	609	618	627
97	636	644	653	662	671	679	688	697	705	714
98	723	732	740	749	758	767	775	784	793	801
99	810	819	827	836	845	854	862	871	880	888
500	897	906	914	923	932	940	949	958	966	975
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	10	9	8
1	1.0	0.9	0.8
2	2.0	1.8	1.6
3	3.0	2.7	2.4
4	4.0	3.6	3.2
5	5.0	4.5	4.0
6	6.0	5.4	4.8
7	7.0	6.3	5.6
8	8.0	7.2	6.4
9	9.0	8.1	7.2

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
500	69 897	906	914	923	932	940	949	958	966	975
01	984	992	*001	*010	*018	*027	*036	*044	*053	*062
02	70 070	079	088	096	105	114	122	131	140	148
03	157	165	174	183	191	200	209	217	226	234
04	243	252	260	269	278	286	295	303	312	321
05	329	338	346	355	364	372	381	389	398	406
06	415	424	432	441	449	458	467	475	484	492
07	501	509	518	526	535	544	552	561	569	578
08	586	595	603	612	621	629	638	646	655	663
09	672	680	689	697	706	714	723	731	740	749
510	757	766	774	783	791	800	808	817	825	834
11	842	851	859	868	876	885	893	902	910	919
12	927	935	944	952	961	969	978	986	995	*003
13	71 012	020	029	037	046	054	063	071	079	088
14	096	105	113	122	130	139	147	155	164	172
15	181	189	198	206	214	223	231	240	248	257
16	265	273	282	290	299	307	315	324	332	341
17	349	357	366	374	383	391	399	408	416	425
18	433	441	450	458	466	475	483	492	500	508
19	517	525	533	542	550	559	567	575	584	592
520	600	609	617	625	634	642	650	659	667	675
21	684	692	700	709	717	725	734	742	750	759
22	767	775	784	792	800	809	817	825	834	842
23	850	858	867	875	883	892	900	908	917	925
24	933	941	950	958	966	975	983	991	999	*008
25	72 016	024	032	041	049	057	066	074	082	090
26	099	107	115	123	132	140	148	156	165	173
27	181	189	198	206	214	222	230	239	247	255
28	263	272	280	288	296	304	313	321	329	337
29	346	354	362	370	378	387	395	403	411	419
530	428	436	444	452	460	469	477	485	493	501
31	509	518	526	534	542	550	558	567	575	583
32	591	599	607	616	624	632	640	648	656	665
33	673	681	689	697	705	713	722	730	738	746
34	754	762	770	779	787	795	803	811	819	827
35	835	843	852	860	868	876	884	892	900	908
36	916	925	933	941	949	957	965	973	981	989
37	997	*006	*014	*022	*030	*038	*046	*054	*062	*070
38	73 078	086	094	102	111	119	127	135	143	151
39	159	167	175	183	191	199	207	215	223	231
540	239	247	255	263	272	280	288	296	304	312
41	320	328	336	344	352	360	368	376	384	392
42	400	408	416	424	432	440	448	456	464	472
43	480	488	496	504	512	520	528	536	544	552
44	560	568	576	584	592	600	608	616	624	632
45	640	648	656	664	672	679	687	695	703	711
46	719	727	735	743	751	759	767	775	783	791
47	799	807	815	823	830	838	846	854	862	870
48	878	886	894	902	910	918	926	933	941	949
49	957	965	973	981	989	997	*005	*013	*020	*028
550	74 036	044	052	060	068	076	084	092	099	107
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	9	8	7
1	0.9	0.8	0.7
2	1.8	1.6	1.4
3	2.7	2.4	2.1
4	3.6	3.2	2.8
5	4.5	4.0	3.5
6	5.4	4.8	4.2
7	6.3	5.6	4.9
8	7.2	6.4	5.6
9	8.1	7.2	6.3

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
550	74 036	044	052	060	068	076	084	092	099	107
51	115	123	131	139	147	155	162	170	178	186
52	194	202	210	218	225	233	241	249	257	265
53	273	280	288	296	304	312	320	327	335	343
54	351	359	367	374	382	390	398	406	414	421
55	429	437	445	453	461	468	476	484	492	500
56	507	515	523	531	539	547	554	562	570	578
57	586	593	601	609	617	624	632	640	648	656
58	663	671	679	687	695	702	710	718	726	733
59	741	749	757	764	772	780	788	796	803	811
560	819	827	834	842	850	858	865	873	881	889
61	896	904	912	920	927	935	943	950	958	966
62	974	981	989	997	*005	*012	*020	*028	*035	*043
63	75 051	059	066	074	082	089	097	105	113	120
64	128	136	143	151	159	166	174	182	189	197
65	205	213	220	228	236	243	251	259	266	274
66	282	289	297	305	312	320	328	335	343	351
67	358	366	374	381	389	397	404	412	420	427
68	435	442	450	458	465	473	481	488	496	504
69	511	519	526	534	542	549	557	565	572	580
570	587	595	603	610	618	626	633	641	648	656
71	664	671	679	686	694	702	709	717	724	732
72	740	747	755	762	770	778	785	793	800	808
73	815	823	831	838	846	853	861	868	876	884
74	891	899	906	914	921	929	937	944	952	959
75	967	974	982	989	997	*005	*012	*020	*027	*035
76	76 042	050	057	065	072	080	087	095	103	110
77	118	125	133	140	148	155	163	170	178	185
78	193	200	208	215	223	230	238	245	253	260
79	268	275	283	290	298	305	313	320	328	335
580	343	350	358	365	373	380	388	395	403	410
81	418	425	433	440	448	455	462	470	477	485
82	492	500	507	515	522	530	537	545	552	559
83	567	574	582	589	597	604	612	619	626	634
84	641	649	656	664	671	678	686	693	701	708
85	716	723	730	738	745	753	760	768	775	782
86	790	797	805	812	819	827	834	842	849	856
87	864	871	879	886	893	901	908	916	923	930
88	938	945	953	960	967	975	982	989	997	*004
89	77 012	019	026	034	041	048	056	063	070	078
590	085	093	100	107	115	122	129	137	144	151
91	159	166	173	181	188	195	203	210	217	225
92	232	240	247	254	262	269	276	283	291	298
93	305	313	320	327	335	342	349	357	364	371
94	379	386	393	401	408	415	422	430	437	444
95	452	459	466	474	481	488	495	503	510	517
96	525	532	539	546	554	561	568	576	583	590
97	597	605	612	619	627	634	641	648	656	663
98	670	677	685	692	699	706	714	721	728	735
99	743	750	757	764	772	779	786	793	801	808
600	815	822	830	837	844	851	859	866	873	880
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	8	7
1	0.8	0.7
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
600	77 815	822	830	837	844	851	859	866	873	880
01	887	895	902	909	916	924	931	938	945	952
02	960	967	974	981	988	996	*003	*010	*017	*025
03	78 032	039	046	053	061	068	075	082	089	097
04	104	111	118	125	132	140	147	154	161	168
05	176	183	190	197	204	211	219	226	233	240
06	247	254	262	269	276	283	290	297	305	312
07	319	326	333	340	347	355	362	369	376	383
08	390	398	405	412	419	426	433	440	447	455
09	462	469	476	483	490	497	504	512	519	526
610	533	540	547	554	561	569	576	583	590	597
11	604	611	618	625	633	640	647	654	661	668
12	675	682	689	696	704	711	718	725	732	739
13	746	753	760	767	774	781	789	796	803	810
14	817	824	831	838	845	852	859	866	873	880
15	888	895	902	909	916	923	930	937	944	951
16	958	965	972	979	986	993	*000	*007	*014	*021
17	79 029	036	043	050	057	064	071	078	085	092
18	099	106	113	120	127	134	141	148	155	162
19	169	176	183	190	197	204	211	218	225	232
620	239	246	253	260	267	274	281	288	295	302
21	309	316	323	330	337	344	351	358	365	372
22	379	386	393	400	407	414	421	428	435	442
23	449	456	463	470	477	484	491	498	505	511
24	518	525	532	539	546	553	560	567	574	581
25	588	595	602	609	616	623	630	637	644	650
26	657	664	671	678	685	692	699	706	713	720
27	727	734	741	748	754	761	768	775	782	789
28	796	803	810	817	824	831	837	844	851	858
29	865	872	879	886	893	900	906	913	920	927
630	934	941	948	955	962	969	975	982	989	996
31	80 003	010	017	024	030	037	044	051	058	065
32	072	079	085	092	099	106	113	120	127	134
33	140	147	154	161	168	175	182	188	195	202
34	209	216	223	229	236	243	250	257	264	271
35	277	284	291	298	305	312	318	325	332	339
36	346	353	359	366	373	380	387	393	400	407
37	414	421	428	434	441	448	455	462	468	475
38	482	489	496	502	509	516	523	530	536	543
39	550	557	564	570	577	584	591	598	604	611
640	618	625	632	638	645	652	659	665	672	679
41	686	693	699	706	713	720	726	733	740	747
42	754	760	767	774	781	787	794	801	808	814
43	821	828	835	841	848	855	862	868	875	882
44	889	895	902	909	916	922	929	936	943	949
45	956	963	969	976	983	990	996	*003	*010	*017
46	81 023	030	037	043	050	057	064	070	077	084
47	090	097	104	111	117	124	131	137	144	151
48	158	164	171	178	184	191	198	204	211	218
49	224	231	238	245	251	258	265	271	278	285
650	291	298	305	311	318	325	331	338	345	351
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	8	7	6
1	0.8	0.7	0.6
2	1.6	1.4	1.2
3	2.4	2.1	1.8
4	3.2	2.8	2.4
5	4.0	3.5	3.0
6	4.8	4.2	3.6
7	5.6	4.9	4.2
8	6.4	5.6	4.8
9	7.2	6.3	5.4

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
650	81 291	298	305	311	318	325	331	338	345	351
51	358	365	371	378	385	391	398	405	411	418
52	425	431	438	445	451	458	465	471	478	485
53	491	498	505	511	518	525	531	538	544	551
54	558	564	571	578	584	591	598	604	611	617
55	624	631	637	644	651	657	664	671	677	684
56	690	697	704	710	717	723	730	737	743	750
57	757	763	770	776	783	790	796	803	809	816
58	823	829	836	842	849	856	862	869	875	882
59	889	895	902	908	915	921	928	935	941	948
660	954	961	968	974	981	987	994	*000	*007	*014
61	82 020	027	033	040	046	053	060	066	073	079
62	086	092	099	105	112	119	125	132	138	145
63	151	158	164	171	178	184	191	197	204	210
64	217	223	230	236	243	249	256	263	269	276
65	282	289	295	302	308	315	321	328	334	341
66	347	354	360	367	373	380	387	393	400	406
67	413	419	426	432	439	445	452	458	465	471
68	478	484	491	497	504	510	517	523	530	536
69	543	549	556	562	569	575	582	588	595	601
670	607	614	620	627	633	640	646	653	659	666
71	672	679	685	692	698	705	711	718	724	730
72	737	743	750	756	763	769	776	782	789	795
73	802	808	814	821	827	834	840	847	853	860
74	866	872	879	885	892	898	905	911	918	924
75	930	937	943	950	956	963	969	975	982	988
76	995	*001	*008	*014	*020	*027	*033	*040	*046	*052
77	83 059	065	072	078	085	091	097	104	110	117
78	123	129	136	142	149	155	161	168	174	181
79	187	193	200	206	213	219	225	232	238	245
680	251	257	264	270	276	283	289	296	302	308
81	315	321	327	334	340	347	353	359	366	372
82	378	385	391	398	404	410	417	423	429	436
83	442	448	455	461	467	474	480	487	493	499
84	506	512	518	525	531	537	544	550	556	563
85	569	575	582	588	594	601	607	613	620	626
86	632	639	645	651	658	664	670	677	683	689
87	696	702	708	715	721	727	734	740	746	753
88	759	765	771	778	784	790	797	803	809	816
89	822	828	835	841	847	853	860	866	872	879
690	885	891	897	904	910	916	923	929	935	942
91	948	954	960	967	973	979	985	992	998	*004
92	84 011	017	023	029	036	042	048	055	061	067
93	073	080	086	092	098	105	111	117	123	130
94	136	142	148	155	161	167	173	180	186	192
95	198	205	211	217	223	230	236	242	248	255
96	261	267	273	280	286	292	298	305	311	317
97	323	330	336	342	348	354	361	367	373	379
98	386	392	398	404	410	417	423	429	435	442
99	448	454	460	466	473	479	485	491	497	504
700	510	516	522	528	535	541	547	553	559	566
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	7	6
1	0.7	0.6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
700	84 510	516	522	528	535	541	547	553	559	566
01	572	578	584	590	597	603	609	615	621	628
02	634	640	646	652	658	665	671	677	683	689
03	696	702	708	714	720	726	733	739	745	751
04	757	763	770	776	782	788	794	800	807	813
05	819	825	831	837	844	850	856	862	868	874
06	880	887	893	899	905	911	917	924	930	936
07	942	948	954	960	967	973	979	985	991	997
08	85 003	009	016	022	028	034	040	046	052	058
09	065	071	077	083	089	095	101	107	114	120
710	126	132	138	144	150	156	163	169	175	181
11	187	193	199	205	211	217	224	230	236	242
12	248	254	260	266	272	278	285	291	297	303
13	309	315	321	327	333	339	345	352	358	364
14	370	376	382	388	394	400	406	412	418	425
15	431	437	443	449	455	461	467	473	479	485
16	491	497	503	509	516	522	528	534	540	546
17	552	558	564	570	576	582	588	594	600	606
18	612	618	625	631	637	643	649	655	661	667
19	673	679	685	691	697	703	709	715	721	727
720	733	739	745	751	757	763	769	775	781	788
21	794	800	806	812	818	824	830	836	842	848
22	854	860	866	872	878	884	890	896	902	908
23	914	920	926	932	938	944	950	956	962	968
24	974	980	986	992	998	*004	*010	*016	*022	*028
25	86 034	040	046	052	058	064	070	076	082	088
26	094	100	106	112	118	124	130	136	141	147
27	153	159	165	171	177	183	189	195	201	207
28	213	219	225	231	237	243	249	255	261	267
29	273	279	285	291	297	303	308	314	320	326
730	332	338	344	350	356	362	368	374	380	386
31	392	398	404	410	415	421	427	433	439	445
32	451	457	463	469	475	481	487	493	499	504
33	510	516	522	528	534	540	546	552	558	564
34	570	576	581	587	593	599	605	611	617	623
35	629	635	641	646	652	658	664	670	676	682
36	688	694	700	705	711	717	723	729	735	741
37	747	753	759	764	770	776	782	788	794	800
38	806	812	817	823	829	835	841	847	853	859
39	864	870	876	882	888	894	900	906	911	917
740	923	929	935	941	947	953	958	964	970	976
41	982	988	994	999	*005	*011	*017	*023	*029	*035
42	87 040	046	052	058	064	070	075	081	087	093
43	099	105	111	116	122	128	134	140	146	151
44	157	163	169	175	181	186	192	198	204	210
45	216	221	227	233	239	245	251	256	262	268
46	274	280	286	291	297	303	309	315	320	326
47	332	338	344	349	355	361	367	373	379	384
48	390	396	402	408	413	419	425	431	437	442
49	448	454	460	466	471	477	483	489	495	500
750	506	512	518	523	529	535	541	547	552	558
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	7	6	5
1	0.7	0.6	0.5
2	1.4	1.2	1.0
3	2.1	1.8	1.5
4	2.8	2.4	2.0
5	3.5	3.0	2.5
6	4.2	3.6	3.0
7	4.9	4.2	3.5
8	5.6	4.8	4.0
9	6.3	5.4	4.5

N.	0	1	2	3	4	5	6	7	8	9
750	87 506	512	518	523	529	535	541	547	552	558
51	564	570	576	581	587	593	599	604	610	616
52	622	628	633	639	645	651	656	662	668	674
53	679	685	691	697	703	708	714	720	726	731
54	737	743	749	754	760	766	772	777	783	789
55	795	800	806	812	818	823	829	835	841	846
56	852	858	864	869	875	881	887	892	898	904
57	910	915	921	927	933	938	944	950	955	961
58	967	973	978	984	990	996	*001	*007	*013	*018
59	88 024	030	036	041	047	053	058	064	070	076
760	081	087	093	098	104	110	116	121	127	133
61	138	144	150	156	161	167	173	178	184	190
62	195	201	207	213	218	224	230	235	241	247
63	252	258	264	270	275	281	287	292	298	304
64	309	315	321	326	332	338	343	349	355	360
65	366	372	377	383	389	395	400	406	412	417
66	423	429	434	440	446	451	457	463	468	474
67	480	485	491	497	502	508	513	519	525	530
68	536	542	547	553	559	564	570	576	581	587
69	593	598	604	610	615	621	627	632	638	643
770	649	655	660	666	672	677	683	689	694	700
71	705	711	717	722	728	734	739	745	750	756
72	762	767	773	779	784	790	795	801	807	812
73	818	824	829	835	840	846	852	857	863	868
74	874	880	885	891	897	902	908	913	919	925
75	930	936	941	947	953	958	964	969	975	981
76	986	992	997	*003	*009	*014	*020	*025	*031	*037
77	89 042	048	053	059	064	070	076	081	087	092
78	098	104	109	115	120	126	131	137	143	148
79	154	159	165	170	176	182	187	193	198	204
780	209	215	221	226	232	237	243	248	254	260
81	265	271	276	282	287	293	298	304	310	315
82	321	326	332	337	343	348	354	360	365	371
83	376	382	387	393	398	404	409	415	421	426
84	432	437	443	448	454	459	465	470	476	481
85	487	492	498	504	509	515	520	526	531	537
86	542	548	553	559	564	570	575	581	586	592
87	597	603	609	614	620	625	631	636	642	647
88	653	658	664	669	675	680	686	691	697	702
89	708	713	719	724	730	735	741	746	752	757
790	763	768	774	779	785	790	796	801	807	812
91	818	823	829	834	840	845	851	856	862	867
92	873	878	883	889	894	900	905	911	916	922
93	927	933	938	944	949	955	960	966	971	977
94	982	988	993	998	*004	*009	*015	*020	*026	*031
95	90 037	042	048	053	059	064	069	075	080	086
96	091	097	102	108	113	119	124	129	135	140
97	146	151	157	162	168	173	179	184	189	195
98	200	206	211	217	222	227	233	238	244	249
99	255	260	266	271	276	282	287	293	298	304
800	309	314	320	325	331	336	342	347	352	358
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	6	5
1	0.6	0.5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
800	90 309	314	320	325	331	336	342	347	352	358
01	363	369	374	380	385	390	396	401	407	412
02	417	423	428	434	439	445	450	455	461	466
03	472	477	482	488	493	499	504	509	515	520
04	526	531	536	542	547	553	558	563	569	574
05	580	585	590	596	601	607	612	617	623	628
06	634	639	644	650	655	660	666	671	677	682
07	687	693	698	703	709	714	720	725	730	736
08	741	747	752	757	763	768	773	779	784	789
09	795	800	806	811	816	822	827	832	838	843
810	849	854	859	865	870	875	881	886	891	897
11	902	907	913	918	924	929	934	940	945	950
12	956	961	966	972	977	982	988	993	998	*004
13	91 009	014	020	025	030	036	041	046	052	057
14	062	068	073	078	084	089	094	100	105	110
15	116	121	126	132	137	142	148	153	158	164
16	169	174	180	185	190	196	201	206	212	217
17	222	228	233	238	243	249	254	259	265	270
18	275	281	286	291	297	302	307	312	318	323
19	328	334	339	344	350	355	360	365	371	376
820	381	387	392	397	403	408	413	418	424	429
21	434	440	445	450	455	461	466	471	477	482
22	487	492	498	503	508	514	519	524	529	535
23	540	545	551	556	561	566	572	577	582	587
24	593	598	603	609	614	619	624	630	635	640
25	645	651	656	661	666	672	677	682	687	693
26	698	703	709	714	719	724	730	735	740	745
27	751	756	761	766	772	777	782	787	793	798
28	803	808	814	819	824	829	834	840	845	850
29	855	861	866	871	876	882	887	892	897	903
830	908	913	918	924	929	934	939	944	950	955
31	960	965	971	976	981	986	991	997	*002	*007
32	92 012	018	023	028	033	038	044	049	054	059
33	065	070	075	080	085	091	096	101	106	111
34	117	122	127	132	137	143	148	153	158	163
35	169	174	179	184	189	195	200	205	210	215
36	221	226	231	236	241	247	252	257	262	267
37	273	278	283	288	293	298	304	309	314	319
38	324	330	335	340	345	350	355	361	366	371
39	376	381	387	392	397	402	407	412	418	423
840	428	433	438	443	449	454	459	464	469	474
41	480	485	490	495	500	505	511	516	521	526
42	531	536	542	547	552	557	562	567	572	578
43	583	588	593	598	603	609	614	619	624	629
44	634	639	645	650	655	660	665	670	675	681
45	686	691	696	701	706	711	716	722	727	732
46	737	742	747	752	758	763	768	773	778	783
47	788	793	799	804	809	814	819	824	829	834
48	840	845	850	855	860	865	870	875	881	886
49	891	896	901	906	911	916	921	927	932	937
850	942	947	952	957	962	967	973	978	983	988
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	6	5
1	0.6	0.5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
850	92 942	947	952	957	962	967	973	978	983	988
51	993	998	*003	*008	*013	*018	*024	*029	*034	*039
52	93 044	049	054	059	064	069	075	080	085	090
53	095	100	105	110	115	120	125	131	136	141
54	146	151	156	161	166	171	176	181	186	192
55	197	202	207	212	217	222	227	232	237	242
56	247	252	258	263	268	273	278	283	288	293
57	298	303	308	313	318	323	328	334	339	344
58	349	354	359	364	369	374	379	384	389	394
59	399	404	409	414	420	425	430	435	440	445
860	450	455	460	465	470	475	480	485	490	495
61	500	505	510	515	520	526	531	536	541	546
62	551	556	561	566	571	576	581	586	591	596
63	601	606	611	616	621	626	631	636	641	646
64	651	656	661	666	671	676	682	687	692	697
65	702	707	712	717	722	727	732	737	742	747
66	752	757	762	767	772	777	782	787	792	797
67	802	807	812	817	822	827	832	837	842	847
68	852	857	862	867	872	877	882	887	892	897
69	902	907	912	917	922	927	932	937	942	947
870	952	957	962	967	972	977	982	987	992	997
71	94 002	007	012	017	022	027	032	037	042	047
72	052	057	062	067	072	077	082	086	091	096
73	101	106	111	116	121	126	131	136	141	146
74	151	156	161	166	171	176	181	186	191	196
75	201	206	211	216	221	226	231	236	240	245
76	250	255	260	265	270	275	280	285	290	295
77	300	305	310	315	320	325	330	335	340	345
78	349	354	359	364	369	374	379	384	389	394
79	399	404	409	414	419	424	429	433	438	443
880	448	453	458	463	468	473	478	483	488	493
81	498	503	507	512	517	522	527	532	537	542
82	547	552	557	562	567	571	576	581	586	591
83	596	601	606	611	616	621	626	630	635	640
84	645	650	655	660	665	670	675	680	685	689
85	694	699	704	709	714	719	724	729	734	738
86	743	748	753	758	763	768	773	778	783	787
87	792	797	802	807	812	817	822	827	832	836
88	841	846	851	856	861	866	871	876	880	885
89	890	895	900	905	910	915	919	924	929	934
890	939	944	949	954	959	963	968	973	978	983
91	988	993	998	*002	*007	*012	*017	*022	*027	*032
92	95 036	041	046	051	056	061	066	071	075	080
93	085	090	095	100	105	109	114	119	124	129
94	134	139	143	148	153	158	163	168	173	177
95	182	187	192	197	202	207	211	216	221	226
96	231	236	240	245	250	255	260	265	270	274
97	279	284	289	294	299	303	308	313	318	323
98	328	332	337	342	347	352	357	361	366	371
99	376	381	386	390	395	400	405	410	415	419
900	424	429	434	439	444	448	453	458	463	468
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	6	5	4
1	0.6	0.5	0.4
2	1.2	1.0	0.8
3	1.8	1.5	1.2
4	2.4	2.0	1.6
5	3.0	2.5	2.0
6	3.6	3.0	2.4
7	4.2	3.5	2.8
8	4.8	4.0	3.2
9	5.4	4.5	3.6

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
900	95 424	429	434	439	444	448	453	458	463	468
01	472	477	482	487	492	497	501	506	511	516
02	521	525	530	535	540	545	550	554	559	564
03	569	574	578	583	588	593	598	602	607	612
04	617	622	626	631	636	641	646	650	655	660
05	665	670	674	679	684	689	694	698	703	708
06	713	718	722	727	732	737	742	746	751	756
07	761	766	770	775	780	785	789	794	799	804
08	809	813	818	823	828	832	837	842	847	852
09	856	861	866	871	875	880	885	890	895	899
910	904	909	914	918	923	928	933	938	942	947
11	952	957	961	966	971	976	980	985	990	995
12	999	*004	*009	*014	*019	*023	*028	*033	*038	*042
13	96 047	052	057	061	066	071	076	080	085	090
14	095	099	104	109	114	118	123	128	133	137
15	142	147	152	156	161	166	171	175	180	185
16	190	194	199	204	209	213	218	223	227	232
17	237	242	246	251	256	261	265	270	275	280
18	284	289	294	298	303	308	313	317	322	327
19	332	336	341	346	350	355	360	365	369	374
920	379	384	388	393	398	402	407	412	417	421
21	426	431	435	440	445	450	454	459	464	468
22	473	478	483	487	492	497	501	506	511	515
23	520	525	530	534	539	544	548	553	558	562
24	567	572	577	581	586	591	595	600	605	609
25	614	619	624	628	633	638	642	647	652	656
26	661	666	670	675	680	685	689	694	699	703
27	708	713	717	722	727	731	736	741	745	750
28	755	759	764	769	774	778	783	788	792	797
29	802	806	811	816	820	825	830	834	839	844
930	848	853	858	862	867	872	876	881	886	890
31	895	900	904	909	914	918	923	928	932	937
32	942	946	951	956	960	965	970	974	979	984
33	988	993	997	*002	*007	*011	*016	*021	*025	*030
34	97 035	039	044	049	053	058	063	067	072	077
35	081	086	090	095	100	104	109	114	118	123
36	128	132	137	142	146	151	155	160	165	169
37	174	179	183	188	192	197	202	206	211	216
38	220	225	230	234	239	243	248	253	257	262
39	267	271	276	280	285	290	294	299	304	308
940	313	317	322	327	331	336	340	345	350	354
41	359	364	368	373	377	382	387	391	396	400
42	405	410	414	419	424	428	433	437	442	447
43	451	456	460	465	470	474	479	483	488	493
44	497	502	506	511	516	520	525	529	534	539
45	543	548	552	557	562	566	571	575	580	585
46	589	594	598	603	607	612	617	621	626	630
47	635	640	644	649	653	658	663	667	672	676
48	681	685	690	695	699	704	708	713	717	722
49	727	731	736	740	745	749	754	759	763	768
950	772	777	782	786	791	795	800	804	809	813
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	5	4
1	0.5	0.4
2	1.0	0.8
3	1.5	1.2
4	2.0	1.6
5	2.5	2.0
6	3.0	2.4
7	3.5	2.8
8	4.0	3.2
9	4.5	3.6

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
950	97 772	777	782	786	791	795	800	804	809	813
51	818	823	827	832	836	841	845	850	855	859
52	864	868	873	877	882	886	891	896	900	905
53	909	914	918	923	928	932	937	941	946	950
54	955	959	964	968	973	978	982	987	991	996
55	98 000	005	009	014	019	023	028	032	037	041
56	046	050	055	059	064	068	073	078	082	087
57	091	096	100	105	109	114	118	123	127	132
58	137	141	146	150	155	159	164	168	173	177
59	182	186	191	195	200	204	209	214	218	223
960	227	232	236	241	245	250	254	259	263	268
61	272	277	281	286	290	295	299	304	308	313
62	318	322	327	331	336	340	345	349	354	358
63	363	367	372	376	381	385	390	394	399	403
64	408	412	417	421	426	430	435	439	444	448
65	453	457	462	466	471	475	480	484	489	493
66	498	502	507	511	516	520	525	529	534	538
67	543	547	552	556	561	565	570	574	579	583
68	588	592	597	601	605	610	614	619	623	628
69	632	637	641	646	650	655	659	664	668	673
970	677	682	686	691	695	700	704	709	713	717
71	722	726	731	735	740	744	749	753	758	762
72	767	771	776	780	784	789	793	798	802	807
73	811	816	820	825	829	834	838	843	847	851
74	856	860	865	869	874	878	883	887	892	896
75	900	905	909	914	918	923	927	932	936	941
76	945	949	954	958	963	967	972	976	981	985
77	989	994	998	*003	*007	*012	*016	*021	*025	*029
78	99 034	038	043	047	052	056	061	065	069	074
79	078	083	087	092	096	100	105	109	114	118
980	123	127	131	136	140	145	149	154	158	162
81	167	171	176	180	185	189	193	198	202	207
82	211	216	220	224	229	233	238	242	247	251
83	255	260	264	269	273	277	282	286	291	295
84	300	304	308	313	317	322	326	330	335	339
85	344	348	352	357	361	366	370	374	379	383
86	388	392	396	401	405	410	414	419	423	427
87	432	436	441	445	449	454	458	463	467	471
88	476	480	484	489	493	498	502	506	511	515
89	520	524	528	533	537	542	546	550	555	559
990	564	568	572	577	581	585	590	594	599	603
91	607	612	616	621	625	629	634	638	642	647
92	651	656	660	664	669	673	677	682	686	691
93	695	699	704	708	712	717	721	726	730	734
94	739	743	747	752	756	760	765	769	774	778
95	782	787	791	795	800	804	808	813	817	822
96	826	830	835	839	843	848	852	856	861	865
97	870	874	878	883	887	891	896	900	904	909
98	913	917	922	926	930	935	939	944	948	952
99	957	961	965	970	974	978	983	987	991	996
1000	00 000	004	009	013	017	022	026	030	035	039
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	5	4
1	0.5	0.4
2	1.0	0.8
3	1.5	1.2
4	2.0	1.6
5	2.5	2.0
6	3.0	2.4
7	3.5	2.8
8	4.0	3.2
9	4.5	3.6

Prop. Pts.

TABLE I a. LOGARITHMS OF IMPORTANT CONSTANTS

N = Number	Value of N	$\log_{10} N$
π	3.14159265	0.49714987
$1 \div \pi$	0.31830989	9.50285013
π^2	9.86960440	0.99429975
$\sqrt{\pi}$	1.77245385	0.24857494
e = Napierian Base	2.71828183	0.43429448
$M = \log_{10} e$	0.43429448	9.63778431
$1 \div M = \log_e 10$	2.30258509	0.36221569
$180 \div \pi$ = degrees in 1 radian	57.2957795	1.75812262
$\pi \div 180$ = radians in 1°	0.01745329	8.24187738
$\pi \div 10800$ = radians in 1′	0.0002908882	6.4637261
$\pi \div 648000$ = radians in 1″	0.000004848136811095	4.68557487
sin 1″	0.000004848136811076	4.68557487
tan 1″	0.000004848136811152	4.68557487
centimeters in 1 ft.	30.480	1.4840158
feet in 1 cm.	0.032808	8.5159842
inches in 1 m.	39.37	1.5951654
pounds in 1 kg.	2.20462	0.3433340
kilograms in 1 lb.	0.453593	9.6566660
g	32.16 ft./sec./sec.	1.5073
	= 981 cm./sec./sec.	2.9916690
weight of 1 cu. ft. of water	62.425 lb. (max. density)	1.7953+
weight of 1 cu. ft. of air	0.0807 lb. (at 32° F.)	8.907
cu. in. in 1 (U. S.) gallon	231.	2.3636120
ft. lb. per sec. in 1 H. P.	550.	2.7403627
kg. m. per sec. in 1 H. P.	76.0404	1.8810445
watts in 1 H. P.	745.957	2.8727135

COMMON LOGARITHMS OF THE FIRST HUNDRED PRIME NUMBERS

N	Logarithm	N	Log	N	Log	N	Log	N	Log
1	0000000000	71	8512583	173	2380461	281	4487063	409	6117233
2	3010299957	73	8633229	179	2528530	283	4517864	419	6222140
3	4771212547	79	8976271	181	2576786	293	4668676	421	6242821
5	6989700043	83	9190781	191	2810334	307	4871384	431	6344773
7	8450980400	89	9493900	193	2855573	311	4927604	433	6364879
11	0413926852	97	9867717	197	2944662	313	4955443	439	6424645
13	1139433523	101	0043214	199	2988531	317	5010593	443	6464037
17	2304489214	103	0128372	211	3242825	331	5198280	449	6522463
19	2787536010	107	0293838	223	3483049	337	5276299	457	6599162
23	3617278360	109	0374265	227	3560259	347	5403295	461	6637009
29	4623979979	113	0530784	229	3598355	349	5428254	463	6655810
31	4913616938	127	1038037	233	3673559	353	5477747	467	6693169
37	5682017241	131	1172713	239	3783979	359	5550944	479	6803355
41	6127838567	137	1367206	241	3820170	367	5646661	487	6875290
43	6334684556	139	1430148	251	3996737	373	5717088	491	6910815
47	6720978579	149	1731863	257	4099331	379	5786392	499	6981005
53	7242758696	151	1789769	263	4199557	383	5831988	503	7015680
59	7708520116	157	1958997	269	4297523	389	5899496	509	7067178
61	7853298350	163	2121876	271	4329693	397	5987905	521	7168377

TABLE II

ACTUAL VALUES

OF THE

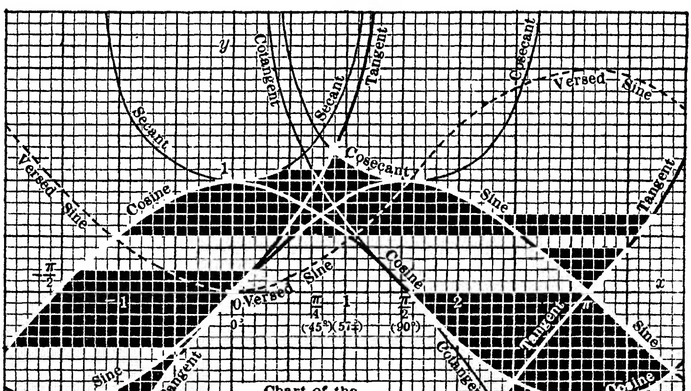
TRIGONOMETRIC FUNCTIONS

FROM

0° TO 90° AT INTERVALS OF ONE MINUTE

TO

FIVE DECIMAL PLACES



′	Sin	Tan	Ctn	Cos	
0	.00000	.00000	——	1.0000	**60**
1	029	029	3437.7	000	59
2	058	058	1718.9	000	58
3	087	087	1145.9	000	57
4	116	116	859.44	000	56
5	.00145	.00145	687.55	1.0000	**55**
6	175	175	572.96	000	54
7	204	204	491.11	000	53
8	233	233	429.72	000	52
9	262	262	381.97	000	51
10	.00291	.00291	343.77	1.0000	**50**
11	320	320	312.52	.99999	49
12	349	349	286.48	999	48
13	378	378	264.44	999	47
14	407	407	245.55	999	46
15	.00436	.00436	229.18	.99999	**45**
16	465	465	214.86	999	44
17	495	495	202.22	999	43
18	524	524	190.98	999	42
19	553	553	180.93	998	41
20	.00582	.00582	171.89	.99998	**40**
21	611	611	163.70	998	39
22	640	640	156.26	998	38
23	669	669	149.47	998	37
24	698	698	143.24	998	36
25	.00727	.00727	137.51	.99997	**35**
26	756	756	132.22	997	34
27	785	785	127.32	997	33
28	814	815	122.77	997	32
29	844	844	118.54	996	31
30	.00873	.00873	114.59	.99996	**30**
31	902	902	110.89	996	29
32	931	931	107.43	996	28
33	960	960	104.17	995	27
34	.00989	.00989	101.11	995	26
35	.01018	.01018	98.218	.99995	**25**
36	047	047	95.489	995	24
37	076	076	92.908	994	23
38	105	105	90.463	994	22
39	134	135	88.144	994	21
40	.01164	.01164	85.940	.99993	**20**
41	193	193	83.844	993	19
42	222	222	81.847	993	18
43	251	251	79.943	992	17
44	280	280	78.126	992	16
45	.01309	.01309	76.390	.99991	**15**
46	338	338	74.729	991	14
47	367	367	73.139	991	13
48	396	396	71.615	990	12
49	425	425	70.153	990	11
50	.01454	.01455	68.750	.99989	**10**
51	483	484	67.402	989	9
52	513	513	66.105	989	8
53	542	542	64.858	988	7
54	571	571	63.657	988	6
55	.01600	.01600	62.499	.99987	**5**
56	629	629	61.383	987	4
57	658	658	60.306	986	3
58	687	687	59.266	986	2
59	716	716	58.261	985	1
60	.01745	.01746	57.290	.99985	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

′	Sin	Tan	Ctn	Cos	
0	.01745	.01746	57.290	.99985	**60**
1	774	775	56.351	984	59
2	803	804	55.442	984	58
3	832	833	54.561	983	57
4	862	862	53.709	983	56
5	.01891	.01891	52.882	.99982	**55**
6	920	920	52.081	982	54
7	949	949	51.303	981	53
8	.01978	.01978	50.549	980	52
9	.02007	.02007	49.816	980	51
10	.02036	.02036	49.104	.99979	**50**
11	065	066	48.412	979	49
12	094	095	47.740	978	48
13	123	124	47.085	977	47
14	152	153	46.449	977	46
15	.02181	.02182	45.829	.99976	**45**
16	211	211	45.226	976	44
17	240	240	44.639	975	43
18	269	269	44.066	974	42
19	298	298	43.508	974	41
20	.02327	.02328	42.964	.99973	**40**
21	356	357	42.433	972	39
22	385	386	41.916	972	38
23	414	415	41.411	971	37
24	443	444	40.917	970	36
25	.02472	.02473	40.436	.99969	**35**
26	501	502	39.965	969	34
27	530	531	39.506	968	33
28	560	560	39.057	967	32
29	589	589	38.618	966	31
30	.02618	.02619	38.188	.99966	**30**
31	647	648	37.769	965	29
32	676	677	37.358	964	28
33	705	706	36.956	963	27
34	734	735	36.563	963	26
35	.02763	.02764	36.178	.99962	**25**
36	792	793	35.801	961	24
37	821	822	35.431	960	23
38	850	851	35.070	959	22
39	879	881	34.715	959	21
40	.02908	.02910	34.368	.99958	**20**
41	938	939	34.027	957	19
42	967	968	33.694	956	18
43	.02996	.02997	33.366	955	17
44	.03025	.03026	33.045	954	16
45	.03054	.03055	32.730	.99953	**15**
46	083	084	32.421	952	14
47	112	114	32.118	952	13
48	141	143	31.821	951	12
49	170	172	31.528	950	11
50	.03199	.03201	31.242	.99949	**10**
51	228	230	30.960	948	9
52	257	259	30.683	947	8
53	286	288	30.412	946	7
54	316	317	30.145	945	6
55	.03345	.03346	29.882	.99944	**5**
56	374	376	29.624	943	4
57	403	405	29.371	942	3
58	432	434	29.122	941	2
59	461	463	28.877	940	1
60	.03490	.03492	28.636	.99939	**0**
	Cos	**Ctn**	**Tan**	**Sin**	

′	Sin	Tan	Ctn	Cos	
0	.03490	.03492	28.636	.99939	**60**
1	519	521	.399	938	59
2	548	550	28.166	937	58
3	577	579	27.937	936	57
4	606	609	.712	935	56
5	.03635	.03638	27.490	.99934	**55**
6	664	667	.271	933	54
7	693	696	27.057	932	53
8	723	725	26.845	931	52
9	752	754	.637	930	51
10	.03781	.03783	26.432	.99929	**50**
11	810	812	.230	927	49
12	839	842	26.031	926	48
13	868	871	25.835	925	47
14	897	900	.642	924	46
15	.03926	.03929	25.452	.99923	**45**
16	955	958	.264	922	44
17	.03984	.03987	25.080	921	43
18	.04013	.04016	24.898	919	42
19	042	046	.719	918	41
20	.04071	.04075	24.542	.99917	**40**
21	100	104	.368	916	39
22	129	133	.196	915	38
23	159	162	24.026	913	37
24	188	191	23.859	912	36
25	.04217	.04220	23.695	.99911	**35**
26	246	250	.532	910	34
27	275	279	.372	909	33
28	304	308	.214	907	32
29	333	337	23.058	906	31
30	.04362	.04366	22.904	.99905	**30**
31	391	395	.752	904	29
32	420	424	.602	902	28
33	449	454	.454	901	27
34	478	483	.308	900	26
35	.04507	.04512	22.164	.99898	**25**
36	536	541	22.022	897	24
37	565	570	21.881	896	23
38	594	599	.743	894	22
39	623	628	.606	893	21
40	.04653	.04658	21.470	.99892	**20**
41	682	687	.337	890	19
42	711	716	.205	889	18
43	740	745	21.075	888	17
44	769	774	20.946	886	16
45	.04798	.04803	20.819	.99885	**15**
46	827	833	.693	883	14
47	856	862	.569	882	13
48	885	891	.446	881	12
49	914	920	.325	879	11
50	.04943	.04949	20.206	.99878	**10**
51	.04972	.04978	20.087	876	9
52	.05001	.05007	19.970	875	8
53	030	037	.855	873	7
54	059	066	.740	872	6
55	.05088	.05095	19.627	.99870	**5**
56	117	124	.516	869	4
57	146	153	.405	867	3
58	175	182	.296	866	2
59	205	212	.188	864	1
60	.05234	.05241	19.081	.99863	**0**
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.05234	.05241	19.081	.99863	**60**
1	263	270	18.976	861	59
2	292	299	.871	860	58
3	321	328	.768	858	57
4	350	357	.666	857	56
5	.05379	.05387	18.564	.99855	**55**
6	408	416	.464	854	54
7	437	445	.366	852	53
8	466	474	.268	851	52
9	495	503	.171	849	51
10	.05524	.05533	18.075	.99847	**50**
11	553	562	17.980	846	49
12	582	591	.886	844	48
13	611	620	.793	842	47
14	640	649	.702	841	46
15	.05669	.05678	17.611	.99839	**45**
16	698	708	.521	838	44
17	727	737	.431	836	43
18	756	766	.343	834	42
19	785	795	.256	833	41
20	.05814	.05824	17.169	.99831	**40**
21	844	854	17.084	829	39
22	873	883	16.999	827	38
23	902	912	.915	826	37
24	931	941	.832	824	36
25	.05960	.05970	16.750	.99822	**35**
26	.05989	.05999	.668	821	34
27	.06018	.06029	.587	819	33
28	047	058	.507	817	32
29	076	087	.428	815	31
30	.06105	.06116	16.350	.99813	**30**
31	134	145	.272	812	29
32	163	175	.195	810	28
33	192	204	.119	808	27
34	221	233	16.043	806	26
35	.06250	.06262	15.969	.99804	**25**
36	279	291	.895	803	24
37	308	321	.821	801	23
38	337	350	.748	799	22
39	366	379	.676	797	21
40	.06395	.06408	15.605	.99795	**20**
41	424	438	.534	793	19
42	453	467	.464	792	18
43	482	496	.394	790	17
44	511	525	.325	788	16
45	.06540	.06554	15.257	.99786	**15**
46	569	584	.189	784	14
47	598	613	.122	782	13
48	627	642	15.056	780	12
49	656	671	14.990	778	11
50	.06685	.06700	14.924	.99776	**10**
51	714	730	.860	774	9
52	743	759	.795	772	8
53	773	788	.732	770	7
54	802	817	.669	768	6
55	.06831	.06847	14.606	.99766	**5**
56	860	876	.544	764	4
57	889	905	.482	762	3
58	918	934	.421	760	2
59	947	963	.361	758	1
60	.06976	.06993	14.301	.99756	**0**
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.06976	.06993	14.301	.99756	**60**
1	.07005	.07022	.241	754	59
2	034	051	.182	752	58
3	063	080	.124	750	57
4	092	110	.065	748	56
5	.07121	.07139	14.008	.99746	**55**
6	150	168	13.951	744	54
7	179	197	.894	742	53
8	208	227	.838	740	52
9	237	256	.782	738	51
10	.07266	.07285	13.727	.99736	**50**
11	295	314	.672	734	49
12	324	344	.617	731	48
13	353	373	.563	729	47
14	382	402	.510	727	46
15	.07411	.07431	13.457	.99725	**45**
16	440	461	.404	723	44
17	469	490	.352	721	43
18	498	519	.300	719	42
19	527	548	.248	716	41
20	.07556	.07578	13.197	.99714	**40**
21	585	607	.146	712	39
22	614	636	.096	710	38
23	643	665	13.046	708	37
24	672	695	12.996	705	36
25	.07701	.07724	12.947	.99703	**35**
26	730	753	.898	701	34
27	759	782	.850	699	33
28	788	812	.801	696	32
29	817	841	.754	694	31
30	.07846	.07870	12.706	.99692	**30**
31	875	899	.659	689	29
32	904	929	.612	687	28
33	933	958	.566	685	27
34	962	.07987	.520	683	26
35	.07991	.08017	12.474	.99680	**25**
36	.08020	046	.429	678	24
37	049	075	.384	676	23
38	078	104	.339	673	22
39	107	134	.295	671	21
40	.08136	.08163	12.251	.99668	**20**
41	165	192	.207	666	19
42	194	221	.163	664	18
43	223	251	.120	661	17
44	252	280	.077	659	16
45	.08281	.08309	12.035	.99657	**15**
46	310	339	11.992	654	14
47	339	368	.950	652	13
48	368	397	.909	649	12
49	397	427	.867	647	11
50	.08426	.08456	11.826	.99644	**10**
51	455	485	.785	642	9
52	484	514	.745	639	8
53	513	544	.705	637	7
54	542	573	.664	635	6
55	.08571	.08602	11.625	.99632	**5**
56	600	632	.585	630	4
57	629	661	.546	627	3
[illegible]	[illegible]	690	.507	625	2
[illegible]	[illegible]	[illegible]	.468	622	1
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	**0**

′	Sin	Tan	Ctn	Cos	
0	.08716	.08749	11.430	.99619	**60**
1	745	778	.392	617	59
2	774	807	.354	614	58
3	803	837	.316	612	57
4	831	866	.279	609	56
5	.08860	.08895	11.242	.99607	**55**
6	889	925	.205	604	54
7	918	954	.168	602	53
8	947	.08983	.132	599	52
9	.08976	.09013	.095	596	51
10	.09005	.09042	11.059	.99594	**50**
11	034	071	11.024	591	49
12	063	101	10.988	588	48
13	092	130	.953	586	47
14	121	159	.918	583	46
15	.09150	.09189	10.883	.99580	**45**
16	179	218	.848	578	44
17	208	247	.814	575	43
18	237	277	.780	572	42
19	266	306	.746	570	41
20	.09295	.09335	10.712	.99567	**40**
21	324	365	.678	564	39
22	353	394	.645	562	38
23	382	423	.612	559	37
24	411	453	.579	556	36
25	.09440	.09482	10.546	.99553	**35**
26	469	511	.514	551	34
27	498	541	.481	548	33
28	527	570	.449	545	32
29	556	600	.417	542	31
30	.09585	.09629	10.385	.99540	**30**
31	614	658	.354	537	29
32	642	688	.322	534	28
33	671	717	.291	531	27
34	700	746	.260	528	26
35	.09729	.09776	10.229	.99526	**25**
36	758	805	.199	523	24
37	787	834	.168	520	23
38	816	864	.138	517	22
39	845	893	.108	514	21
40	.09874	.09923	10.078	.99511	**20**
41	903	952	.048	508	19
42	932	.09981	10.019	506	18
43	961	.10011	9.9893	503	17
44	.09990	040	.9601	500	16
45	.10019	.10069	9.9310	.99497	**15**
46	048	099	.9021	494	14
47	077	128	.8734	491	13
48	106	158	.8448	488	12
49	135	187	.8164	485	11
50	.10164	.10216	9.7882	.99482	**10**
51	192	246	.7601	479	9
52	221	275	.7322	476	8
53	250	305	.7044	473	7
54	279	334	.6768	470	6
55	.10308	.10363	9.6493	.99467	**5**
56	337	393	.6220	464	4
57	366	422	.5949	461	3
58	395	452	.5679	458	2
59	424	481	.5411	455	1
60	.10453	.10510	9.5144	.99452	**0**

′	Sin	Tan	Ctn	Cos	
0	.10453	.10510	9.5144	.99452	**60**
1	482	540	.4878	449	59
2	511	569	.4614	446	58
3	540	599	.4352	443	57
4	569	628	.4090	440	56
5	.10597	.10657	9.3831	.99437	**55**
6	626	687	.3572	434	54
7	655	716	.3315	431	53
8	684	746	.3060	428	52
9	713	775	.2806	424	51
10	.10742	.10805	9.2553	.99421	**50**
11	771	834	.2302	418	49
12	800	863	.2052	415	48
13	829	893	.1803	412	47
14	858	922	.1555	409	46
15	.10887	.10952	9.1309	.99406	**45**
16	916	.10981	.1065	402	44
17	945	.11011	.0821	399	43
18	.10973	040	.0579	396	42
19	.11002	070	.0338	393	41
20	.11031	.11099	9.0098	.99390	**40**
21	060	128	8.9860	386	39
22	089	158	.9623	383	38
23	118	187	.9387	380	37
24	147	217	.9152	377	36
25	.11176	.11246	8.8919	.99374	**35**
26	205	276	.8686	370	34
27	234	305	.8455	367	33
28	263	335	.8225	364	32
29	291	364	.7996	360	31
30	.11320	.11394	8.7769	.99357	**30**
31	349	423	.7542	354	29
32	378	452	.7317	351	28
33	407	482	.7093	347	27
34	436	511	.6870	344	26
35	.11465	.11541	8.6648	.99341	**25**
36	494	570	.6427	337	24
37	523	600	.6208	334	23
38	552	629	.5989	331	22
39	580	659	.5772	327	21
40	.11609	.11688	8.5555	.99324	**20**
41	638	718	.5340	320	19
42	667	747	.5126	317	18
43	696	777	.4913	314	17
44	725	806	.4701	310	16
45	.11754	.11836	8.4490	.99307	**15**
46	783	865	.4280	303	14
47	812	895	.4071	300	13
48	840	924	.3863	297	12
49	869	954	.3656	293	11
50	.11898	.11983	8.3450	.99290	**10**
51	927	.12013	.3245	286	9
52	956	042	.3041	283	8
53	.11985	072	.2838	279	7
54	.12014	101	.2636	276	6
55	.12043	.12131	8.2434	.99272	**5**
56	071	160	.2234	269	4
57	100	190	.2035	265	3
58	129	219	.1837	262	2
59	158	249	.1640	258	1
60	.12187	.12278	8.1443	.99255	**0**
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.12187	.12278	8.1443	.99255	**60**
1	216	308	.1248	251	59
2	245	338	.1054	248	58
3	274	367	.0860	244	57
4	302	397	.0667	240	56
5	.12331	.12426	8.0476	.99237	**55**
6	360	456	.0285	233	54
7	389	485	8.0095	230	53
8	418	515	7.9906	226	52
9	447	544	.9718	222	51
10	.12476	.12574	7.9530	.99219	**50**
11	504	603	.9344	215	49
12	533	633	.9158	211	48
13	562	662	.8973	208	47
14	591	692	.8789	204	46
15	.12620	.12722	7.8606	.99200	**45**
16	649	751	.8424	197	44
17	678	781	.8243	193	43
18	706	810	.8062	189	42
19	735	840	.7882	186	41
20	.12764	.12869	7.7704	.99182	**40**
21	793	899	.7525	178	39
22	822	929	.7348	175	38
23	851	958	.7171	171	37
24	880	.12988	.6996	167	36
25	.12908	.13017	7.6821	.99163	**35**
26	937	047	.6647	160	34
27	966	076	.6473	156	33
28	.12995	106	.6301	152	32
29	.13024	136	.6129	148	31
30	.13053	.13165	7.5958	.99144	**30**
31	081	195	.5787	141	29
32	110	224	.5618	137	28
33	139	254	.5449	133	27
34	168	284	.5281	129	26
35	.13197	.13313	7.5113	.99125	**25**
36	226	343	.4947	122	24
37	254	372	.4781	118	23
38	283	402	.4615	114	22
39	312	432	.4451	110	21
40	.13341	.13461	7.4287	.99106	**20**
41	370	491	.4124	102	19
42	399	521	.3962	098	18
43	427	550	.3800	094	17
44	456	580	.3639	091	16
45	.13485	.13609	7.3479	.99087	**15**
46	514	639	.3319	083	14
47	543	669	.3160	079	13
48	572	698	.3002	075	12
49	600	728	.2844	071	11
50	.13629	.13758	7.2687	.99067	**10**
51	658	787	.2531	063	9
52	687	817	.2375	059	8
53	716	846	.2220	055	7
54	744	876	.2066	051	6
55	.13773	.13906	7.1912	.99047	**5**
56	802	935	.1759	043	4
57	831	965	.1607	039	3
58	860	.13995	.1455	035	2
59	889	.14024	.1304	031	1
60	.13917	.14054	7.1154	.99027	**0**
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.13917	.14054	7.1154	.99027	**60**
1	946	084	.1004	023	59
2	.13975	113	.0855	019	58
3	.14004	143	.0706	015	57
4	033	173	.0558	011	56
5	.14061	.14202	7.0410	.99006	**55**
6	090	232	.0264	.99002	54
7	119	262	7.0117	.98998	53
8	148	291	6.9972	994	52
9	177	321	.9827	990	51
10	.14205	.14351	6.9682	.98986	**50**
11	234	381	.9538	982	49
12	263	410	.9395	978	48
13	292	440	.9252	973	47
14	320	470	.9110	969	46
15	.14349	.14499	6.8969	.98965	**45**
16	378	529	.8828	961	44
17	407	559	.8687	957	43
18	436	588	.8548	953	42
19	464	618	.8408	948	41
20	.14493	.14648	6.8269	.98944	**40**
21	522	678	.8131	940	39
22	551	707	.7994	936	38
23	580	737	.7856	931	37
24	608	767	.7720	927	36
25	.14637	.14796	6.7584	.98923	**35**
26	666	826	.7448	919	34
27	695	856	.7313	914	33
28	723	886	.7179	910	32
29	752	915	.7045	906	31
30	.14781	.14945	6.6912	.98902	**30**
31	810	.14975	.6779	897	29
32	838	.15005	.6646	893	28
33	867	034	.6514	889	27
34	896	064	.6383	884	26
35	.14925	.15094	6.6252	.98880	**25**
36	954	124	.6122	876	24
37	.14982	153	.5992	871	23
38	.15011	183	.5863	867	22
39	040	213	.5734	863	21
40	.15069	.15243	6.5606	.98858	**20**
41	097	272	.5478	854	19
42	126	302	.5350	849	18
43	155	332	.5223	845	17
44	184	362	.5097	841	16
45	.15212	.15391	6.4971	.98836	**15**
46	241	421	.4846	832	14
47	270	451	.4721	827	13
48	299	481	.4596	823	12
49	327	511	.4472	818	11
50	.15356	.15540	6.4348	.98814	**10**
51	385	570	.4225	809	9
52	414	600	.4103	805	8
53	442	630	.3980	800	7
54	471	660	.3859	796	6
55	.15500	.15689	6.3737	.98791	**5**
56	529	719	.3617	787	4
57	557	749	.3496	782	3
58	586	779	.3376	778	2
59	615	809	.3257	773	1
60	.15643	.15838	6.3138	.98769	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

′	Sin	Tan	Ctn	Cos	
0	.15643	.15838	6.3138	.98769	**60**
1	672	868	.3019	764	59
2	701	898	.2901	760	58
3	730	928	.2783	755	57
4	758	958	.2666	751	56
5	.15787	.15988	6.2549	.98746	**55**
6	816	.16017	.2432	741	54
7	845	047	.2316	737	53
8	873	077	.2200	732	52
9	902	107	.2085	728	51
10	.15931	.16137	6.1970	.98723	**50**
11	959	167	.1856	718	49
12	.15988	196	.1742	714	48
13	.16017	226	.1628	709	47
14	046	256	.1515	704	46
15	.16074	.16286	6.1402	.98700	**45**
16	103	316	.1290	695	44
17	132	346	.1178	690	43
18	160	376	.1066	686	42
19	189	405	.0955	681	41
20	.16218	.16435	6.0844	.98676	**40**
21	246	465	.0734	671	39
22	275	495	.0624	667	38
23	304	525	.0514	662	37
24	333	555	.0405	657	36
25	.16361	.16585	6.0296	.98652	**35**
26	390	615	.0188	648	34
27	419	645	6.0080	643	33
28	447	674	5.9972	638	32
29	476	704	.9865	633	31
30	.16505	.16734	5.9758	.98629	**30**
31	533	764	.9651	624	29
32	562	794	.9545	619	28
33	591	824	.9439	614	27
34	620	854	.9333	609	26
35	.16648	.16884	5.9228	.98604	**25**
36	677	914	.9124	600	24
37	706	944	.9019	595	23
38	734	.16974	.8915	590	22
39	763	.17004	.8811	585	21
40	.16792	.17033	5.8708	.98580	**20**
41	820	063	.8605	575	19
42	849	093	.8502	570	18
43	878	123	.8400	565	17
44	906	153	.8298	561	16
45	.16935	.17183	5.8197	.98556	**15**
46	964	213	.8095	551	14
47	.16992	243	.7994	546	13
48	.17021	273	.7894	541	12
49	050	303	.7794	536	11
50	.17078	.17333	5.7694	.98531	**10**
51	107	363	.7594	526	9
52	136	393	.7495	521	8
53	164	423	.7396	516	7
54	193	453	.7297	511	6
55	.17222	.17483	5.7199	.98506	**5**
56	250	513	.7101	501	4
57	279	543	.7004	496	3
58	308	573	.6906	491	2
59	336	603	.6809	486	1
60	.17365	.17633	5.6713	.98481	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

′	Sin	Tan	Ctn	Cos	
0	.17365	.17633	5.6713	.98481	**60**
1	393	663	.6617	476	59
2	422	693	.6521	471	58
3	451	723	.6425	466	57
4	479	753	.6329	461	56
5	.17508	.17783	5.6234	.98455	**55**
6	537	813	.6140	450	54
7	565	843	.6045	445	53
8	594	873	.5951	440	52
9	623	903	.5857	435	51
10	.17651	.17933	5.5764	.98430	**50**
11	680	963	.5671	425	49
12	708	.17993	.5578	420	48
13	737	.18023	.5485	414	47
14	766	053	.5393	409	46
15	.17794	.18083	5.5301	.98404	**45**
16	823	113	.5209	399	44
17	852	143	.5118	394	43
18	880	173	.5026	389	42
19	909	203	.4936	383	41
20	.17937	.18233	5.4845	.98378	**40**
21	966	263	.4755	373	39
22	.17995	293	.4665	368	38
23	.18023	323	.4575	362	37
24	052	353	.4486	357	36
25	.18081	.18384	5.4397	.98352	**35**
26	109	414	.4308	347	34
27	138	444	.4219	341	33
28	166	474	.4131	336	32
29	195	504	.4043	331	31
30	.18224	.18534	5.3955	.98325	**30**
31	252	564	.3868	320	29
32	281	594	.3781	315	28
33	309	624	.3694	310	27
34	338	654	.3607	304	26
35	.18367	.18684	5.3521	.98299	**25**
36	395	714	.3435	294	24
37	424	745	.3349	288	23
38	452	775	.3263	283	22
39	481	805	.3178	277	21
40	.18509	.18835	5.3093	.98272	**20**
41	538	865	.3008	267	19
42	567	895	.2924	261	18
43	595	925	.2839	256	17
44	624	955	.2755	250	16
45	.18652	.18986	5.2672	.98245	**15**
46	681	.19016	.2588	240	14
47	710	046	.2505	234	13
48	738	076	.2422	229	12
49	767	106	.2339	223	11
50	.18795	.19136	5.2257	.98218	**10**
51	824	166	.2174	212	9
52	852	197	.2092	207	8
53	881	227	.2011	201	7
54	910	257	.1929	196	6
55	.18938	.19287	5.1848	.98190	**5**
56	967	317	.1767	185	4
57	.18995	347	.1686	179	3
58	.19024	378	.1606	174	2
59	052	408	.1526	168	1
60	.19081	.19438	5.1446	.98163	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

′	Sin	Tan	Ctn	Cos	
0	.19081	.19438	5.1446	.98163	**60**
1	109	468	.1366	157	59
2	138	498	.1286	152	58
3	167	529	.1207	146	57
4	195	559	.1128	140	56
5	.19224	.19589	5.1049	.98135	**55**
6	252	619	.0970	129	54
7	281	649	.0892	124	53
8	309	680	.0814	118	52
9	338	710	.0736	112	51
10	.19366	.19740	5.0658	.98107	**50**
11	395	770	.0581	101	49
12	423	801	.0504	096	48
13	452	831	.0427	090	47
14	481	861	.0350	084	46
15	.19509	.19891	5.0273	.98079	**45**
16	538	921	.0197	073	44
17	566	952	.0121	067	43
18	595	.19982	5.0045	061	42
19	623	.20012	4.9969	056	41
20	.19652	.20042	4.9894	.98050	**40**
21	680	073	.9819	044	39
22	709	103	.9744	039	38
23	737	133	.9669	033	37
24	766	164	.9594	027	36
25	.19794	.20194	4.9520	.98021	**35**
26	823	224	.9446	016	34
27	851	254	.9372	010	33
28	880	285	.9298	.98004	32
29	908	315	.9225	.97998	31
30	.19937	.20345	4.9152	.97992	**30**
31	965	376	.9078	987	29
32	.19994	406	.9006	981	28
33	.20022	436	.8933	975	27
34	051	466	.8860	969	26
35	.20079	.20497	4.8788	.97963	**25**
36	108	527	.8716	958	24
37	136	557	.8644	952	23
38	165	588	.8573	946	22
39	193	618	.8501	940	21
40	.20222	.20648	4.8430	.97934	**20**
41	250	679	.8359	928	19
42	279	709	.8288	922	18
43	307	739	.8218	916	17
44	336	770	.8147	910	16
45	.20364	.20800	4.8077	.97905	**15**
46	393	830	.8007	899	14
47	421	861	.7937	893	13
48	450	891	.7867	887	12
49	478	921	.7798	881	11
50	.20507	.20952	4.7729	.97875	**10**
51	535	.20982	.7659	869	9
52	563	.21013	.7591	863	8
53	592	043	.7522	857	7
54	620	073	.7453	851	6
55	.20649	.21104	4.7385	.97845	**5**
56	677	134	.7317	839	4
57	706	164	.7249	833	3
58	734	195	.7181	827	2
59	763	225	.7114	821	1
60	.20791	.21256	4.7046	.97815	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

′	Sin	Tan	Ctn	Cos	
0	.20791	.21256	4.7046	.97815	**60**
1	820	286	.6979	809	59
2	848	316	.6912	803	58
3	877	347	.6845	797	57
4	905	377	.6779	791	56
5	.20933	.21408	4.6712	.97784	**55**
6	962	438	.6646	778	54
7	.20990	469	.6580	772	53
8	.21019	499	.6514	766	52
9	047	529	.6448	760	51
10	.21076	.21560	4.6382	.97754	**50**
11	104	590	.6317	748	49
12	132	621	.6252	742	48
13	161	651	.6187	735	47
14	189	682	.6122	729	46
15	.21218	.21712	4.6057	.97723	**45**
16	246	743	.5993	717	44
17	275	773	.5928	711	43
18	303	804	.5864	705	42
19	331	834	.5800	698	41
20	.21360	.21864	4.5736	.97692	**40**
21	388	895	.5673	686	39
22	417	925	.5609	680	38
23	445	956	.5546	673	37
24	474	.21986	.5483	667	36
25	.21502	.22017	4.5420	.97661	**35**
26	530	047	.5357	655	34
27	559	078	.5294	648	33
28	587	108	.5232	642	32
29	616	139	.5169	636	31
30	.21644	.22169	4.5107	.97630	**30**
31	672	200	.5045	623	29
32	701	231	.4983	617	28
33	729	261	.4922	611	27
34	758	292	.4860	604	26
35	.21786	.22322	4.4799	.97598	**25**
36	814	353	.4737	592	24
37	843	383	.4676	585	23
38	871	414	.4615	579	22
39	899	444	.4555	573	21
40	.21928	.22475	4.4494	.97566	**20**
41	956	505	.4434	560	19
42	.21985	536	.4373	553	18
43	.22013	567	.4313	547	17
44	041	597	.4253	541	16
45	.22070	.22628	4.4194	.97534	**15**
46	098	658	.4134	528	14
47	126	689	.4075	521	13
48	155	719	.4015	515	12
49	183	750	.3956	508	11
50	.22212	.22781	4.3897	.97502	**10**
51	240	811	.3838	496	9
52	268	842	.3779	489	8
53	297	872	.3721	483	7
54	325	903	.3662	476	6
55	.22353	.22934	4.3604	.97470	**5**
56	382	964	.3546	463	4
57	410	.22995	.3488	457	3
58	438	.23026	.3430	450	2
59	467	056	.3372	444	1
60	.22495	.23087	4.3315	.97437	**0**
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.22495	.23087	4.3315	.97437	**60**
1	523	117	.3257	430	59
2	552	148	.3200	424	58
3	580	179	.3143	417	57
4	608	209	.3086	411	56
5	.22637	.23240	4.3029	.97404	**55**
6	665	271	.2972	398	54
7	693	301	.2916	391	53
8	722	332	.2859	384	52
9	750	363	.2803	378	51
10	.22778	.23393	4.2747	.97371	**50**
11	807	424	.2691	365	49
12	835	455	.2635	358	48
13	863	485	.2580	351	47
14	892	516	.2524	345	46
15	.22920	.23547	4.2468	.97338	**45**
16	948	578	.2413	331	44
17	.22977	608	.2358	325	43
18	.23005	639	.2303	318	42
19	033	670	.2248	311	41
20	.23062	.23700	4.2193	.97304	**40**
21	090	731	.2139	298	39
22	118	762	.2084	291	38
23	146	793	.2030	284	37
24	175	823	.1976	278	36
25	.23203	.23854	4.1922	.97271	**35**
26	231	885	.1868	264	34
27	260	916	.1814	257	33
28	288	946	.1760	251	32
29	316	.23977	.1706	244	31
30	.23345	.24008	4.1653	.97237	**30**
31	373	039	.1600	230	29
32	401	069	.1547	223	28
33	429	100	.1493	217	27
34	458	131	.1441	210	26
35	.23486	.24162	4.1388	.97203	**25**
36	514	193	.1335	196	24
37	542	223	.1282	189	23
38	571	254	.1230	182	22
39	599	285	.1178	176	21
40	.23627	.24316	4.1126	.97169	**20**
41	656	347	.1074	162	19
42	684	377	.1022	155	18
43	712	408	.0970	148	17
44	740	439	.0918	141	16
45	.23769	.24470	4.0867	.97134	**15**
46	797	501	.0815	127	14
47	825	532	.0764	120	13
48	853	562	.0713	113	12
49	882	593	.0662	106	11
50	.23910	.24624	4.0611	.97100	**10**
51	938	655	.0560	093	9
52	966	686	.0509	086	8
53	.23995	717	.0459	079	7
54	.24023	747	.0408	072	6
55	.24051	.24778	4.0358	.97065	**5**
56	079	809	.0308	058	4
57	108	840	.0257	051	3
58	136	871	.0207	044	2
59	164	902	.0158	037	1
60	.24192	.24933	4.0108	.97030	**0**
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.24192	.24933	4.0108	.97030	60
1	220	964	.0058	023	59
2	249	.24995	4.0009	015	58
3	277	.25026	3.9959	008	57
4	305	056	.9910	.97001	56
5	.24333	.25087	3.9861	.96994	55
6	362	118	.9812	987	54
7	390	149	.9763	980	53
8	418	180	.9714	973	52
9	446	211	.9665	966	51
10	.24474	.25242	3.9617	.96959	50
11	503	273	.9568	952	49
12	531	304	.9520	945	48
13	559	335	.9471	937	47
14	587	366	.9423	930	46
15	.24615	.25397	3.9375	.96923	45
16	644	428	.9327	916	44
17	672	459	.9279	909	43
18	700	490	.9232	902	42
19	728	521	.9184	894	41
20	.24756	.25552	3.9136	.96887	40
21	784	583	.9089	880	39
22	813	614	.9042	873	38
23	841	645	.8995	866	37
24	869	676	.8947	858	36
25	.24897	.25707	3.8900	.96851	35
26	925	738	.8854	844	34
27	954	769	.8807	837	33
28	.24982	800	.8760	829	32
29	.25010	831	.8714	822	31
30	.25038	.25862	3.8667	.96815	30
31	066	893	.8621	807	29
32	094	924	.8575	800	28
33	122	955	.8528	793	27
34	151	.25986	.8482	786	26
35	.25179	.26017	3.8436	.96778	25
36	207	048	.8391	771	24
37	235	079	.8345	764	23
38	263	110	.8299	756	22
39	291	141	.8254	749	21
40	.25320	.26172	3.8208	.96742	20
41	348	203	.8163	734	19
42	376	235	.8118	727	18
43	404	266	.8073	719	17
44	432	297	.8028	712	16
45	.25460	.26328	3.7983	.96705	15
46	488	359	.7938	697	14
47	516	390	.7893	690	13
48	545	421	.7848	682	12
49	573	452	.7804	675	11
50	.25601	.26483	3.7760	.96667	10
51	629	515	.7715	660	9
52	657	546	.7671	653	8
53	685	577	.7627	645	7
54	713	608	.7583	638	6
55	.25741	.26639	3.7539	.96630	5
56	769	670	.7495	623	4
57	798	701	.7451	615	3
58	826	733	.7408	608	2
59	854	764	.7364	600	1
60	.25882	.26795	3.7321	.96593	0
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.25882	.26795	3.7321	.96593	60
1	910	826	.7277	585	59
2	938	857	.7234	578	58
3	966	888	.7191	570	57
4	.25994	920	.7148	562	56
5	.26022	.26951	3.7105	.96555	55
6	050	.26982	.7062	547	54
7	079	27013	.7019	540	53
8	107	044	.6976	532	52
9	135	076	.6933	524	51
10	.26163	.27107	3.6891	.96517	50
11	191	138	.6848	509	49
12	219	169	.6806	502	48
13	247	201	.6764	494	47
14	275	232	.6722	486	46
15	.26303	.27263	3.6680	.96479	45
16	331	294	.6638	471	44
17	359	326	.6596	463	43
18	387	357	.6554	456	42
19	415	388	.6512	448	41
20	.26443	.27419	3.6470	.96440	40
21	471	451	.6429	433	39
22	500	482	.6387	425	38
23	528	513	.6346	417	37
24	556	545	.6305	410	36
25	.26584	.27576	3.6264	.96402	35
26	612	607	.6222	394	34
27	640	638	.6181	386	33
28	668	670	.6140	379	32
29	696	701	.6100	371	31
30	.26724	.27732	3.6059	.96363	30
31	752	764	.6018	355	29
32	780	795	.5978	347	28
33	808	826	.5937	340	27
34	836	858	.5897	332	26
35	.26864	.27889	3.5856	.96324	25
36	892	921	.5816	316	24
37	920	952	.5776	308	23
38	948	.27983	.5736	301	22
39	.26976	.28015	.5696	293	21
40	.27004	.28046	3.5656	.96285	20
41	032	077	.5616	277	19
42	060	109	.5576	269	18
43	088	140	.5536	261	17
44	116	172	.5497	253	16
45	.27144	.28203	3.5457	.96246	15
46	172	234	.5418	238	14
47	200	266	.5379	230	13
48	228	297	.5339	222	12
49	256	329	.5300	214	11
50	.27284	.28360	3.5261	.96206	10
51	312	391	.5222	198	9
52	340	423	.5183	190	8
53	368	454	.5144	182	7
54	396	486	.5105	174	6
55	.27424	.28517	3.5067	.96166	5
56	452	549	.5028	158	4
57	480	580	.4989	150	3
58	508	612	.4951	142	2
59	536	643	.4912	134	1
60	.27564	.28675	3.4874	.96126	0
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.27564	.28675	3.4874	.96126	**60**
1	592	706	.4836	118	59
2	620	738	.4798	110	58
3	648	769	.4760	102	57
4	676	801	.4722	094	56
5	.27704	.28832	3.4684	.96086	**55**
6	731	864	.4646	078	54
7	759	895	.4608	070	53
8	787	927	.4570	062	52
9	815	958	.4533	054	51
10	.27843	.28990	3.4495	.96046	**50**
11	871	.29021	.4458	037	49
12	899	053	.4420	029	48
13	927	084	.4383	021	47
14	955	116	.4346	013	46
15	.27983	.29147	3.4308	.96005	**45**
16	.28011	179	.4271	.95997	44
17	039	210	.4234	989	43
18	067	242	.4197	981	42
19	095	274	.4160	972	41
20	.28123	.29305	3.4124	.95964	**40**
21	150	337	.4087	956	39
22	178	368	.4050	948	38
23	206	400	.4014	940	37
24	234	432	.3977	931	36
25	.28262	.29463	3.3941	.95923	**35**
26	290	495	.3904	915	34
27	318	526	.3868	907	33
28	346	558	.3832	898	32
29	374	590	.3796	890	31
30	.28402	.29621	3.3759	.95882	**30**
31	429	653	.3723	874	29
32	457	685	.3687	865	28
33	485	716	.3652	857	27
34	513	748	.3616	849	26
35	.28541	.29780	3.3580	.95841	**25**
36	569	811	.3544	832	24
37	597	843	.3509	824	23
38	625	875	.3473	816	22
39	652	906	.3438	807	21
40	.28680	.29938	3.3402	.95799	**20**
41	708	.29970	.3367	791	19
42	736	.30001	.3332	782	18
43	764	033	.3297	774	17
44	792	065	.3261	766	16
45	.28820	.30097	3.3226	.95757	**15**
46	847	128	.3191	749	14
47	875	160	.3156	740	13
48	903	192	.3122	732	12
49	931	224	.3087	724	11
50	.28959	.30255	3.3052	.95715	**10**
51	.28987	287	.3017	707	9
52	.29015	319	.2983	698	8
53	042	351	.2948	690	7
54	070	382	.2914	681	6
55	.29098	.30414	3.2879	.95673	**5**
56	126	446	.2845	664	4
57	154	478	.2811	656	3
58	182	509	.2777	647	2
59	209	541	.2743	639	1
60	.29237	.30573	3.2709	.95630	**0**
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.29237	.30573	3.2709	.95630	**60**
1	265	605	.2675	622	59
2	293	637	.2641	613	58
3	321	669	.2607	605	57
4	348	700	.2573	596	56
5	.29376	.30732	3.2539	.95588	**55**
6	404	764	.2506	579	54
7	432	796	.2472	571	53
8	460	828	.2438	562	52
9	487	860	.2405	554	51
10	.29515	.30891	3.2371	.95545	**50**
11	543	923	.2338	536	49
12	571	955	.2305	528	48
13	599	.30987	.2272	519	47
14	626	.31019	.2238	511	46
15	.29654	.31051	3.2205	.95502	**45**
16	682	083	.2172	493	44
17	710	115	.2139	485	43
18	737	147	.2106	476	42
19	765	178	.2073	467	41
20	.29793	.31210	3.2041	.95459	**40**
21	821	242	.2008	450	39
22	849	274	.1975	441	38
23	876	306	.1943	433	37
24	904	338	.1910	424	36
25	.29932	.31370	3.1878	.95415	**35**
26	960	402	.1845	407	34
27	.29987	434	.1813	398	33
28	.30015	466	.1780	389	32
29	043	498	.1748	380	31
30	.30071	.31530	3.1716	.95372	**30**
31	098	562	.1684	363	29
32	126	594	.1652	354	28
33	154	626	.1620	345	27
34	182	658	.1588	337	26
35	.30209	.31690	3.1556	.95328	**25**
36	237	722	.1524	319	24
37	265	754	.1492	310	23
38	292	786	.1460	301	22
39	320	818	.1429	293	21
40	.30348	.31850	3.1397	.95284	**20**
41	376	882	.1366	275	19
42	403	914	.1334	266	18
43	431	946	.1303	257	17
44	459	.31978	.1271	248	16
45	.30486	.32010	3.1240	.95240	**15**
46	514	042	.1209	231	14
47	542	074	.1178	222	13
48	570	106	.1146	213	12
49	597	139	.1115	204	11
50	.30625	.32171	3.1084	.95195	**10**
51	653	203	.1053	186	9
52	680	235	.1022	177	8
53	708	267	.0991	168	7
54	736	299	.0961	159	6
55	.30763	.32331	3.0930	.95150	**5**
56	791	363	.0899	142	4
57	819	396	.0868	133	3
58	846	428	.0838	124	2
59	874	460	.0807	115	1
60	.30902	.32492	3.0777	.95106	**0**
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.30902	.32492	3.0777	.95106	**60**
1	929	524	.0746	097	59
2	957	556	.0716	088	58
3	.30985	588	.0686	079	57
4	.31012	621	.0655	070	56
5	.31040	.32653	3.0625	.95061	**55**
6	068	685	.0595	052	54
7	096	717	.0565	043	53
8	123	749	.0535	033	52
9	151	782	.0505	024	51
10	.31178	.32814	3.0475	.95015	**50**
11	206	846	.0445	.95006	49
12	233	878	.0415	.94997	48
13	261	911	.0385	988	47
14	289	943	.0356	979	46
15	.31316	.32975	3.0326	.94970	**45**
16	344	.33007	.0296	961	44
17	372	040	.0267	952	43
18	399	072	.0237	943	42
19	427	104	.0208	933	41
20	.31454	.33136	3.0178	.94924	**40**
21	482	169	.0149	915	39
22	510	201	.0120	906	38
23	537	233	.0090	897	37
24	565	266	.0061	888	36
25	.31593	.33298	3.0032	.94878	**35**
26	620	330	3.0003	869	34
27	648	363	2.9974	860	33
28	675	395	.9945	851	32
29	703	427	.9916	842	31
30	.31730	.33460	2.9887	.94832	**30**
31	758	492	.9858	823	29
32	786	524	.9829	814	28
33	813	557	.9800	805	27
34	841	589	.9772	795	26
35	.31868	.33621	2.9743	.94786	**25**
36	896	654	.9714	777	24
37	923	686	.9686	768	23
38	951	718	.9657	758	22
39	.31979	751	.9629	749	21
40	.32006	.33783	2.9600	.94740	**20**
41	034	816	.9572	730	19
42	061	848	.9544	721	18
43	089	881	.9515	712	17
44	116	913	.9487	702	16
45	.32144	.33945	2.9459	.94693	**15**
46	171	.33978	.9431	684	14
47	199	.34010	.9403	674	13
48	227	043	.9375	665	12
49	254	075	.9347	656	11
50	.32282	.34108	2.9319	.94646	**10**
51	309	140	.9291	637	9
52	337	173	.9263	627	8
53	364	205	.9235	618	7
54	392	238	.9208	609	6
55	.32419	.34270	2.9180	.94599	**5**
56	447	303	.9152	590	4
57	474	335	.9125	580	3
58	502	368	.9097	571	2
59	529	400	.9070	561	1
60	.32557	.34433	2.9042	.94552	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

′	Sin	Tan	Ctn	Cos	
0	.32557	.34433	2.9042	.94552	**60**
1	584	465	.9015	542	59
2	612	498	.8987	533	58
3	639	530	.8960	523	57
4	667	563	.8933	514	56
5	.32694	.34596	2.8905	.94504	**55**
6	722	628	.8878	495	54
7	749	661	.8851	485	53
8	777	693	.8824	476	52
9	804	726	.8797	466	51
10	.32832	.34758	2.8770	.94457	**50**
11	859	791	.8743	447	49
12	887	824	.8716	438	48
13	914	856	.8689	428	47
14	942	889	.8662	418	46
15	.32969	.34922	2.8636	.94409	**45**
16	.32997	954	.8609	399	44
17	.33024	.34987	.8582	390	43
18	051	.35020	.8556	380	42
19	079	052	.8529	370	41
20	.33106	.35085	2.8502	.94361	**40**
21	134	118	.8476	351	39
22	161	150	.8449	342	38
23	189	183	.8423	332	37
24	216	216	.8397	322	36
25	.33244	.35248	2.8370	.94313	**35**
26	271	281	.8344	303	34
27	298	314	.8318	293	33
28	326	346	.8291	284	32
29	353	379	.8265	274	31
30	.33381	.35412	2.8239	.94264	**30**
31	408	445	.8213	254	29
32	436	477	.8187	245	28
33	463	510	.8161	235	27
34	490	543	.8135	225	26
35	.33518	.35576	2.8109	.94215	**25**
36	545	608	.8083	206	24
37	573	641	.8057	196	23
38	600	674	.8032	186	22
39	627	707	.8006	176	21
40	.33655	.35740	2.7980	.94167	**20**
41	682	772	.7955	157	19
42	710	805	.7929	147	18
43	737	838	.7903	137	17
44	764	871	.7878	127	16
45	.33792	.35904	2.7852	.94118	**15**
46	819	937	.7827	108	14
47	846	.35969	.7801	098	13
48	874	.36002	.7776	088	12
49	901	035	.7751	078	11
50	.33929	.36068	2.7725	.94068	**10**
51	956	101	.7700	058	9
52	.33983	134	.7675	049	8
53	.34011	167	.7650	039	7
54	038	199	.7625	029	6
55	.34065	.36232	2.7600	.94019	**5**
56	093	265	.7575	.94009	4
57	120	298	.7550	.93999	3
58	147	331	.7525	989	2
59	175	364	.7500	979	1
60	.34202	.36397	2.7475	.93969	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

′	Sin	Tan	Ctn	Cos	
0	.34202	.36397	2.7475	.93969	**60**
1	229	430	.7450	959	59
2	257	463	.7425	949	58
3	284	496	.7400	939	57
4	311	529	.7376	929	56
5	.34339	.36562	2.7351	.93919	**55**
6	366	595	.7326	909	54
7	393	628	.7302	899	53
8	421	661	.7277	889	52
9	448	694	.7253	879	51
10	.34475	.36727	2.7228	.93869	**50**
11	503	760	.7204	859	49
12	530	793	.7179	849	48
13	557	826	.7155	839	47
14	584	859	.7130	829	46
15	.34612	.36892	2.7106	.93819	**45**
16	639	925	.7082	809	44
17	666	958	.7058	799	43
18	694	.36991	.7034	789	42
19	721	.37024	.7009	779	41
20	.34748	.37057	2.6985	.93769	**40**
21	775	090	.6961	759	39
22	803	123	.6937	748	38
23	830	157	.6913	738	37
24	857	190	.6889	728	36
25	.34884	.37223	2.6865	.93718	**35**
26	912	256	.6841	708	34
27	939	289	.6818	698	33
28	966	322	.6794	688	32
29	.34993	355	.6770	677	31
30	.35021	.37388	2.6746	.93667	**30**
31	048	422	.6723	657	29
32	075	455	.6699	647	28
33	102	488	.6675	637	27
34	130	521	.6652	626	26
35	.35157	.37554	2.6628	.93616	**25**
36	184	588	.6605	606	24
37	211	621	.6581	596	23
38	239	654	.6558	585	22
39	266	687	.6534	575	21
40	.35293	.37720	2.6511	.93565	**20**
41	320	754	.6488	555	19
42	347	787	.6464	544	18
43	375	820	.6441	534	17
44	402	853	.6418	524	16
45	.35429	.37887	2.6395	.93514	**15**
46	456	920	.6371	503	14
47	484	953	.6348	493	13
48	511	.37986	.6325	483	12
49	538	.38020	.6302	472	11
50	.35565	.38053	2.6279	.93462	**10**
51	592	086	.6256	452	9
52	619	120	.6233	441	8
53	647	153	.6210	431	7
54	674	186	.6187	420	6
55	.35701	.38220	2.6165	.93410	**5**
56	728	253	.6142	400	4
57	755	286	.6119	389	3
58	782	320	.6096	379	2
59	810	353	.6074	368	1
60	.35837	.38386	2 6051	.93358	**0**
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.35837	.38386	2.6051	.93358	**60**
1	864	420	.6028	348	59
2	891	453	.6006	337	58
3	918	487	.5983	327	57
4	945	520	.5961	316	56
5	.35973	.38553	2.5938	.93306	**55**
6	.36000	587	.5916	295	54
7	027	620	.5893	285	53
8	054	654	.5871	274	52
9	081	687	.5848	264	51
10	.36108	.38721	2.5826	.93253	**50**
11	135	754	.5804	243	49
12	162	787	.5782	232	48
13	190	821	.5759	222	47
14	217	854	.5737	211	46
15	.36244	.38888	2.5715	.93201	**45**
16	271	921	.5693	190	44
17	298	955	.5671	180	43
18	325	.38988	.5649	169	42
19	352	.39022	.5627	159	41
20	.36379	.39055	2.5605	.93148	**40**
21	406	089	.5583	137	39
22	434	122	.5561	127	38
23	461	156	.5539	116	37
24	488	190	.5517	106	36
25	.36515	.39223	2.5495	.93095	**35**
26	542	257	.5473	084	34
27	569	290	.5452	074	33
28	596	324	.5430	063	32
29	623	357	.5408	052	31
30	.36650	.39391	2.5386	.93042	**30**
31	677	425	.5365	031	29
32	704	458	.5343	020	28
33	731	492	.5322	.93010	27
34	758	526	.5300	.92999	26
35	.36785	.39559	2.5279	.92988	**25**
36	812	593	.5257	978	24
37	839	626	.5236	967	23
38	867	660	.5214	956	22
39	894	694	.5193	945	21
40	.36921	.39727	2.5172	.92935	**20**
41	948	761	.5150	924	19
42	.36975	795	.5129	913	18
43	.37002	829	.5108	902	17
44	029	862	.5086	892	16
45	.37056	.39896	2.5065	.92881	**15**
46	083	930	.5044	870	14
47	110	963	.5023	859	13
48	137	.39997	.5002	849	12
49	164	.40031	.4981	838	11
50	.37191	.40065	2.4960	.92827	**10**
51	218	098	.4939	816	9
52	245	132	.4918	805	8
53	272	166	.4897	794	7
54	299	200	.4876	784	6
55	.37326	.40234	2.4855	.92773	**5**
56	353	267	.4834	762	4
57	380	301	.4813	751	3
58	407	335	.4792	740	2
59	434	369	.4772	729	1
60	.37461	.40403	2.4751	.92718	**0**
	Cos	Ctn	Tan	Sin	′

'	Sin	Tan	Ctn	Cos	
0	.37461	.40403	2.4751	.92718	**60**
1	488	436	.4730	707	59
2	515	470	.4709	697	58
3	542	504	.4689	686	57
4	569	538	.4668	675	56
5	.37595	.40572	2.4648	.92664	**55**
6	622	606	.4627	653	54
7	649	640	.4606	642	53
8	676	674	.4586	631	52
9	703	707	.4566	620	51
10	.37730	.40741	2.4545	.92609	**50**
11	757	775	.4525	598	49
12	784	809	.4504	587	48
13	811	843	.4484	576	47
14	838	877	.4464	565	46
15	.37865	.40911	2.4443	.92554	**45**
16	892	945	.4423	543	44
17	919	.40979	.4403	532	43
18	946	.41013	.4383	521	42
19	973	047	.4362	510	41
20	.37999	.41081	2.4342	.92499	**40**
21	.38026	115	.4322	488	39
22	053	149	.4302	477	38
23	080	183	.4282	466	37
24	107	217	.4262	455	36
25	.38134	.41251	2.4242	.92444	**35**
26	161	285	.4222	432	34
27	188	319	.4202	421	33
28	215	353	.4182	410	32
29	241	387	.4162	399	31
30	.38268	.41421	2.4142	.92388	**30**
31	295	455	.4122	377	29
32	322	490	.4102	366	28
33	349	524	.4083	355	27
34	376	558	.4063	343	26
35	.38403	.41592	2.4043	.92332	**25**
36	430	626	.4023	321	24
37	456	660	.4004	310	23
38	483	694	.3984	299	22
39	510	728	.3964	287	21
40	.38537	.41763	2.3945	.92276	**20**
41	564	797	.3925	265	19
42	591	831	.3906	254	18
43	617	865	.3886	243	17
44	644	899	.3867	231	16
45	.38671	.41933	2.3847	.92220	**15**
46	698	.41968	.3828	209	14
47	725	.42002	.3808	198	13
48	752	036	.3789	186	12
49	778	070	.3770	175	11
50	.38805	.42105	2.3750	.92164	**10**
51	832	139	.3731	152	9
52	859	173	.3712	141	8
53	886	207	.3693	130	7
54	912	242	.3673	119	6
55	.38939	.42276	2.3654	.92107	**5**
56	966	310	.3635	096	4
57	.38993	345	.3616	085	3
58	.39020	379	.3597	073	2
59	046	413	.3578	062	1
60	.39073	.42447	2.3559	.92050	**0**
	Cos	Ctn	Tan	Sin	'

'	Sin	Tan	Ctn	Cos	
0	.39073	.42447	2.3559	.92050	**60**
1	100	482	.3539	039	59
2	127	516	.3520	028	58
3	153	551	.3501	016	57
4	180	585	.3483	.92005	56
5	.39207	.42619	2.3464	.91994	**55**
6	234	654	.3445	982	54
7	260	688	.3426	971	53
8	287	722	.3407	959	52
9	314	757	.3388	948	51
10	.39341	.42791	2.3369	.91936	**50**
11	367	826	.3351	925	49
12	394	860	.3332	914	48
13	421	894	.3313	902	47
14	448	929	.3294	891	46
15	.39474	.42963	2.3276	.91879	**45**
16	501	.42998	.3257	868	44
17	528	.43032	.3238	856	43
18	555	067	.3220	845	42
19	581	101	.3201	833	41
20	.39608	.43136	2.3183	.91822	**40**
21	635	170	.3164	810	39
22	661	205	.3146	799	38
23	688	239	.3127	787	37
24	715	274	.3109	775	36
25	.39741	.43308	2.3090	.91764	**35**
26	768	343	.3072	752	34
27	795	378	.3053	741	33
28	822	412	.3035	729	32
29	848	447	.3017	718	31
30	.39875	.43481	2.2998	.91706	**30**
31	902	516	.2980	694	29
32	928	550	.2962	683	28
33	955	585	.2944	671	27
34	.39982	620	.2925	660	26
35	.40008	.43654	2.2907	.91648	**25**
36	035	689	.2889	636	24
37	062	724	.2871	625	23
38	088	758	.2853	613	22
39	115	793	.2835	601	21
40	.40141	.43828	2.2817	.91590	**20**
41	168	862	.2799	578	19
42	195	897	.2781	566	18
43	221	932	.2763	555	17
44	248	.43966	.2745	543	16
45	.40275	.44001	2.2727	.91531	**15**
46	301	036	.2709	519	14
47	328	071	.2691	508	13
48	355	105	.2673	496	12
49	381	140	.2655	484	11
50	.40408	.44175	2.2637	.91472	**10**
51	434	210	.2620	461	9
52	461	244	.2602	449	8
53	488	279	.2584	437	7
54	514	314	.2566	425	6
55	.40541	.44349	2.2549	.91414	**5**
56	567	384	.2531	402	4
57	594	418	.2513	390	3
58	621	453	.2496	378	2
59	647	488	.2478	366	1
60	.40674	.44523	2.2460	.91355	**0**
	Cos	Ctn	Tan	Sin	'

′	Sin	Tan	Ctn	Cos	
0	.40674	.44523	2.2460	.91355	60
1	700	558	.2443	343	59
2	727	593	.2425	331	58
3	753	627	.2408	319	57
4	780	662	.2390	307	56
5	.40806	.44697	2.2373	.91295	55
6	833	732	.2355	283	54
7	860	767	.2338	272	53
8	886	802	.2320	260	52
9	913	837	.2303	248	51
10	.40939	.44872	2.2286	.91236	50
11	966	907	.2268	224	49
12	.40992	942	.2251	212	48
13	.41019	.44977	.2234	200	47
14	045	.45012	.2216	188	46
15	.41072	.45047	2.2199	.91176	45
16	098	082	.2182	164	44
17	125	117	.2165	152	43
18	151	152	.2148	140	42
19	178	187	.2130	128	41
20	.41204	.45222	2.2113	.91116	40
21	231	257	.2096	104	39
22	257	292	.2079	092	38
23	284	327	.2062	080	37
24	310	362	.2045	068	36
25	.41337	.45397	2.2028	.91056	35
26	363	432	.2011	044	34
27	390	467	.1994	032	33
28	416	502	.1977	020	32
29	443	538	.1960	.91008	31
30	.41469	.45573	2.1943	.90996	30
31	496	608	.1926	984	29
32	522	643	.1909	972	28
33	549	678	.1892	960	27
34	575	713	.1876	948	26
35	.41602	.45748	2.1859	.90936	25
36	628	784	.1842	924	24
37	655	819	.1825	911	23
38	681	854	.1808	899	22
39	707	889	.1792	887	21
40	.41734	.45924	2.1775	.90875	20
41	760	960	.1758	863	19
42	787	.45995	.1742	851	18
43	813	.46030	.1725	839	17
44	840	065	.1708	826	16
45	.41866	.46101	2.1692	.90814	15
46	892	136	.1675	802	14
47	919	171	.1659	790	13
48	945	206	.1642	778	12
49	972	242	.1625	766	11
50	.41998	.46277	2.1609	.90753	10
51	.42024	312	.1592	741	9
52	051	348	.1576	729	8
53	077	383	.1560	717	7
54	104	418	.1543	704	6
55	.42130	.46454	2.1527	.90692	5
56	156	489	.1510	680	4
57	183	525	.1494	668	3
58	209	560	.1478	655	2
59	235	595	.1461	643	1
60	.42262	.46631	2.1445	.90631	0
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.42262	.46631	2.1445	.90631	60
1	288	666	.1429	618	59
2	315	702	.1413	606	58
3	341	737	.1396	594	57
4	367	772	.1380	582	56
5	.42394	.46808	2.1364	.90569	55
6	420	843	.1348	557	54
7	446	879	.1332	545	53
8	473	914	.1315	532	52
9	499	950	.1299	520	51
10	.42525	.46985	2.1283	.90507	50
11	552	.47021	.1267	495	49
12	578	056	.1251	483	48
13	604	092	.1235	470	47
14	631	128	.1219	458	46
15	.42657	.47163	2.1203	.90446	45
16	683	199	.1187	433	44
17	709	234	.1171	421	43
18	736	270	.1155	408	42
19	762	305	.1139	396	41
20	.42788	.47341	2.1123	.90383	40
21	815	377	.1107	371	39
22	841	412	.1092	358	38
23	867	448	.1076	346	37
24	894	483	.1060	334	36
25	.42920	.47519	2.1044	.90321	35
26	946	555	.1028	309	34
27	972	590	.1013	296	33
28	.42999	626	.0997	284	32
29	.43025	662	.0981	271	31
30	.43051	.47698	2.0965	.90259	30
31	077	733	.0950	246	29
32	104	769	.0934	233	28
33	130	805	.0918	221	27
34	156	840	.0903	208	26
35	.43182	.47876	2.0887	.90196	25
36	209	912	.0872	183	24
37	235	948	.0856	171	23
38	261	.47984	.0840	158	22
39	287	.48019	.0825	146	21
40	.43313	.48055	2.0809	.90133	20
41	340	091	.0794	120	19
42	366	127	.0778	108	18
43	392	163	.0763	095	17
44	418	198	.0748	082	16
45	.43445	.48234	2.0732	.90070	15
46	471	270	.0717	057	14
47	497	306	.0701	045	13
48	523	342	.0686	032	12
49	549	378	.0671	019	11
50	.43575	.48414	2.0655	.90007	10
51	602	450	.0640	.89994	9
52	628	486	.0625	981	8
53	654	521	.0609	968	7
54	680	557	.0594	956	6
55	.43706	.48593	2.0579	.89943	5
56	733	629	.0564	930	4
57	759	665	.0549	918	3
58	785	701	.0533	905	2
59	811	737	.0518	892	1
60	.43837	.48773	2 0503	.89879	0
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.43837	.48773	2.0503	.89879	**60**
1	863	809	.0488	867	59
2	889	845	.0473	854	58
3	916	881	.0458	841	57
4	942	917	.0443	828	56
5	.43968	.48953	2.0428	.89816	**55**
6	.43994	.48989	.0413	803	54
7	.44020	.49026	.0398	790	53
8	046	062	.0383	777	52
9	072	098	.0368	764	51
10	.44098	.49134	2.0353	.89752	**50**
11	124	170	.0338	739	49
12	151	206	.0323	726	48
13	177	242	.0308	713	47
14	203	278	.0293	700	46
15	.44229	.49315	2.0278	.89687	**45**
16	255	351	.0263	674	44
17	281	387	.0248	662	43
18	307	423	.0233	649	42
19	333	459	.0219	636	41
20	.44359	.49495	2.0204	.89623	**40**
21	385	532	.0189	610	39
22	411	568	.0174	597	38
23	437	604	.0160	584	37
24	464	640	.0145	571	36
25	.44490	.49677	2.0130	.89558	**35**
26	516	713	.0115	545	34
27	542	749	.0101	532	33
28	568	786	.0086	519	32
29	594	822	.0072	506	31
30	.44620	.49858	2.0057	.89493	**30**
31	646	894	.0042	480	29
32	672	931	.0028	467	28
33	698	.49967	2.0013	454	27
34	724	.50004	1.9999	441	26
35	.44750	.50040	1.9984	.89428	**25**
36	776	076	.9970	415	24
37	802	113	.9955	402	23
38	828	149	.9941	389	22
39	854	185	.9926	376	21
40	.44880	.50222	1.9912	.89363	**20**
41	906	258	.9897	350	19
42	932	295	.9883	337	18
43	958	331	.9868	324	17
44	.44984	368	.9854	311	16
45	.45010	.50404	1.9840	.89298	**15**
46	036	441	.9825	285	14
47	062	477	.9811	272	13
48	088	514	.9797	259	12
49	114	550	.9782	245	11
50	.45140	.50587	1.9768	.89232	**10**
51	166	623	.9754	219	9
52	192	660	.9740	206	8
53	218	696	.9725	193	7
54	243	733	.9711	180	6
55	.45269	.50769	1.9697	.89167	**5**
56	295	806	.9683	153	4
57	321	843	.9669	140	3
58	347	879	.9654	127	2
59	373	916	.9640	114	1
60	.45399	.50953	1.9626	.89101	**0**
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.45399	.50953	1.9626	.89101	**60**
1	425	.50989	.9612	087	59
2	451	.51026	.9598	074	58
3	477	063	.9584	061	57
4	503	099	.9570	048	56
5	.45529	.51136	1.9556	.89035	**55**
6	554	173	.9542	021	54
7	580	209	.9528	.89008	53
8	606	246	.9514	.88995	52
9	632	283	.9500	981	51
10	.45658	.51319	1.9486	.88968	**50**
11	684	356	.9472	955	49
12	710	393	.9458	942	48
13	736	430	.9444	928	47
14	762	467	.9430	915	46
15	.45787	.51503	1.9416	.88902	**45**
16	813	540	.9402	888	44
17	839	577	.9388	875	43
18	865	614	.9375	862	42
19	891	651	.9361	848	41
20	.45917	.51688	1.9347	.88835	**40**
21	942	724	.9333	822	39
22	968	761	.9319	808	38
23	.45994	798	.9306	795	37
24	.46020	835	.9292	782	36
25	.46046	.51872	1.9278	.88768	**35**
26	072	909	.9265	755	34
27	097	946	.9251	741	33
28	123	.51983	.9237	728	32
29	149	.52020	.9223	715	31
30	.46175	.52057	1.9210	.88701	**30**
31	201	094	.9196	688	29
32	226	131	.9183	674	28
33	252	168	.9169	661	27
34	278	205	.9155	647	26
35	.46304	.52242	1.9142	.88634	**25**
36	330	279	.9128	620	24
37	355	316	.9115	607	23
38	381	353	.9101	593	22
39	407	390	.9088	580	21
40	.46433	.52427	1.9074	.88566	**20**
41	458	464	.9061	553	19
42	484	501	.9047	539	18
43	510	538	.9034	526	17
44	536	575	.9020	512	16
45	.46561	.52613	1.9007	.88499	**15**
46	587	650	.8993	485	14
47	613	687	.8980	472	13
48	639	724	.8967	458	12
49	664	761	.8953	445	11
50	.46690	.52798	1.8940	.88431	**10**
51	716	836	.8927	417	9
52	742	873	.8913	404	8
53	767	910	.8900	390	7
54	793	947	.8887	377	6
55	.46819	.52985	1.8873	.88363	**5**
56	844	.53022	.8860	349	4
57	870	059	.8847	336	3
58	896	096	.8834	322	2
59	921	134	.8820	308	1
60	.46947	.53171	1.8807	.88295	**0**
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.46947	.53171	1.8807	.88295	**60**
1	973	208	.8794	281	59
2	.46999	246	.8781	267	58
3	.47024	283	.8768	254	57
4	050	320	.8755	240	56
5	.47076	.53358	1.8741	.88226	**55**
6	101	395	.8728	213	54
7	127	432	.8715	199	53
8	153	470	.8702	185	52
9	178	507	.8689	172	51
10	.47204	.53545	1.8676	.88158	**50**
11	229	582	.8663	144	49
12	255	620	.8650	130	48
13	281	657	.8637	117	47
14	306	694	.8624	103	46
15	.47332	.53732	1.8611	.88089	**45**
16	358	769	.8598	075	44
17	383	807	.8585	062	43
18	409	844	.8572	048	42
19	434	882	.8559	034	41
20	.47460	.53920	1.8546	.88020	**40**
21	486	957	.8533	.88006	39
22	511	.53995	.8520	.87993	38
23	537	.54032	.8507	979	37
24	562	070	.8495	965	36
25	.47588	.54107	1.8482	.87951	**35**
26	614	145	.8469	937	34
27	639	183	.8456	923	33
28	665	220	.8443	909	32
29	690	258	.8430	896	31
30	.47716	.54296	1.8418	.87882	**30**
31	741	333	.8405	868	29
32	767	371	.8392	854	28
33	793	409	.8379	840	27
34	818	446	.8367	826	26
35	.47844	.54484	1.8354	.87812	**25**
36	869	522	.8341	798	24
37	895	560	.8329	784	23
38	920	597	.8316	770	22
39	946	635	.8303	756	21
40	.47971	.54673	1.8291	.87743	**20**
41	.47997	711	.8278	729	19
42	.48022	748	.8265	715	18
43	048	786	.8253	701	17
44	073	824	.8240	687	16
45	.48099	.54862	1.8228	.87673	**15**
46	124	900	.8215	659	14
47	150	938	.8202	645	13
48	175	.54975	.8190	631	12
49	201	.55013	.8177	617	11
50	.48226	.55051	1.8165	.87603	**10**
51	252	089	.8152	589	9
52	277	127	.8140	575	8
53	303	165	.8127	561	7
54	328	203	.8115	546	6
55	.48354	.55241	1.8103	.87532	**5**
56	379	279	.8090	518	4
57	405	317	.8078	504	3
58	430	355	.8065	490	2
59	456	393	.8053	476	1
60	.48481	.55431	1.8040	.87462	**0**
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.48481	.55431	1.8040	.87462	**60**
1	506	469	.8028	448	59
2	532	507	.8016	434	58
3	557	545	.8003	420	57
4	583	583	.7991	406	56
5	.48608	.55621	1.7979	.87391	**55**
6	634	659	.7966	377	54
7	659	697	.7954	363	53
8	684	736	.7942	349	52
9	710	774	.7930	335	51
10	.48735	.55812	1.7917	.87321	**50**
11	761	850	.7905	306	49
12	786	888	.7893	292	48
13	811	926	.7881	278	47
14	837	.55964	.7868	264	46
15	.48862	.56003	1.7856	.87250	**45**
16	888	041	.7844	235	44
17	913	079	.7832	221	43
18	938	117	.7820	207	42
19	964	156	.7808	193	41
20	.48989	.56194	1.7796	.87178	**40**
21	.49014	232	.7783	164	39
22	040	270	.7771	150	38
23	065	309	.7759	136	37
24	090	347	.7747	121	36
25	.49116	.56385	1.7735	.87107	**35**
26	141	424	.7723	093	34
27	166	462	.7711	079	33
28	192	501	.7699	064	32
29	217	539	.7687	050	31
30	.49242	.56577	1.7675	.87036	**30**
31	268	616	.7663	021	29
32	293	654	.7651	.87007	28
33	318	693	.7639	.86993	27
34	344	731	.7627	978	26
35	.49369	.56769	1.7615	.86964	**25**
36	394	808	.7603	949	24
37	419	846	.7591	935	23
38	445	885	.7579	921	22
39	470	923	.7567	906	21
40	.49495	.56962	1.7556	.86892	**20**
41	521	.57000	.7544	878	19
42	546	039	.7532	863	18
43	571	078	.7520	849	17
44	596	116	.7508	834	16
45	.49622	.57155	1.7496	.86820	**15**
46	647	193	.7485	805	14
47	672	232	.7473	791	13
48	697	271	.7461	777	12
49	723	309	.7449	762	11
50	.49748	.57348	1.7437	.86748	**10**
51	773	386	.7426	733	9
52	798	425	.7414	719	8
53	824	464	.7402	704	7
54	849	503	.7391	690	6
55	.49874	.57541	1.7379	.86675	**5**
56	899	580	.7367	661	4
57	924	619	.7355	646	3
58	950	657	.7344	632	2
59	.49975	696	.7332	617	1
60	.50000	.57735	1.7321	.86603	**0**
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.50000	.57735	1.7321	.86603	**60**
1	025	774	.7309	588	59
2	050	813	.7297	573	58
3	076	851	.7286	559	57
4	101	890	.7274	544	56
5	.50126	.57929	1.7262	.86530	**55**
6	151	.57968	.7251	515	54
7	176	.58007	.7239	501	53
8	201	046	.7228	486	52
9	227	085	.7216	471	51
10	.50252	.58124	1.7205	.86457	**50**
11	277	162	.7193	442	49
12	302	201	.7182	427	48
13	327	240	.7170	413	47
14	352	279	.7159	398	46
15	.50377	.58318	1.7147	.86384	**45**
16	403	357	.7136	369	44
17	428	396	.7124	354	43
18	453	435	.7113	340	42
19	478	474	.7102	325	41
20	.50503	.58513	1.7090	.86310	**40**
21	528	552	.7079	295	39
22	553	591	.7067	281	38
23	578	631	.7056	266	37
24	603	670	.7045	251	36
25	.50628	.58709	1.7033	.86237	**35**
26	654	748	.7022	222	34
27	679	787	.7011	207	33
28	704	826	.6999	192	32
29	729	865	.6988	178	31
30	.50754	.58905	1.6977	.86163	**30**
31	779	944	.6965	148	29
32	804	.58983	.6954	133	28
33	829	.59022	.6943	119	27
34	854	061	.6932	104	26
35	.50879	.59101	1.6920	.86089	**25**
36	904	140	.6909	074	24
37	929	179	.6898	059	23
38	954	218	.6887	045	22
39	.50979	258	.6875	030	21
40	.51004	.59297	1.6864	.86015	**20**
41	029	336	.6853	.86000	19
42	054	376	.6842	.85985	18
43	079	415	.6831	970	17
44	104	454	.6820	956	16
45	.51129	.59494	1.6808	.85941	**15**
46	154	533	.6797	926	14
47	179	573	.6786	911	13
48	204	612	.6775	896	12
49	229	651	.6764	881	11
50	.51254	.59691	1.6753	.85866	**10**
51	279	730	.6742	851	9
52	304	770	.6731	836	8
53	329	809	.6720	821	7
54	354	849	.6709	806	6
55	.51379	.59888	1.6698	.85792	**5**
56	404	928	.6687	777	4
57	429	.59967	.6676	762	3
58	454	.60007	.6665	747	2
59	479	046	.6654	732	1
60	.51504	.60086	1.6643	.85717	**0**
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.51504	.60086	1.6643	.85717	**60**
1	529	126	.6632	702	59
2	554	165	.6621	687	58
3	579	205	.6610	672	57
4	604	245	.6599	657	56
5	.51628	.60284	1.6588	.85642	**55**
6	653	324	.6577	627	54
7	678	364	.6566	612	53
8	703	403	.6555	597	52
9	728	443	.6545	582	51
10	.51753	.60483	1.6534	.85567	**50**
11	778	522	.6523	551	49
12	803	562	.6512	536	48
13	828	602	.6501	521	47
14	852	642	.6490	506	46
15	.51877	.60681	1.6479	.85491	**45**
16	902	721	.6469	476	44
17	927	761	.6458	461	43
18	952	801	.6447	446	42
19	.51977	841	.6436	431	41
20	.52002	.60881	1.6426	.85416	**40**
21	026	921	.6415	401	39
22	051	.60960	.6404	385	38
23	076	.61000	.6393	370	37
24	101	040	.6383	355	36
25	.52126	.61080	1.6372	.85340	**35**
26	151	120	.6361	325	34
27	175	160	.6351	310	33
28	200	200	.6340	294	32
29	225	240	.6329	279	31
30	.52250	.61280	1.6319	.85264	**30**
31	275	320	.6308	249	29
32	299	360	.6297	234	28
33	324	400	.6287	218	27
34	349	440	.6276	203	26
35	.52374	.61480	1.6265	.85188	**25**
36	399	520	.6255	173	24
37	423	561	.6244	157	23
38	448	601	.6234	142	22
39	473	641	.6223	127	21
40	.52498	.61681	1.6212	.85112	**20**
41	522	721	.6202	096	19
42	547	761	.6191	081	18
43	572	801	.6181	066	17
44	597	842	.6170	051	16
45	.52621	.61882	1.6160	.85035	**15**
46	646	922	.6149	020	14
47	671	.61962	.6139	.85005	13
48	696	.62003	.6128	.84989	12
49	720	043	.6118	974	11
50	.52745	.62083	1.6107	.84959	**10**
51	770	124	.6097	943	9
52	794	164	.6087	928	8
53	819	204	.6076	913	7
54	844	245	.6066	897	6
55	.52869	.62285	1.6055	.84882	**5**
56	893	325	.6045	866	4
57	918	366	.6034	851	3
58	943	406	.6024	836	2
59	967	446	.6014	820	1
60	.52992	.62487	1.6003	.84805	**0**
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.52992	.62487	1.6003	.84805	60
1	.53017	527	.5993	789	59
2	041	568	.5983	774	58
3	066	608	.5972	759	57
4	091	649	.5962	743	56
5	.53115	.62689	1.5952	.84728	55
6	140	730	.5941	712	54
7	164	770	.5931	697	53
8	189	811	.5921	681	52
9	214	852	.5911	666	51
10	.53238	.62892	1.5900	.84650	50
11	263	933	.5890	635	49
12	288	.62973	.5880	619	48
13	312	.63014	.5869	604	47
14	337	055	.5859	588	46
15	.53361	.63095	1.5849	.84573	45
16	386	136	.5839	557	44
17	411	177	.5829	542	43
18	435	217	.5818	526	42
19	460	258	.5808	511	41
20	.53484	.63299	1.5798	.84495	40
21	509	340	.5788	480	39
22	534	380	.5778	464	38
23	558	421	.5768	448	37
24	583	462	.5757	433	36
25	.53607	.63503	1.5747	.84417	35
26	632	544	.5737	402	34
27	656	584	.5727	386	33
28	681	625	.5717	370	32
29	705	666	.5707	355	31
30	.53730	.63707	1.5697	.84339	30
31	754	748	.5687	324	29
32	779	789	.5677	308	28
33	804	830	.5667	292	27
34	828	871	.5657	277	26
35	.53853	.63912	1.5647	.84261	25
36	877	953	.5637	245	24
37	902	.63994	.5627	230	23
38	926	.64035	.5617	214	22
39	951	076	.5607	198	21
40	.53975	.64117	1.5597	.84182	20
41	.54000	158	.5587	167	19
42	024	199	.5577	151	18
43	049	240	.5567	135	17
44	073	281	.5557	120	16
45	.54097	.64322	1.5547	.84104	15
46	122	363	.5537	088	14
47	146	404	.5527	072	13
48	171	446	.5517	057	12
49	195	487	.5507	041	11
50	.54220	.64528	1.5497	.84025	10
51	244	569	.5487	.84009	9
52	269	610	.5477	.83994	8
53	293	652	.5468	978	7
54	317	693	.5458	962	6
55	.54342	.64734	1.5448	.83946	5
56	366	775	.5438	930	4
57	391	817	.5428	915	3
58	415	858	.5418	899	2
59	440	899	.5408	883	1
60	.54464	.64941	1.5399	.83867	0
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.54464	.64941	1.5399	.83867	60
1	488	.64982	.5389	851	59
2	513	.65024	.5379	835	58
3	537	065	.5369	819	57
4	561	106	.5359	804	56
5	.54586	.65148	1.5350	.83788	55
6	610	189	.5340	772	54
7	635	231	.5330	756	53
8	659	272	.5320	740	52
9	683	314	.5311	724	51
10	.54708	.65355	1.5301	.83708	50
11	732	397	.5291	692	49
12	756	438	.5282	676	48
13	781	480	.5272	660	47
14	805	521	.5262	645	46
15	.54829	.65563	1.5253	.83629	45
16	854	604	.5243	613	44
17	878	646	.5233	597	43
18	902	688	.5224	581	42
19	927	729	.5214	565	41
20	.54951	.65771	1.5204	.83549	40
21	975	813	.5195	533	39
22	.54999	854	.5185	517	38
23	.55024	896	.5175	501	37
24	048	938	.5166	485	36
25	.55072	.65980	1.5156	.83469	35
26	097	.66021	.5147	453	34
27	121	063	.5137	437	33
28	145	105	.5127	421	32
29	169	147	.5118	405	31
30	.55194	.66189	1.5108	.83389	30
31	218	230	.5099	373	29
32	242	272	.5089	356	28
33	266	314	.5080	340	27
34	291	356	.5070	324	26
35	.55315	.66398	1.5061	.83308	25
36	339	440	.5051	292	24
37	363	482	.5042	276	23
38	388	524	.5032	260	22
39	412	566	.5023	244	21
40	.55436	.66608	1.5013	.83228	20
41	460	650	.5004	212	19
42	484	692	.4994	195	18
43	509	734	.4985	179	17
44	533	776	.4975	163	16
45	.55557	.66818	1.4966	.83147	15
46	581	860	.4957	131	14
47	605	902	.4947	115	13
48	630	944	.4938	098	12
49	654	.66986	.4928	082	11
50	.55678	.67028	1.4919	.83066	10
51	702	071	.4910	050	9
52	726	113	.4900	034	8
53	750	155	.4891	017	7
54	775	197	.4882	.83001	6
55	.55799	.67239	1.4872	.82985	5
56	823	282	.4863	969	4
57	847	324	.4854	953	3
58	871	366	.4844	936	2
59	895	409	.4835	920	1
60	.55919	.67451	1.4826	.82904	0
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.55919	.67451	1.4826	.82904	**60**
1	943	493	.4816	887	59
2	968	536	.4807	871	58
3	.55992	578	.4798	855	57
4	.56016	620	.4788	839	56
5	.56040	.67663	1.4779	.82822	**55**
6	064	705	.4770	806	54
7	088	748	.4761	790	53
8	112	790	.4751	773	52
9	136	832	.4742	757	51
10	.56160	.67875	1.4733	.82741	**50**
11	184	917	.4724	724	49
12	208	.67960	.4715	708	48
13	232	.68002	.4705	692	47
14	256	045	.4696	675	46
15	.56280	.68088	1.4687	.82659	**45**
16	305	130	.4678	643	44
17	329	173	.4669	626	43
18	353	215	.4659	610	42
19	377	258	.4650	593	41
20	.56401	.68301	1.4641	.82577	**40**
21	425	343	.4632	561	39
22	449	386	.4623	544	38
23	473	429	.4614	528	37
24	497	471	.4605	511	36
25	.56521	.68514	1.4596	.82495	**35**
26	545	557	.4586	478	34
27	569	600	.4577	462	33
28	593	642	.4568	446	32
29	617	685	.4559	429	31
30	.56641	.68728	1.4550	.82413	**30**
31	665	771	.4541	396	29
32	689	814	.4532	380	28
33	713	857	.4523	363	27
34	736	900	.4514	347	26
35	.56760	.68942	1.4505	.82330	**25**
36	784	.68985	.4496	314	24
37	808	.69028	.4487	297	23
38	832	071	.4478	281	22
39	856	114	.4469	264	21
40	.56880	.69157	1.4460	.82248	**20**
41	904	200	.4451	231	19
42	928	243	.4442	214	18
43	952	286	.4433	198	17
44	.56976	329	.4424	181	16
45	.57000	.69372	1.4415	.82165	**15**
46	024	416	.4406	148	14
47	047	459	.4397	132	13
48	071	502	.4388	115	12
49	095	545	.4379	098	11
50	.57119	.69588	1.4370	.82082	**10**
51	143	631	.4361	065	9
52	167	675	.4352	048	8
53	191	718	.4344	032	7
54	215	761	.4335	.82015	6
55	.57238	.69804	1.4326	.81999	**5**
56	262	847	.4317	982	4
57	286	891	.4308	965	3
58	310	934	.4299	949	2
59	334	.69977	.4290	932	1
60	.57358	.70021	1.4281	.81915	**0**
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.57358	.70021	1.4281	.81915	**60**
1	381	064	.4273	899	59
2	405	107	.4264	882	58
3	429	151	.4255	865	57
4	453	194	.4246	848	56
5	.57477	.70238	1.4237	.81832	**55**
6	501	281	.4229	815	54
7	524	325	.4220	798	53
8	548	368	.4211	782	52
9	572	412	.4202	765	51
10	.57596	.70455	1.4193	.81748	**50**
11	619	499	.4185	731	49
12	643	542	.4176	714	48
13	667	586	.4167	698	47
14	691	629	.4158	681	46
15	.57715	.70673	1.4150	.81664	**45**
16	738	717	.4141	647	44
17	762	760	.4132	631	43
18	786	804	.4124	614	42
19	810	848	.4115	597	41
20	.57833	.70891	1.4106	.81580	**40**
21	857	935	.4097	563	39
22	881	.70979	.4089	546	38
23	904	.71023	.4080	530	37
24	928	066	.4071	513	36
25	.57952	.71110	1.4063	.81496	**35**
26	976	154	.4054	479	34
27	.57999	198	.4045	462	33
28	.58023	242	.4037	445	32
29	047	285	.4028	428	31
30	.58070	.71329	1.4019	.81412	**30**
31	094	373	.4011	395	29
32	118	417	.4002	378	28
33	141	461	.3994	361	27
34	165	505	.3985	344	26
35	.58189	.71549	1.3976	.81327	**25**
36	212	593	.3968	310	24
37	236	637	.3959	293	23
38	260	681	.3951	276	22
39	283	725	.3942	259	21
40	.58307	.71769	1.3934	.81242	**20**
41	330	813	.3925	225	19
42	354	857	.3916	208	18
43	378	901	.3908	191	17
44	401	946	.3899	174	16
45	.58425	.71990	1.3891	.81157	**15**
46	449	.72034	.3882	140	14
47	472	078	.3874	123	13
48	496	122	.3865	106	12
49	519	167	.3857	089	11
50	.58543	.72211	1.3848	.81072	**10**
51	567	255	.3840	055	9
52	590	299	.3831	038	8
53	614	344	.3823	021	7
54	637	388	.3814	.81004	6
55	.58661	.72432	1.3806	.80987	**5**
56	684	477	.3798	970	4
57	708	521	.3789	953	3
58	731	565	.3781	936	2
59	755	610	.3772	919	1
60	.58779	.72654	1.3764	.80902	**0**
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.58779	.72654	1.3764	.80902	**60**
1	802	699	.3755	885	59
2	826	743	.3747	867	58
3	849	788	.3739	850	57
4	873	832	.3730	833	56
5	.58896	.72877	1.3722	.80816	**55**
6	920	921	.3713	799	54
7	943	.72966	.3705	782	53
8	967	.73010	.3697	765	52
9	.58990	055	.3688	748	51
10	.59014	.73100	1.3680	.80730	**50**
11	037	144	.3672	713	49
12	061	189	.3663	696	48
13	084	234	.3655	679	47
14	108	278	.3647	662	46
15	.59131	.73323	1.3638	.80644	**45**
16	154	368	.3630	627	44
17	178	413	.3622	610	43
18	201	457	.3613	593	42
19	225	502	.3605	576	41
20	.59248	.73547	1.3597	.80558	**40**
21	272	592	.3588	541	39
22	295	637	.3580	524	38
23	318	681	.3572	507	37
24	342	726	.3564	489	36
25	.59365	.73771	1.3555	.80472	**35**
26	389	816	.3547	455	34
27	412	861	.3539	438	33
28	436	906	.3531	420	32
29	459	951	.3522	403	31
30	.59482	.73996	1.3514	.80386	**30**
31	506	.74041	.3506	368	29
32	529	086	.3498	351	28
33	552	131	.3490	334	27
34	576	176	.3481	316	26
35	.59599	.74221	1.3473	.80299	**25**
36	622	267	.3465	282	24
37	646	312	.3457	264	23
38	669	357	.3449	247	22
39	693	402	.3440	230	21
40	.59716	.74447	1.3432	.80212	**20**
41	739	492	.3424	195	19
42	763	538	.3416	178	18
43	786	583	.3408	160	17
44	809	628	.3400	143	16
45	.59832	.74674	1.3392	.80125	**15**
46	856	719	.3384	108	14
47	879	764	.3375	091	13
48	902	810	.3367	073	12
49	926	855	.3359	056	11
50	.59949	.74900	1.3351	.80038	**10**
51	972	946	.3343	021	9
52	.59995	.74991	.3335	.80003	8
53	.60019	.75037	.3327	.79986	7
54	042	082	.3319	968	6
55	.60065	.75128	1.3311	.79951	**5**
56	089	173	.3303	934	4
57	112	219	.3295	916	3
58	135	264	.3287	899	2
59	158	310	.3278	881	1
60	.60182	.75355	1.3270	.79864	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

′	Sin	Tan	Ctn	Cos	
0	.60182	.75355	1.3270	.79864	**60**
1	205	401	.3262	846	59
2	228	447	.3254	829	58
3	251	492	.3246	811	57
4	274	538	.3238	793	56
5	.60298	.75584	1.3230	.79776	**55**
6	321	629	.3222	758	54
7	344	675	.3214	741	53
8	367	721	.3206	723	52
9	390	767	.3198	706	51
10	.60414	.75812	1.3190	.79688	**50**
11	437	858	.3182	671	49
12	460	904	.3175	653	48
13	483	950	.3167	635	47
14	506	.75996	.3159	618	46
15	.60529	.76042	1.3151	.79600	**45**
16	553	088	.3143	583	44
17	576	134	.3135	565	43
18	599	180	.3127	547	42
19	622	226	.3119	530	41
20	.60645	.76272	1.3111	.79512	**40**
21	668	318	.3103	494	39
22	691	364	.3095	477	38
23	714	410	.3087	459	37
24	738	456	.3079	441	36
25	.60761	.76502	1.3072	.79424	**35**
26	784	548	.3064	406	34
27	807	594	.3056	388	33
28	830	640	.3048	371	32
29	853	686	.3040	353	31
30	.60876	.76733	1.3032	.79335	**30**
31	899	779	.3024	318	29
32	922	825	.3017	300	28
33	945	871	.3009	282	27
34	968	918	.3001	264	26
35	.60991	.76964	1.2993	.79247	**25**
36	.61015	.77010	.2985	229	24
37	038	057	.2977	211	23
38	061	103	.2970	193	22
39	084	149	.2962	176	21
40	.61107	.77196	1.2954	.79158	**20**
41	130	242	.2946	140	19
42	153	289	.2938	122	18
43	176	335	.2931	105	17
44	199	382	.2923	087	16
45	.61222	.77428	1.2915	.79069	**15**
46	245	475	.2907	051	14
47	268	521	.2900	033	13
48	291	568	.2892	.79016	12
49	314	615	.2884	.78998	11
50	.61337	.77661	1.2876	.78980	**10**
51	360	708	.2869	962	9
52	383	754	.2861	944	8
53	406	801	.2853	926	7
54	429	848	.2846	908	6
55	.61451	.77895	1.2838	.78891	**5**
56	474	941	.2830	873	4
57	497	.77988	.2822	855	3
58	520	.78035	.2815	837	2
59	543	082	.2807	819	1
60	.61566	.78129	1.2799	.78801	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

′	Sin	Tan	Ctn	Cos	
0	.61566	.78129	1.2799	.78801	**60**
1	589	175	.2792	783	59
2	612	222	.2784	765	58
3	635	269	.2776	747	57
4	658	316	.2769	729	56
5	.61681	.78363	1.2761	.78711	**55**
6	704	410	.2753	694	54
7	726	457	.2746	676	53
8	749	504	.2738	658	52
9	772	551	.2731	640	51
10	.61795	.78598	1.2723	.78622	**50**
11	818	645	.2715	604	49
12	841	692	.2708	586	48
13	864	739	.2700	568	47
14	887	786	.2693	550	46
15	.61909	.78834	1.2685	.78532	**45**
16	932	881	.2677	514	44
17	955	928	.2670	496	43
18	.61978	.78975	.2662	478	42
19	.62001	.79022	.2655	460	41
20	.62024	.79070	1.2647	.78442	**40**
21	046	117	.2640	424	39
22	069	164	.2632	405	38
23	092	212	.2624	387	37
24	115	259	.2617	369	36
25	.62138	.79306	1.2609	.78351	**35**
26	160	354	.2602	333	34
27	183	401	.2594	315	33
28	206	449	.2587	297	32
29	229	496	.2579	279	31
30	.62251	.79544	1.2572	.78261	**30**
31	274	591	.2564	243	29
32	297	639	.2557	225	28
33	320	686	.2549	206	27
34	342	734	.2542	188	26
35	.62365	.79781	1.2534	.78170	**25**
36	388	829	.2527	152	24
37	411	877	.2519	134	23
38	433	924	.2512	116	22
39	456	.79972	.2504	098	21
40	.62479	.80020	1.2497	.78079	**20**
41	502	067	.2489	061	19
42	524	115	.2482	043	18
43	547	163	.2475	025	17
44	570	211	.2467	.78007	16
45	.62592	.80258	1.2460	.77988	**15**
46	615	306	.2452	970	14
47	638	354	.2445	952	13
48	660	402	.2437	934	12
49	683	450	.2430	916	11
50	.62706	.80498	1.2423	.77897	**10**
51	728	546	.2415	879	9
52	751	594	.2408	861	8
53	774	642	.2401	843	7
54	796	690	.2393	824	6
55	.62819	.80738	1.2386	.77806	**5**
56	842	786	.2378	788	4
57	864	834	.2371	769	3
58	887	882	.2364	751	2
59	909	930	.2356	733	1
60	.62932	.80978	1.2349	.77715	**0**
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.62932	.80978	1.2349	.77715	**60**
1	955	.81027	.2342	696	59
2	.62977	075	.2334	678	58
3	.63000	123	.2327	660	57
4	022	171	.2320	641	56
5	.63045	.81220	1.2312	.77623	**55**
6	068	268	.2305	605	54
7	090	316	.2298	586	53
8	113	364	.2290	568	52
9	135	413	.2283	550	51
10	.63158	.81461	1.2276	.77531	**50**
11	180	510	.2268	513	49
12	203	558	.2261	494	48
13	225	606	.2254	476	47
14	248	655	.2247	458	46
15	.63271	.81703	1.2239	.77439	**45**
16	293	752	.2232	421	44
17	316	800	.2225	402	43
18	338	849	.2218	384	42
19	361	898	.2210	366	41
20	.63383	.81946	1.2203	.77347	**40**
21	406	.81995	.2196	329	39
22	428	.82044	.2189	310	38
23	451	092	.2181	292	37
24	473	141	.2174	273	36
25	.63496	.82190	1.2167	.77255	**35**
26	518	238	.2160	236	34
27	540	287	.2153	218	33
28	563	336	.2145	199	32
29	585	385	.2138	181	31
30	.63608	.82434	1.2131	.77162	**30**
31	630	483	.2124	144	29
32	653	531	.2117	125	28
33	675	580	.2109	107	27
34	698	629	.2102	088	26
35	.63720	.82678	1.2095	.77070	**25**
36	742	727	.2088	051	24
37	765	776	.2081	033	23
38	787	825	.2074	.77014	22
39	810	874	.2066	.76996	21
40	.63832	.82923	1.2059	.76977	**20**
41	854	.82972	.2052	959	19
42	877	.83022	.2045	940	18
43	899	071	.2038	921	17
44	922	120	.2031	903	16
45	.63944	.83169	1.2024	.76884	**15**
46	966	218	.2017	866	14
47	.63989	268	.2009	847	13
48	.64011	317	.2002	828	12
49	033	366	.1995	810	11
50	.64056	.83415	1.1988	.76791	**10**
51	078	465	.1981	772	9
52	100	514	.1974	754	8
53	123	564	.1967	735	7
54	145	613	.1960	717	6
55	.64167	.83662	1.1953	.76698	**5**
56	190	712	.1946	679	4
57	212	761	.1939	661	3
58	234	811	.1932	642	2
59	256	860	.1925	623	1
60	.64279	.83910	1.1918	.76604	**0**
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.64279	.83910	1.1918	.76604	**60**
1	301	.83960	.1910	586	59
2	323	.84009	.1903	567	58
3	346	059	.1896	548	57
4	368	108	.1889	530	56
5	.64390	.84158	1.1882	.76511	**55**
6	412	208	.1875	492	54
7	435	258	.1868	473	53
8	457	307	.1861	455	52
9	479	357	.1854	436	51
10	.64501	.84407	1.1847	.76417	**50**
11	524	457	.1840	398	49
12	546	507	.1833	380	48
13	568	556	.1826	361	47
14	590	606	.1819	342	46
15	.64612	.84656	1.1812	.76323	**45**
16	635	706	.1806	304	44
17	657	756	.1799	286	43
18	679	806	.1792	267	42
19	701	856	.1785	248	41
20	.64723	.84906	1.1778	.76229	**40**
21	746	.84956	.1771	210	39
22	768	.85006	.1764	192	38
23	790	057	.1757	173	37
24	812	107	.1750	154	36
25	.64834	.85157	1.1743	.76135	**35**
26	856	207	.1736	116	34
27	878	257	.1729	097	33
28	901	308	.1722	078	32
29	923	358	.1715	059	31
30	.64945	.85408	1.1708	.76041	**30**
31	967	458	.1702	022	29
32	.64989	509	.1695	.76003	28
33	.65011	559	.1688	.75984	27
34	033	609	.1681	965	26
35	.65055	.85660	1.1674	.75946	**25**
36	077	710	.1667	927	24
37	100	761	.1660	908	23
38	122	811	.1653	889	22
39	144	862	.1647	870	21
40	.65166	.85912	1.1640	.75851	**20**
41	188	.85963	.1633	832	19
42	210	.86014	.1626	813	18
43	232	064	.1619	794	17
44	254	115	.1612	775	16
45	.65276	.86166	1.1606	.75756	**15**
46	298	216	.1599	738	14
47	320	267	.1592	719	13
48	342	318	.1585	700	12
49	364	368	.1578	680	11
50	.65386	.86419	1.1571	.75661	**10**
51	408	470	.1565	642	9
52	430	521	.1558	623	8
53	452	572	.1551	604	7
54	474	623	.1544	585	6
55	.65496	.86674	1.1538	.75566	**5**
56	518	725	.1531	547	4
57	540	776	.1524	528	3
58	562	827	.1517	509	2
59	584	878	.1510	490	1
60	.65606	.86929	1.1504	.75471	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

′	Sin	Tan	Ctn	Cos	
0	.65606	.86929	1.1504	.75471	**60**
1	628	.86980	.1497	452	59
2	650	.87031	.1490	433	58
3	672	082	.1483	414	57
4	694	133	.1477	395	56
5	.65716	.87184	1.1470	.75375	**55**
6	738	236	.1463	356	54
7	759	287	.1456	337	53
8	781	338	.1450	318	52
9	803	389	.1443	299	51
10	.65825	.87441	1.1436	.75280	**50**
11	847	492	.1430	261	49
12	869	543	.1423	241	48
13	891	595	.1416	222	47
14	913	646	.1410	203	46
15	.65935	.87698	1.1403	.75184	**45**
16	956	749	.1396	165	44
17	.65978	801	.1389	146	43
18	.66000	852	.1383	126	42
19	022	904	.1376	107	41
20	.66044	.87955	1.1369	.75088	**40**
21	066	.88007	.1363	069	39
22	088	059	.1356	050	38
23	109	110	.1349	030	37
24	131	162	.1343	.75011	36
25	.66153	.88214	1.1336	.74992	**35**
26	175	265	.1329	973	34
27	197	317	.1323	953	33
28	218	369	.1316	934	32
29	240	421	.1310	915	31
30	.66262	.88473	1.1303	.74896	**30**
31	284	524	.1296	876	29
32	306	576	.1290	857	28
33	327	628	.1283	838	27
34	349	680	.1276	818	26
35	.66371	.88732	1.1270	.74799	**25**
36	393	784	.1263	780	24
37	414	836	.1257	760	23
38	436	888	.1250	741	22
39	458	940	.1243	722	21
40	.66480	.88992	1.1237	.74703	**20**
41	501	.89045	.1230	683	19
42	523	097	.1224	664	18
43	545	149	.1217	644	17
44	566	201	.1211	625	16
45	.66588	.89253	1.1204	.74606	**15**
46	610	306	.1197	586	14
47	632	358	.1191	567	13
48	653	410	.1184	548	12
49	675	463	.1178	528	11
50	.66697	.89515	1.1171	.74509	**10**
51	718	567	.1165	489	9
52	740	620	.1158	470	8
53	762	672	.1152	451	7
54	783	725	.1145	431	6
55	.66805	.89777	1.1139	.74412	**5**
56	827	830	.1132	392	4
57	848	883	.1126	373	3
58	870	935	.1119	353	2
59	891	.89988	.1113	334	1
60	.66913	.90040	1.1106	.74314	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

′	Sin	Tan	Ctn	Cos	
0	.66913	.90040	1.1106	.74314	**60**
1	935	093	.1100	295	59
2	956	146	.1093	276	58
3	978	199	.1087	256	57
4	.66999	251	.1080	237	56
5	.67021	.90304	1.1074	.74217	**55**
6	043	357	.1067	198	54
7	064	410	.1061	178	53
8	086	463	.1054	159	52
9	107	516	.1048	139	51
10	.67129	.90569	1.1041	.74120	**50**
11	151	621	.1035	100	49
12	172	674	.1028	080	48
13	194	727	.1022	061	47
14	215	781	.1016	041	46
15	.67237	.90834	1.1009	.74022	**45**
16	258	887	.1003	.74002	44
17	280	940	.0996	.73983	43
18	301	.90993	.0990	963	42
19	323	.91046	.0983	944	41
20	.67344	.91099	1.0977	.73924	**40**
21	366	153	.0971	904	39
22	387	206	.0964	885	38
23	409	259	.0958	865	37
24	430	313	.0951	846	36
25	.67452	.91366	1.0945	.73826	**35**
26	473	419	.0939	806	34
27	495	473	.0932	787	33
28	516	526	.0926	767	32
29	538	580	.0919	747	31
30	.67559	.91633	1.0913	.73728	**30**
31	580	687	.0907	708	29
32	602	740	.0900	688	28
33	623	794	.0894	669	27
34	645	847	.0888	649	26
35	.67666	.91901	1.0881	.73629	**25**
36	688	.91955	.0875	610	24
37	709	.92008	.0869	590	23
38	730	062	.0862	570	22
39	752	116	.0856	551	21
40	.67773	.92170	1.0850	.73531	**20**
41	795	224	.0843	511	19
42	816	277	.0837	491	18
43	837	331	.0831	472	17
44	859	385	.0824	452	16
45	.67880	.92439	1.0818	.73432	**15**
46	901	493	.0812	413	14
47	923	547	.0805	393	13
48	944	601	.0799	373	12
49	965	655	.0793	353	11
50	.67987	.92709	1.0786	.73333	**10**
51	.68008	763	.0780	314	9
52	029	817	.0774	294	8
53	051	872	.0768	274	7
54	072	926	.0761	254	6
55	.68093	.92980	1.0755	.73234	**5**
56	115	.93034	.0749	215	4
57	136	088	.0742	195	3
58	157	143	.0736	175	2
59	179	197	.0730	155	1
60	.68200	.93252	1.0724	.73135	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

′	Sin	Tan	Ctn	Cos	
0	.68200	.93252	1.0724	.73135	**60**
1	221	306	.0717	116	59
2	242	360	.0711	096	58
3	264	415	.0705	076	57
4	285	469	.0699	056	56
5	.68306	.93524	1.0692	.73036	**55**
6	327	578	.0686	.73016	54
7	349	633	.0680	.72996	53
8	370	688	.0674	976	52
9	391	742	.0668	957	51
10	.68412	.93797	1.0661	.72937	**50**
11	434	852	.0655	917	49
12	455	906	.0649	897	48
13	476	.93961	.0643	877	47
14	497	.94016	.0637	857	46
15	.68518	.94071	1.0630	.72837	**45**
16	539	125	.0624	817	44
17	561	180	.0618	797	43
18	582	235	.0612	777	42
19	603	290	.0606	757	41
20	.68624	.94345	1.0599	.72737	**40**
21	645	400	.0593	717	39
22	666	455	.0587	697	38
23	688	510	.0581	677	37
24	709	565	.0575	657	36
25	.68730	.94620	1.0569	.72637	**35**
26	751	676	.0562	617	34
27	772	731	.0556	597	33
28	793	786	.0550	577	32
29	814	841	.0544	557	31
30	.68835	.94896	1.0538	.72537	**30**
31	857	.94952	.0532	517	29
32	878	.95007	.0526	497	28
33	899	062	.0519	477	27
34	920	118	.0513	457	26
35	.68941	.95173	1.0507	.72437	**25**
36	962	229	.0501	417	24
37	.68983	284	.0495	397	23
38	.69004	340	.0489	377	22
39	025	395	.0483	357	21
40	.69046	.95451	1.0477	.72337	**20**
41	067	506	.0470	317	19
42	088	562	.0464	297	18
43	109	618	.0458	277	17
44	130	673	.0452	257	16
45	.69151	.95729	1.0446	.72236	**15**
46	172	785	.0440	216	14
47	193	841	.0434	196	13
48	214	897	.0428	176	12
49	235	.95952	.0422	156	11
50	.69256	.96008	1.0416	.72136	**10**
51	277	064	.0410	116	9
52	298	120	.0404	095	8
53	319	176	.0398	075	7
54	340	232	.0392	055	6
55	.69361	.96288	1.0385	.72035	**5**
56	382	344	.0379	.72015	4
57	403	400	.0373	.71995	3
58	424	457	.0367	974	2
59	445	513	.0361	954	1
60	.69466	.96569	1.0355	.71934	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

′	Sin	Tan	Ctn	Cos	
0	.69466	.96569	1.0355	.71934	**60**
1	487	625	.0349	914	59
2	508	681	.0343	894	58
3	529	738	.0337	873	57
4	549	794	.0331	853	56
5	.69570	.96850	1.0325	.71833	**55**
6	591	907	.0319	813	54
7	612	.96963	.0313	792	53
8	633	.97020	.0307	772	52
9	654	076	.0301	752	51
10	.69675	.97133	1.0295	.71732	**50**
11	696	189	.0289	711	49
12	717	246	.0283	691	48
13	737	302	.0277	671	47
14	758	359	.0271	650	46
15	.69779	.97416	1.0265	.71630	**45**
16	800	472	.0259	610	44
17	821	529	.0253	590	43
18	842	586	.0247	569	42
19	862	643	.0241	549	41
20	.69883	.97700	1.0235	.71529	**40**
21	904	756	.0230	508	39
22	925	813	.0224	488	38
23	946	870	.0218	468	37
24	966	927	.0212	447	36
25	.69987	.97984	1.0206	.71427	**35**
26	.70008	.98041	.0200	407	34
27	029	098	.0194	386	33
28	049	155	.0188	366	32
29	070	213	.0182	345	31
30	.70091	.98270	1.0176	.71325	**30**
31	112	327	.0170	305	29
32	132	384	.0164	284	28
33	153	441	.0158	264	27
34	174	499	.0152	243	26
35	.70195	.98556	1.0147	.71223	**25**
36	215	613	.0141	203	24
37	236	671	.0135	182	23
38	257	728	.0129	162	22
39	277	786	.0123	141	21
40	.70298	.98843	1.0117	.71121	**20**
41	319	901	.0111	100	19
42	339	.98958	.0105	080	18
43	360	.99016	.0099	059	17
44	381	073	.0094	039	16
45	.70401	.99131	1.0088	.71019	**15**
46	422	189	.0082	.70998	14
47	443	247	.0076	978	13
48	463	304	.0070	957	12
49	484	362	.0064	937	11
50	.70505	.99420	1.0058	.70916	**10**
51	525	478	.0052	896	9
52	546	536	.0047	875	8
53	567	594	.0041	855	7
54	587	652	.0035	834	6
55	.70608	.99710	1.0029	.70813	**5**
56	628	768	.0023	793	4
57	649	826	.0017	772	3
58	670	884	.0012	752	2
59	690	.99942	.0006	731	1
60	.70711	1.0000	1.0000	.70711	**0**
	Cos	Ctn	Tan	Sin	′

TABLE III

COMMON LOGARITHMS

OF THE

TRIGONOMETRIC FUNCTIONS

FROM

0° TO 90° AT INTERVALS OF ONE MINUTE

TO

FIVE DECIMAL PLACES

NOTE: *To find* $\log \sin \alpha$ *and* $\log \tan \alpha$ *more precisely than by ordinary interpolation, for small values of* α, *if* α *is not a tabulated angle.*

Let t be the first tabulated angle below α. Express both α and t in the same unit (minutes, or seconds, or any other convenient unit). Then

$$\log \sin \alpha - \log \sin t = \log \alpha - \log t,$$

approximately, at least to five decimal places if $\alpha < 3°$ and $\alpha - t < 1'$.

Now $\log \alpha$ and $\log t$ can be found from Table I, and $\log \sin t$ is tabulated in Table III; hence $\log \sin \alpha$ can be found. Thus to find $\log \sin 1° 12'.4$, write $1° 12'.4 = 72'.4$, and arrange the computation as follows:

$$\begin{array}{rll}
\log 72.4 = & 1.85974 & \text{(Table I)} \\
\log 72.0 = & \underline{1.85733} & \text{(Table I)} \\
\text{(subtract)} & 0.00241 & \\
\log \sin 1° 12' = \log \sin 72' = & \underline{8.32103 - 10} & \text{(Table III)} \\
\log \sin 1° 12'.4 = \log \sin 72'.4 = & 8.32344 - 10 & \text{(Required)}
\end{array}$$

Likewise $\qquad \log \tan \alpha - \log \tan t = \log \alpha - \log t,$

approximately, at least to five decimal places if $\alpha < 3°$ and $\alpha - t < 1'$. The method of calculation is exactly as above.

The cosines and cotangents of angles near 90° can be found by first reducing them to sines and tangents of angles near 0°. Above 3° ordinary interpolation

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	——		——		——	0.00 000	**60**
1	6.46 373	30103	6.46 373	30103	3.53 627	0.00 000	59
2	6.76 476	17609	6.76 476	17609	3.23 524	0.00 000	58
3	6.94 085	12494	6.94 085	12494	3.05 915	0.00 000	57
4	7.06 579	9691	7.06 579	9691	2.93 421	0.00 000	56
5	7.16 270	7918	7.16 270	7918	2.83 730	0.00 000	**55**
6	7.24 188	6694	7.24 188	6694	2.75 812	0.00 000	54
7	7.30 882	5800	7.30 882	5800	2.69 118	0.00 000	53
8	7.36 682	5115	7.36 682	5115	2.63 318	0.00 000	52
9	7.41 797	4576	7.41 797	4576	2.58 203	0.00 000	51
10	7.46 373	4139	7.46 373	4139	2.53 627	0.00 000	**50**
11	7.50 512	3779	7.50 512	3779	2.49 488	0.00 000	49
12	7.54 291	3476	7.54 291	3476	2.45 709	0.00 000	48
13	7.57 767	3218	7.57 767	3219	2.42 233	0.00 000	47
14	7.60 985	2997	7.60 986	2996	2.39 014	0.00 000	46
15	7.63 982	2802	7.63 982	2803	2.36 018	0.00 000	**45**
16	7.66 784	2633	7.66 785	2633	2.33 215	0.00 000	44
17	7.69 417	2483	7.69 418	2482	2.30 582	9.99 999	43
18	7.71 900	2348	7.71 900	2348	2.28 100	9.99 999	42
19	7.74 248	2227	7.74 248	2228	2.25 752	9.99 999	41
20	7.76 475	2119	7.76 476	2119	2.23 524	9.99 999	**40**
21	7.78 594	2021	7.78 595	2020	2.21 405	9.99 999	39
22	7.80 615	1930	7.80 615	1931	2.19 385	9.99 999	38
23	7.82 545	1848	7.82 546	1848	2.17 454	9.99 999	37
24	7.84 393	1773	7.84 394	1773	2.15 606	9.99 999	36
25	7.86 166	1704	7.86 167	1704	2.13 833	9.99 999	**35**
26	7.87 870	1639	7.87 871	1639	2.12 129	9.99 999	34
27	7.89 509	1579	7.89 510	1579	2.10 490	9.99 999	33
28	7.91 088	1524	7.91 089	1524	2.08 911	9.99 999	32
29	7.92 612	1472	7.92 613	1473	2.07 387	9.99 998	31
30	7.94 084	1424	7.94 086	1424	2.05 914	9.99 998	**30**
31	7.95 508	1379	7.95 510	1379	2.04 490	9.99 998	29
32	7.96 887	1336	7.96 889	1336	2.03 111	9.99 998	28
33	7.98 223	1297	7.98 225	1297	2.01 775	9.99 998	27
34	7.99 520	1259	7.99 522	1259	2.00 478	9.99 998	26
35	8.00 779	1223	8.00 781	1223	1.99 219	9.99 998	**25**
36	8.02 002	1190	8.02 004	1190	1.97 996	9.99 998	24
37	8.03 192	1158	8.03 194	1159	1.96 806	9.99 997	23
38	8.04 350	1128	8.04 353	1128	1.95 647	9.99 997	22
39	8.05 478	1100	8.05 481	1100	1.94 519	9.99 997	21
40	8.06 578	1072	8.06 581	1072	1.93 419	9.99 997	**20**
41	8.07 650	1046	8.07 653	1047	1.92 347	9.99 997	19
42	8.08 696	1022	8.08 700	1022	1.91 300	9.99 997	18
43	8.09 718	999	8.09 722	998	1.90 278	9.99 997	17
44	8.10 717	976	8.10 720	976	1.89 280	9.99 996	16
45	8.11 693	954	8.11 696	955	1.88 304	9.99 996	**15**
46	8.12 647	934	8.12 651	934	1.87 349	9.99 996	14
47	8.13 581	914	8.13 585	915	1.86 415	9.99 996	13
48	8.14 495	896	8.14 500	895	1.85 500	9.99 996	12
49	8.15 391	877	8.15 395	878	1.84 605	9.99 996	11
50	8.16 268	860	8.16 273	860	1.83 727	9.99 995	**10**
51	8.17 128	843	8.17 133	843	1.82 867	9.99 995	9
52	8.17 971	827	8.17 976	828	1.82 024	9.99 995	8
53	8.18 798	812	8.18 804	812	1.81 196	9.99 995	7
54	8.19 610	797	8.19 616	797	1.80 384	9.99 995	6
55	8.20 407	782	8.20 413	782	1.79 587	9.99 994	**5**
56	8.21 189	769	8.21 195	769	1.78 805	9.99 994	4
57	8.21 958	755	8.21 964	756	1.78 036	9.99 994	3
58	8.22 713	743	8.22 720	742	1.77 280	9.99 994	2
59	8.23 456	730	8.23 462	730	1.76 538	9.99 994	1
60	8.24 186		8.24 192		1.75 808	9.99 993	**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

For logarithms of sines or tangents of angles less than 3° (or logarithms of cosines or cotangents of angles greater than 87°), see Note on interpolation, p. 45.

When the tabular differences are large, that method is usually better. The proportional parts stated for 1° and 2° in this table are sufficient when great accuracy is not required, even if the ordinary method of interpolation is used.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	8.24 186	717	8.24 192	718	1.75 808	9.99 993	**60**
1	8.24 903	706	8.24 910	706	1.75 090	9.99 993	59
2	8.25 609	695	8.25 616	696	1.74 384	9.99 993	58
3	8.26 304	684	8.26 312	684	1.73 688	9.99 993	57
4	8.26 988	673	8.26 996	673	1.73 004	9.99 992	56
5	8.27 661	663	8.27 669	663	1.72 331	9.99 992	**55**
6	8.28 324	653	8.28 332	654	1.71 668	9.99 992	54
7	8.28 977	644	8.28 986	643	1.71 014	9.99 992	53
8	8.29 621	634	8.29 629	634	1.70 371	9.99 992	52
9	8.30 255	624	8.30 263	625	1.69 737	9.99 991	51
10	8.30 879	616	8.30 888	617	1.69 112	9.99 991	**50**
11	8.31 495	608	8.31 505	607	1.68 495	9.99 991	49
12	8.32 103	599	8.32 112	599	1.67 888	9.99 990	48
13	8.32 702	590	8.32 711	591	1.67 289	9.99 990	47
14	8.33 292	583	8.33 302	584	1.66 698	9.99 990	46
15	8.33 875	575	8.33 886	575	1.66 114	9.99 990	**45**
16	8.34 450	568	8.34 461	568	1.65 539	9.99 989	44
17	8.35 018	560	8.35 029	561	1.64 971	9.99 989	43
18	8.35 578	553	8.35 590	553	1.64 410	9.99 989	42
19	8.36 131	547	8.36 143	546	1.63 857	9.99 989	41
20	8.36 678	539	8.36 689	540	1.63 311	9.99 988	**40**
21	8.37 217	533	8.37 229	533	1.62 771	9.99 988	39
22	8.37 750	526	8.37 762	527	1.62 238	9.99 988	38
23	8.38 276	520	8.38 289	520	1.61 711	9.99 987	37
24	8.38 796	514	8.38 809	514	1.61 191	9.99 987	36
25	8.39 310	508	8.39 323	509	1.60 677	9.99 987	**35**
26	8.39 818	502	8.39 832	502	1.60 168	9.99 986	34
27	8.40 320	496	8.40 334	496	1.59 666	9.99 986	33
28	8.40 816	491	8.40 830	491	1.59 170	9.99 986	32
29	8.41 307	485	8.41 321	486	1.58 679	9.99 985	31
30	8.41 792	480	8.41 807	480	1.58 193	9.99 985	**30**
31	8.42 272	474	8.42 287	475	1.57 713	9.99 985	29
32	8.42 746	470	8.42 762	470	1.57 238	9.99 984	28
33	8.43 216	464	8.43 232	464	1.56 768	9.99 984	27
34	8.43 680	459	8.43 696	460	1.56 304	9.99 984	26
35	8.44 139	455	8.44 156	455	1.55 844	9.99 983	**25**
36	8.44 594	450	8.44 611	450	1.55 389	9.99 983	24
37	8.45 044	445	8.45 061	446	1.54 939	9.99 983	23
38	8.45 489	441	8.45 507	441	1.54 493	9.99 982	22
39	8.45 930	436	8.45 948	437	1.54 052	9.99 982	21
40	8.46 366	433	8.46 385	432	1.53 615	9.99 982	**20**
41	8.46 799	427	8.46 817	428	1.53 183	9.99 981	19
42	8.47 226	424	8.47 245	424	1.52 755	9.99 981	18
43	8.47 650	419	8.47 669	420	1.52 331	9.99 981	17
44	8.48 069	416	8.48 089	416	1.51 911	9.99 980	16
45	8.48 485	411	8.48 505	412	1.51 495	9.99 980	**15**
46	8.48 896	408	8.48 917	408	1.51 083	9.99 979	14
47	8.49 304	404	8.49 325	404	1.50 675	9.99 979	13
48	8.49 708	400	8.49 729	401	1.50 271	9.99 979	12
49	8.50 108	396	8.50 130	397	1.49 870	9.99 978	11
50	8.50 504	393	8.50 527	393	1.49 473	9.99 978	**10**
51	8.50 897	390	8.50 920	390	1.49 080	9.99 977	9
52	8.51 287	386	8.51 310	386	1.48 690	9.99 977	8
53	8.51 673	382	8.51 696	383	1.48 304	9.99 977	7
54	8.52 055	379	8.52 079	380	1.47 921	9.99 976	6
55	8.52 434	376	8.52 459	376	1.47 541	9.99 976	**5**
56	8.52 810	373	8.52 835	373	1.47 165	9.99 975	4
57	8.53 183	369	8.53 208	370	1.46 792	9.99 975	3
58	8.53 552	367	8.53 578	367	1.46 422	9.99 974	2
59	8.53 919	363	8.53 945	363	1.46 055	9.99 974	1
60	8.54 282		8.54 308		1.45 692	9.99 974	**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop Pts.

	720	710	690	680	670
2	144	142	138	136	134
3	216	213	207	204	201
4	288	284	276	272	268
5	360	355	345	340	335
6	432	426	414	408	402
7	504	497	483	476	469
8	576	568	552	544	536
9	648	639	621	612	603

	660	650	640	630	620
2	132	130	128	126	124
3	198	195	192	189	186
4	264	260	256	252	248
5	330	325	320	315	310
6	396	390	384	378	372
7	462	455	448	441	434
8	528	520	512	504	496
9	594	585	576	567	558

	610	600	590	580	570
2	122	120	118	116	114
3	183	180	177	174	171
4	244	240	236	232	228
5	305	300	295	290	285
6	366	360	354	348	342
7	427	420	413	406	399
8	488	480	472	464	456
9	549	540	531	522	513

	560	550	540	530	520
2	112	110	108	106	104
3	168	165	162	159	156
4	224	220	216	212	208
5	280	275	270	265	260
6	336	330	324	318	312
7	392	385	378	371	364
8	448	440	432	424	416
9	504	495	486	477	468

	510	500	490	480	470
2	102	100	98	96	94
3	153	150	147	144	141
4	204	200	196	192	188
5	255	250	245	240	235
6	306	300	294	288	282
7	357	350	343	336	329
8	408	400	392	384	376
9	459	450	441	432	423

	460	450	440	430	420
2	92	90	88	86	84
3	138	135	132	129	126
4	184	180	176	172	168
5	230	225	220	215	210
6	276	270	264	258	252
7	322	315	308	301	294
8	368	360	352	344	336
9	414	405	396	387	378

	410	400	395	390	385
2	82	80	79.0	78	77.0
3	123	120	118.5	117	115.5
4	164	160	158.0	156	154.0
5	205	200	197.5	195	192.5
6	246	240	237.0	234	231.0
7	287	280	276.5	273	269.5
8	328	320	316.0	312	308.0
9	369	360	355.5	351	346.5

	380	375	370	365	360
2	76	75.0	74	73.0	72
3	114	112.5	111	109.5	108
4	152	150.0	148	146.0	144
5	190	187.5	185	182.5	180
6	228	225.0	222	219.0	216
7	266	262.5	259	255.5	252
8	304	300.0	296	292.0	288
9	342	337.5	333	328.5	324

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	8.54 282	360	8.54 308	361	1.45 692	9.99 974	**60**
1	8.54 642	357	8.54 669	358	1.45 331	9.99 973	59
2	8.54 999	355	8.55 027	355	1.44 973	9.99 973	58
3	8.55 354	351	8.55 382	352	1.44 618	9.99 972	57
4	8.55 705	349	8.55 734	349	1.44 266	9.99 972	56
5	8.56 054	346	8.56 083	346	1.43 917	9.99 971	**55**
6	8.56 400	343	8.56 429	344	1.43 571	9.99 971	54
7	8.56 743	341	8.56 773	341	1.43 227	9.99 970	53
8	8.57 084	337	8.57 114	338	1.42 886	9.99 970	52
9	8.57 421	336	8.57 452	336	1.42 548	9.99 969	51
10	8.57 757	332	8.57 788	333	1.42 212	9.99 969	**50**
11	8.58 089	330	8.58 121	330	1.41 879	9.99 968	49
12	8.58 419	328	8.58 451	328	1.41 549	9.99 968	48
13	8.58 747	325	8.58 779	326	1.41 221	9.99 967	47
14	8.59 072	323	8.59 105	323	1.40 895	9.99 967	46
15	8.59 395	320	8.59 428	321	1.40 572	9.99 967	**45**
16	8.59 715	318	8.59 749	319	1.40 251	9.99 966	44
17	8.60 033	316	8.60 068	316	1.39 932	9.99 966	43
18	8.60 349	313	8.60 384	314	1.39 616	9.99 965	42
19	8.60 662	311	8.60 698	311	1.39 302	9.99 964	41
20	8.60 973	309	8.61 009	310	1.38 991	9.99 964	**40**
21	8.61 282	307	8.61 319	307	1.38 681	9.99 963	39
22	8.61 589	305	8.61 626	305	1.38 374	9.99 963	38
23	8.61 894	302	8.61 931	303	1.38 069	9.99 962	37
24	8.62 196	301	8.62 234	301	1.37 766	9.99 962	36
25	8.62 497	298	8.62 535	299	1.37 465	9.99 961	**35**
26	8.62 795	296	8.62 834	297	1.37 166	9.99 961	34
27	8.63 091	294	8.63 131	295	1.36 869	9.99 960	33
28	8.63 385	293	8.63 426	292	1.36 574	9.99 960	32
29	8.63 678	290	8.63 718	291	1.36 282	9.99 959	31
30	8.63 968	288	8.64 009	289	1.35 991	9.99 959	**30**
31	8.64 256	287	8.64 298	287	1.35 702	9.99 958	29
32	8.64 543	284	8.64 585	285	1.35 415	9.99 958	28
33	8.64 827	283	8.64 870	284	1.35 130	9.99 957	27
34	8.65 110	281	8.65 154	281	1.34 846	9.99 956	26
35	8.65 391	279	8.65 435	280	1.34 565	9.99 956	**25**
36	8.65 670	277	8.65 715	278	1.34 285	9.99 955	24
37	8.65 947	276	8.65 993	276	1.34 007	9.99 955	23
38	8.66 223	274	8.66 269	274	1.33 731	9.99 954	22
39	8.66 497	272	8.66 543	273	1.33 457	9.99 954	21
40	8.66 769	270	8.66 816	271	1.33 184	9.99 953	**20**
41	8.67 039	269	8.67 087	269	1.32 913	9.99 952	19
42	8.67 308	267	8.67 356	268	1.32 644	9.99 952	18
43	8.67 575	266	8.67 624	266	1.32 376	9.99 951	17
44	8.67 841	263	8.67 890	264	1.32 110	9.99 951	16
45	8.68 104	263	8.68 154	263	1.31 846	9.99 950	**15**
46	8.68 367	260	8.68 417	261	1.31 583	9.99 949	14
47	8.68 627	259	8.68 678	260	1.31 322	9.99 949	13
48	8.68 886	258	8.68 938	258	1.31 062	9.99 948	12
49	8.69 144	256	8.69 196	257	1.30 804	9.99 948	11
50	8.69 400	254	8.69 453	255	1.30 547	9.99 947	**10**
51	8.69 654	253	8.69 708	254	1.30 292	9.99 946	9
52	8.69 907	252	8.69 962	252	1.30 038	9.99 946	8
53	8.70 159	250	8.70 214	251	1.29 786	9.99 945	7
54	8.70 409	249	8.70 465	249	1.29 535	9.99 944	6
55	8.70 658	247	8.70 714	248	1.29 286	9.99 944	**5**
56	8.70 905	246	8.70 962	246	1.29 038	9.99 943	4
57	8.71 151	244	8.71 208	245	1.28 792	9.99 942	3
58	8.71 395	243	8.71 453	244	1.28 547	9.99 942	2
59	8.71 638	242	8.71 697	243	1.28 303	9.99 941	1
60	8.71 880		8.71 940		1.28 060	9.99 940	**0**
′	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	360	355	350	345
2	72	71.0	70	69.0
3	108	106.5	105	103.5
4	144	142.0	140	138.0
5	180	177.5	175	172.5
6	216	213.0	210	207.0
7	252	248.5	245	241.5
8	288	284.0	280	276.0
9	324	319.5	315	310.5

	340	335	330	325
2	68	67.0	66	65.0
3	102	100.5	99	97.5
4	136	134.0	132	130.0
5	170	167.5	165	162.5
6	204	201.0	198	195.0
7	238	234.5	231	227.5
8	272	268.0	264	260.0
9	306	301.5	297	292.5

	320	315	310	305
2	64	63.0	62	61.0
3	96	94.5	93	91.5
4	128	126.0	124	122.0
5	160	157.5	155	152.5
6	192	189.0	186	183.0
7	224	220.5	217	213.5
8	256	252.0	248	244.0
9	288	283.5	279	274.5

	300	295	290	285
2	60	59.0	58	57.0
3	90	88.5	87	85.5
4	120	118.0	116	114.0
5	150	147.5	145	142.5
6	180	177.0	174	171.0
7	210	206.5	203	199.5
8	240	236.0	232	228.0
9	270	265.5	261	256.5

	280	275	270	265
2	56	55.0	54	53.0
3	84	82.5	81	79.5
4	112	110.0	108	106.0
5	140	137.5	135	132.5
6	168	165.0	162	159.0
7	196	192.5	189	185.5
8	224	220.0	216	212.0
9	252	247.5	243	238.5

	260	255	250	245
2	52	51.0	50	49.0
3	78	76.5	75	73.5
4	104	102.0	100	98.0
5	130	127.5	125	122.5
6	156	153.0	150	147.0
7	182	178.5	175	171.5
8	208	204.0	200	196.0
9	234	229.5	225	220.5

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	8.71 880	240	8.71 940	241	1.28 060	9.99 940	60
1	8.72 120	239	8.72 181	239	1.27 819	9.99 940	59
2	8.72 359	238	8.72 420	239	1.27 580	9.99 939	58
3	8.72 597	237	8.72 659	237	1.27 341	9.99 938	57
4	8.72 834	235	8.72 896	236	1.27 104	9.99 938	56
5	8.73 069	234	8.73 132	234	1.26 868	9.99 937	55
6	8.73 303	232	8.73 366	234	1.26 634	9.99 936	54
7	8.73 535	232	8.73 600	232	1.26 400	9.99 936	53
8	8.73 767	230	8.73 832	231	1.26 168	9.99 935	52
9	8.73 997	229	8.74 063	229	1.25 937	9.99 934	51
10	8.74 226	228	8.74 292	229	1.25 708	9.99 934	50
11	8.74 454	226	8.74 521	227	1.25 479	9.99 933	49
12	8.74 680	226	8.74 748	226	1.25 252	9.99 932	48
13	8.74 906	224	8.74 974	225	1.25 026	9.99 932	47
14	8.75 130	223	8.75 199	224	1.24 801	9.99 931	46
15	8.75 353	222	8.75 423	222	1.24 577	9.99 930	45
16	8.75 575	220	8.75 645	222	1.24 355	9.99 929	44
17	8.75 795	220	8.75 867	220	1.24 133	9.99 929	43
18	8.76 015	219	8.76 087	219	1.23 913	9.99 928	42
19	8.76 234	217	8.76 306	219	1.23 694	9.99 927	41
20	8.76 451	216	8.76 525	217	1.23 475	9.99 926	40
21	8.76 667	216	8.76 742	216	1.23 258	9.99 926	39
22	8.76 883	214	8.76 958	215	1.23 042	9.99 925	38
23	8.77 097	213	8.77 173	214	1.22 827	9.99 924	37
24	8.77 310	212	8.77 387	213	1.22 613	9.99 923	36
25	8.77 522	211	8.77 600	211	1.22 400	9.99 923	35
26	8.77 733	210	8.77 811	211	1.22 189	9.99 922	34
27	8.77 943	209	8.78 022	210	1.21 978	9.99 921	33
28	8.78 152	208	8.78 232	209	1.21 768	9.99 920	32
29	8.78 360	208	8.78 441	208	1.21 559	9.99 920	31
30	8.78 568	206	8.78 649	206	1.21 351	9.99 919	30
31	8.78 774	205	8.78 855	206	1.21 145	9.99 918	29
32	8.78 979	204	8.79 061	205	1.20 939	9.99 917	28
33	8.79 183	203	8.79 266	204	1.20 734	9.99 917	27
34	8.79 386	202	8.79 470	203	1.20 530	9.99 916	26
35	8.79 588	201	8.79 673	202	1.20 327	9.99 915	25
36	8.79 789	201	8.79 875	201	1.20 125	9.99 914	24
37	8.79 990	199	8.80 076	201	1.19 924	9.99 913	23
38	8.80 189	199	8.80 277	199	1.19 723	9.99 913	22
39	8.80 388	197	8.80 476	198	1.19 524	9.99 912	21
40	8.80 585	197	8.80 674	198	1.19 326	9.99 911	20
41	8.80 782	196	8.80 872	196	1.19 128	9.99 910	19
42	8.80 978	195	8.81 068	196	1.18 932	9.99 909	18
43	8.81 173	194	8.81 264	195	1.18 736	9.99 909	17
44	8.81 367	193	8.81 459	194	1.18 541	9.99 908	16
45	8.81 560	192	8.81 653	193	1.18 347	9.99 907	15
46	8.81 752	192	8.81 846	192	1.18 154	9.99 906	14
47	8.81 944	190	8.82 038	192	1.17 962	9.99 905	13
48	8.82 134	190	8.82 230	190	1.17 770	9.99 904	12
49	8.82 324	189	8.82 420	190	1.17 580	9.99 904	11
50	8.82 513	188	8.82 610	189	1.17 390	9.99 903	10
51	8.82 701	187	8.82 799	188	1.17 201	9.99 902	9
52	8.82 888	187	8.82 987	188	1.17 013	9.99 901	8
53	8.83 075	186	8.83 175	186	1.16 825	9.99 900	7
54	8.83 261	185	8.83 361	186	1.16 639	9.99 899	6
55	8.83 446	184	8.83 547	185	1.16 453	9.99 898	5
56	8.83 630	183	8.83 732	184	1.16 268	9.99 898	4
57	8.83 813	183	8.83 916	184	1.16 084	9.99 897	3
58	8.83 996	181	8.84 100	182	1.15 900	9.99 896	2
59	8.84 177	181	8.84 282	182	1.15 718	9.99 895	1
60	8.84 358		8.84 464		1.15 536	9.99 894	0
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	241	239	237	235
2	48.2	47.8	47.4	47.0
3	72.3	71.7	71.1	70.5
4	96.4	95.6	94.8	94.0
5	120.5	119.5	118.5	117.5
6	144.6	143.4	142.2	141.0
7	168.7	167.3	165.9	164.5
8	192.8	191.2	189.6	188.0
9	216.9	215.1	213.3	211.5

	234	232	229	227
2	46.8	46.4	45.8	45.4
3	70.2	69.6	68.7	68.1
4	93.6	92.8	91.6	90.8
5	117.0	116.0	114.5	113.5
6	140.4	139.2	137.4	136.2
7	163.8	162.4	160.3	158.9
8	187.2	185.6	183.2	181.6
9	210.6	208.8	206.1	204.3

	226	224	222	220
2	45.2	44.8	44.4	44.0
3	67.8	67.2	66.6	66.0
4	90.4	89.6	88.8	88.0
5	113.0	112.0	111.0	110.0
6	135.6	134.4	133.2	132.0
7	158.2	156.8	155.4	154.0
8	180.8	179.2	177.6	176.0
9	203.4	201.6	199.8	198.0

	219	217	215	213
2	43.8	43.4	43.0	42.6
3	65.7	65.1	64.5	63.9
4	87.6	86.8	86.0	85.2
5	109.5	108.5	107.5	106.5
6	131.4	130.2	129.0	127.8
7	153.3	151.9	150.5	149.1
8	175.2	173.6	172.0	170.4
9	197.1	195.3	193.5	191.7

	211	208	206	203
2	42.2	41.6	41.2	40.6
3	63.3	62.4	61.8	60.9
4	84.4	83.2	82.4	81.2
5	105.5	104.0	103.0	101.5
6	126.6	124.8	123.6	121.8
7	147.7	145.6	144.2	142.1
8	168.8	166.4	164.8	162.4
9	189.9	187.2	185.4	182.7

	201	199	197	195
2	40.2	39.8	39.4	39.0
3	60.3	59.7	59.1	58.5
4	80.4	79.6	78.8	78.0
5	100.5	99.5	98.5	97.5
6	120.6	119.4	118.2	117.0
7	140.7	139.3	137.9	136.5
8	160.8	159.2	157.6	156.0
9	180.9	179.1	177.3	175.5

	193	192	190	188
2	38.6	38.4	38.0	37.6
3	57.9	57.6	57.0	56.4
4	77.2	76.8	76.0	75.2
5	96.5	96.0	95.0	94.0
6	115.8	115.2	114.0	112.8
7	135.1	134.4	133.0	131.6
8	154.4	153.6	152.0	150.4
9	173.7	172.8	171.0	169.2

	186	184	182	181
2	37.2	36.8	36.4	36.2
3	55.8	55.2	54.6	54.3
4	74.4	73.6	72.8	72.4
5	93.0	92.0	91.0	90.5
6	111.6	110.4	109.2	108.6
7	130.2	128.8	127.4	126.7
8	148.8	147.2	145.6	144.8
9	167.4	165.6	163.8	162.9

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	8.84 358	181	8.84 464	182	1.15 536	9.99 894	**60**
1	8.84 539	179	8.84 646	180	1.15 354	9.99 893	59
2	8.84 718	179	8.84 826	180	1.15 174	9.99 892	58
3	8.84 897	178	8.85 006	179	1.14 994	9.99 891	57
4	8.85 075	177	8.85 185	178	1.14 815	9.99 891	56
5	8.85 252	177	8.85 363	177	1.14 637	9.99 890	**55**
6	8.85 429	176	8.85 540	177	1.14 460	9.99 889	54
7	8.85 605	175	8.85 717	176	1.14 283	9.99 888	53
8	8.85 780	175	8.85 893	176	1.14 107	9.99 887	52
9	8.85 955	173	8.86 069	174	1.13 931	9.99 886	51
10	8.86 128	173	8.86 243	174	1.13 757	9.99 885	**50**
11	8.86 301	173	8.86 417	174	1.13 583	9.99 884	49
12	8.86 474	171	8.86 591	172	1.13 409	9.99 883	48
13	8.86 645	171	8.86 763	172	1.13 237	9.99 882	47
14	8.86 816	171	8.86 935	171	1.13 065	9.99 881	46
15	8.86 987	169	8.87 106	171	1.12 894	9.99 880	**45**
16	8.87 156	169	8.87 277	170	1.12 723	9.99 879	44
17	8.87 325	169	8.87 447	169	1.12 553	9.99 879	43
18	8.87 494	167	8.87 616	169	1.12 384	9.99 878	42
19	8.87 661	168	8.87 785	168	1.12 215	9.99 877	41
20	8.87 829	166	8.87 953	167	1.12 047	9.99 876	**40**
21	8.87 995	166	8.88 120	167	1.11 880	9.99 875	39
22	8.88 161	165	8.88 287	166	1.11 713	9.99 874	38
23	8.88 326	164	8.88 453	165	1.11 547	9.99 873	37
24	8.88 490	164	8.88 618	165	1.11 382	9.99 872	36
25	8.88 654	163	8.88 783	165	1.11 217	9.99 871	**35**
26	8.88 817	163	8.88 948	163	1.11 052	9.99 870	34
27	8.88 980	162	8.89 111	163	1.10 889	9.99 869	33
28	8.89 142	162	8.89 274	163	1.10 726	9.99 868	32
29	8.89 304	160	8.89 437	161	1.10 563	9.99 867	31
30	8.89 464	161	8.89 598	162	1.10 402	9.99 866	**30**
31	8.89 625	159	8.89 760	160	1.10 240	9.99 865	29
32	8.89 784	159	8.89 920	160	1.10 080	9.99 864	28
33	8.89 943	159	8.90 080	160	1.09 920	9.99 863	27
34	8.90 102	158	8.90 240	159	1.09 760	9.99 862	26
35	8.90 260	157	8.90 399	158	1.09 601	9.99 861	**25**
36	8.90 417	157	8.90 557	158	1.09 443	9.99 860	24
37	8.90 574	156	8.90 715	157	1.09 285	9.99 859	23
38	8.90 730	155	8.90 872	157	1.09 128	9.99 858	22
39	8.90 885	155	8.91 029	156	1.08 971	9.99 857	21
40	8.91 040	155	8.91 185	155	1.08 815	9.99 856	**20**
41	8.91 195	154	8.91 340	155	1.08 660	9.99 855	19
42	8.91 349	153	8.91 495	155	1.08 505	9.99 854	18
43	8.91 502	153	8.91 650	153	1.08 350	9.99 853	17
44	8.91 655	152	8.91 803	154	1.08 197	9.99 852	16
45	8.91 807	152	8.91 957	153	1.08 043	9.99 851	**15**
46	8.91 959	151	8.92 110	152	1.07 890	9.99 850	14
47	8.92 110	151	8.92 262	152	1.07 738	9.99 848	13
48	8.92 261	150	8.92 414	151	1.07 586	9.99 847	12
49	8.92 411	150	8.92 565	151	1.07 435	9.99 846	11
50	8.92 561	149	8.92 716	150	1.07 284	9.99 845	**10**
51	8.92 710	149	8.92 866	150	1.07 134	9.99 844	9
52	8.92 859	148	8.93 016	149	1.06 984	9.99 843	8
53	8.93 007	147	8.93 165	148	1.06 835	9.99 842	7
54	8.93 154	147	8.93 313	149	1.06 687	9.99 841	6
55	8.93 301	147	8.93 462	147	1.06 538	9.99 840	**5**
56	8.93 448	146	8.93 609	147	1.06 391	9.99 839	4
57	8.93 594	146	8.93 756	147	1.06 244	9.99 838	3
58	8.93 740	145	8.93 903	146	1.06 097	9.99 837	2
59	8.93 885	145	8.94 049	146	1.05 951	9.99 836	1
60	8.94 030		8.94 195		1.05 805	9.99 834	**0**
′	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	182	181	180	179
2	36.4	36.2	36.0	35.8
3	54.6	54.3	54.0	53.7
4	72.8	72.4	72.0	71 6
5	91.0	90.5	90.0	89.5
6	109.2	108.6	108.0	107.4
7	127.4	126.7	126.0	125.3
8	145.6	144.8	144.0	143 2
9	163.8	162.9	162.0	161.1

	178	177	176	175
2	35.6	35.4	35.2	35.0
3	53.4	53.1	52.8	52.5
4	71.2	70.8	70.4	70.0
5	89.0	88.5	88.0	87.5
6	106.8	106.2	105.6	105.0
7	124.6	123.9	123.2	122.5
8	142.4	141.6	140.8	140.0
9	160.2	159.3	158.4	157.5

	174	173	172	171
2	34.8	34.6	34.4	34.2
3	52.2	51.9	51.6	51.3
4	69.6	69.2	68.8	68.4
5	87.0	86.5	86.0	85.5
6	104.4	103.8	103.2	102.6
7	121.8	121.1	120.4	119.7
8	139 2	138.4	137.6	136.8
9	156.6	155.7	154.8	153.9

	170	169	168	167
2	34.0	33.8	33.6	33.4
3	51.0	50.7	50.4	50.1
4	68.0	67.6	67.2	66.8
5	85.0	84.5	84.0	83.5
6	102.0	101.4	100.8	100.2
7	119.0	118.3	117.6	116.9
8	136 0	135.2	134.4	133.6
9	153.0	152.1	151.2	150.3

	166	165	164	163
2	33.2	33.0	32.8	32.6
3	49.8	49.5	49 2	48.9
4	66.4	66.0	65.6	65.2
5	83.0	82.5	82.0	81.5
6	99 6	99.0	98.4	97.8
7	116.2	115.5	114.8	114.1
8	132.8	132.0	131.2	130.4
9	149.4	148.5	147.6	146.7

	162	161	160	159
2	32.4	32.2	32.0	31.8
3	48.6	48.3	48.0	47.7
4	64.8	64.4	64.0	63.6
5	81.0	80.5	80.0	79.5
6	97.2	96.6	96.0	95.4
7	113.4	112.7	112.0	111.3
8	129.6	128.8	128.0	127 2
9	145.8	144.9	144.0	143.1

	158	157	156	155
2	31.6	31.4	31.2	31.0
3	47.4	47.1	46.8	46.5
4	63.2	62.8	62.4	62 0
5	79.0	78.5	78.0	77.5
6	94.8	94.2	93.6	93.0
7	110.6	109.9	109 2	108.5
8	126.4	125.6	124.8	124.0
9	142.2	141.3	140.4	139.5

	154	153	152	151
2	30.8	30.6	30.4	30.2
3	46.2	45.9	45.6	45.3
4	61.6	61.2	60.8	60.4
5	77.0	76.5	76.0	75.5
6	92.4	91.8	91.2	90.6
7	107.8	107.1	106.4	105.7
8	123.2	122.4	121.6	120.8
9	138.6	137.7	136.8	135.9

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	8.94 030	144	8.94 195	145	1.05 805	9.99 834	60
1	8.94 174	143	8.94 340	145	1.05 660	9.99 833	59
2	8.94 317	144	8.94 485	145	1.05 515	9.99 832	58
3	8.94 461	142	8.94 630	143	1.05 370	9.99 831	57
4	8.94 603	143	8.94 773	144	1.05 227	9.99 830	56
5	8.94 746	141	8.94 917	143	1.05 083	9.99 829	55
6	8.94 887	142	8.95 060	142	1.04 940	9.99 828	54
7	8.95 029	141	8.95 202	142	1.04 798	9.99 827	53
8	8.95 170	140	8.95 344	142	1.04 656	9.99 825	52
9	8.95 310	140	8.95 486	141	1.04 514	9.99 824	51
10	8.95 450	139	8.95 627	140	1.04 373	9.99 823	50
11	8.95 589	139	8.95 767	141	1.04 233	9.99 822	49
12	8.95 728	139	8.95 908	139	1.04 092	9.99 821	48
13	8.95 867	138	8.96 047	140	1.03 953	9.99 820	47
14	8.96 005	138	8.96 187	138	1.03 813	9.99 819	46
15	8.96 143	137	8.96 325	139	1.03 675	9.99 817	45
16	8.96 280	137	8.96 464	138	1.03 536	9.99 816	44
17	8.96 417	136	8.96 602	137	1.03 398	9.99 815	43
18	8.96 553	136	8.96 739	138	1.03 261	9.99 814	42
19	8.96 689	136	8.96 877	136	1.03 123	9.99 813	41
20	8.96 825	135	8.97 013	137	1.02 987	9.99 812	40
21	8.96 960	135	8.97 150	135	1.02 850	9.99 810	39
22	8.97 095	134	8.97 285	136	1.02 715	9.99 809	38
23	8.97 229	134	8.97 421	135	1.02 579	9.99 808	37
24	8.97 363	133	8.97 556	135	1.02 444	9.99 807	36
25	8.97 496	133	8.97 691	134	1.02 309	9.99 806	35
26	8.97 629	133	8.97 825	134	1.02 175	9.99 804	34
27	8.97 762	132	8.97 959	133	1.02 041	9.99 803	33
28	8.97 894	132	8.98 092	133	1.01 908	9.99 802	32
29	8.98 026	131	8.98 225	133	1.01 775	9.99 801	31
30	8.98 157	131	8.98 358	132	1.01 642	9.99 800	30
31	8.98 288	131	8.98 490	132	1.01 510	9.99 798	29
32	8.98 419	130	8.98 622	131	1.01 378	9.99 797	28
33	8.98 549	130	8.98 753	131	1.01 247	9.99 796	27
34	8.98 679	129	8.98 884	131	1.01 116	9.99 795	26
35	8.98 808	129	8.99 015	130	1.00 985	9.99 793	25
36	8.98 937	129	8.99 145	130	1.00 855	9.99 792	24
37	8.99 066	128	8.99 275	130	1.00 725	9.99 791	23
38	8.99 194	128	8.99 405	129	1.00 595	9.99 790	22
39	8.99 322	128	8.99 534	128	1.00 466	9.99 788	21
40	8.99 450	127	8.99 662	129	1.00 338	9.99 787	20
41	8.99 577	127	8.99 791	128	1.00 209	9.99 786	19
42	8.99 704	126	8.99 919	127	1.00 081	9.99 785	18
43	8.99 830	126	9.00 046	128	0.99 954	9.99 783	17
44	8.99 956	126	9.00 174	127	0.99 826	9.99 782	16
45	9.00 082	125	9.00 301	126	0.99 699	9.99 781	15
46	9.00 207	125	9.00 427	126	0.99 573	9.99 780	14
47	9.00 332	124	9.00 553	126	0.99 447	9.99 778	13
48	9.00 456	125	9.00 679	126	0.99 321	9.99 777	12
49	9.00 581	123	9.00 805	125	0.99 195	9.99 776	11
50	9.00 704	124	9.00 930	125	0.99 070	9.99 775	10
51	9.00 828	123	9.01 055	124	0.98 945	9.99 773	9
52	9.00 951	123	9.01 179	124	0.98 821	9.99 772	8
53	9.01 074	122	9.01 303	124	0.98 697	9.99 771	7
54	9.01 196	122	9.01 427	123	0.98 573	9.99 769	6
55	9.01 318	122	9.01 550	123	0.98 450	9.99 768	5
56	9.01 440	121	9.01 673	123	0.98 327	9.99 767	4
57	9.01 561	121	9.01 796	122	0.98 204	9.99 765	3
58	9.01 682	121	9.01 918	122	0.98 082	9.99 764	2
59	9.01 803	120	9.02 040	122	0.97 960	9.99 763	1
60	9.01 923		9.02 162		0.97 838	9.99 761	0
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	150	149	148	147
2	30.0	29.8	29.6	29.4
3	45.0	44.7	44.4	44.1
4	60.0	59.6	59.2	58.8
5	75.0	74.5	74.0	73.5
6	90.0	89.4	88.8	88.2
7	105.0	104.3	103.6	102.9
8	120.0	119.2	118.4	117.6
9	135.0	134.1	133.2	132.3

	146	145	144	143
2	29.2	29.0	28.8	28.6
3	43.8	43.5	43.2	42.9
4	58.4	58.0	57.6	57.2
5	73.0	72.5	72.0	71.5
6	87.6	87.0	86.4	85.8
7	102.2	101.5	100.8	100.1
8	116.8	116.0	115.2	114.4
9	131.4	130.5	129.6	128.7

	142	141	140	139
2	28.4	28.2	28.0	27.8
3	42.6	42.3	42.0	41.7
4	56.8	56.4	56.0	55.6
5	71.0	70.5	70.0	69.5
6	85.2	84.6	84.0	83.4
7	99.4	98.7	98.0	97.3
8	113.6	112.8	112.0	111.2
9	127.8	126.9	126.0	125.1

	138	137	136	135
2	27.6	27.4	27.2	27.0
3	41.4	41.1	40.8	40.5
4	55.2	54.8	54.4	54.0
5	69.0	68.5	68.0	67.5
6	82.8	82.2	81.6	81.0
7	96.6	95.9	95.2	94.5
8	110.4	109.6	108.8	108.0
9	124.2	123.3	122.4	121.5

	134	133	132	131
2	26.8	26.6	26.4	26.2
3	40.2	39.9	39.6	39.3
4	53.6	53.2	52.8	52.4
5	67.0	66.5	66.0	65.5
6	80.4	79.8	79.2	78.6
7	93.8	93.1	92.4	91.7
8	107.2	106.4	105.6	104.8
9	120.6	119.7	118.8	117.9

	130	129	128	127
2	26.0	25.8	25.6	25.4
3	39.0	38.7	38.4	38.1
4	52.0	51.6	51.2	50.8
5	65.0	64.5	64.0	63.5
6	78.0	77.4	76.8	76.2
7	91.0	90.3	89.6	88.9
8	104.0	103.2	102.4	101.6
9	117.0	116.1	115.2	114.3

	126	125	124	123
2	25.2	25.0	24.8	24.6
3	37.8	37.5	37.2	36.9
4	50.4	50.0	49.6	49.2
5	63.0	62.5	62.0	61.5
6	75.6	75.0	74.4	73.8
7	88.2	87.5	86.8	86.1
8	100.8	100.0	99.2	98.4
9	113.4	112.5	111.6	110.7

	122	121	120
2	24.4	24.2	24.0
3	36.6	36.3	36.0
4	48.8	48.4	48.0
5	61.0	60.5	60.0
6	73.2	72.6	72.0
7	85.4	84.7	84.0
8	97.6	96.8	96.0
9	109.8	108.9	108.0

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	9.01 923	120	9.02 162	121	0.97 838	9.99 761	**60**
1	9.02 043	120	9.02 283	121	0.97 717	9.99 760	59
2	9.02 163	120	9.02 404	121	0.97 596	9.99 759	58
3	9.02 283	119	9.02 525	120	0.97 475	9.99 757	57
4	9.02 402	118	9.02 645	121	0.97 355	9.99 756	56
5	9.02 520	119	9.02 766	119	0.97 234	9.99 755	**55**
6	9.02 639	118	9.02 885	120	0.97 115	9.99 753	54
7	9.02 757	117	9.03 005	119	0.96 995	9.99 752	53
8	9.02 874	118	9.03 124	118	0.96 876	9.99 751	52
9	9.02 992	117	9.03 242	119	0.96 758	9.99 749	51
10	9.03 109	117	9.03 361	118	0.96 639	9.99 748	**50**
11	9.03 226	116	9.03 479	118	0.96 521	9.99 747	49
12	9.03 342	116	9.03 597	117	0.96 403	9.99 745	48
13	9.03 458	116	9.03 714	118	0.96 286	9.99 744	47
14	9.03 574	116	9.03 832	116	0.96 168	9.99 742	46
15	9.03 690	115	9.03 948	117	0.96 052	9.99 741	**45**
16	9.03 805	115	9.04 065	116	0.95 935	9.99 740	44
17	9.03 920	114	9.04 181	116	0.95 819	9.99 738	43
18	9.04 034	115	9.04 297	116	0.95 703	9.99 737	42
19	9.04 149	113	9.04 413	115	0.95 587	9.99 736	41
20	9.04 262	114	9.04 528	115	0.95 472	9.99 734	**40**
21	9.04 376	114	9.04 643	115	0.95 357	9.99 733	39
22	9.04 490	113	9.04 758	115	0.95 242	9.99 731	38
23	9.04 603	112	9.04 873	114	0.95 127	9.99 730	37
24	9.04 715	113	9.04 987	114	0.95 013	9.99 728	36
25	9.04 828	112	9.05 101	113	0.94 899	9.99 727	**35**
26	9.04 940	112	9.05 214	114	0.94 786	9.99 726	34
27	9.05 052	112	9.05 328	113	0.94 672	9.99 724	33
28	9.05 164	111	9.05 441	112	0.94 559	9.99 723	32
29	9.05 275	111	9.05 553	113	0.94 447	9.99 721	31
30	9.05 386	111	9.05 666	112	0.94 334	9.99 720	**30**
31	9.05 497	110	9.05 778	112	0.94 222	9.99 718	29
32	9.05 607	110	9.05 890	112	0.94 110	9.99 717	28
33	9.05 717	110	9.06 002	111	0.93 998	9.99 716	27
34	9.05 827	110	9.06 113	111	0.93 887	9.99 714	26
35	9.05 937	109	9.06 224	111	0.93 776	9.99 713	**25**
36	9.06 046	109	9.06 335	110	0.93 665	9.99 711	24
37	9.06 155	109	9.06 445	111	0.93 555	9.99 710	23
38	9.06 264	108	9.06 556	110	0.93 444	9.99 708	22
39	9.06 372	109	9.06 666	109	0.93 334	9.99 707	21
40	9.06 481	108	9.06 775	110	0.93 225	9.99 705	**20**
41	9.06 589	107	9.06 885	109	0.93 115	9.99 704	19
42	9.06 696	108	9.06 994	109	0.93 006	9.99 702	18
43	9.06 804	107	9.07 103	108	0.92 897	9.99 701	17
44	9.06 911	107	9.07 211	109	0.92 789	9.99 699	16
45	9.07 018	106	9.07 320	108	0.92 680	9.99 698	**15**
46	9.07 124	107	9.07 428	108	0.92 572	9.99 696	14
47	9.07 231	106	9.07 536	107	0.92 464	9.99 695	13
48	9.07 337	105	9.07 643	108	0.92 357	9.99 693	12
49	9.07 442	106	9.07 751	107	0.92 249	9.99 692	11
50	9.07 548	105	9.07 858	106	0.92 142	9.99 690	**10**
51	9.07 653	105	9.07 964	107	0.92 036	9.99 689	9
52	9.07 758	105	9.08 071	106	0.91 929	9.99 687	8
53	9.07 863	105	9.08 177	106	0.91 823	9.99 686	7
54	9.07 968	104	9.08 283	106	0.91 717	9.99 684	6
55	9.08 072	104	9.08 389	106	0.91 611	9.99 683	**5**
56	9.08 176	104	9.08 495	105	0.91 505	9.99 681	4
57	9.08 280	103	9.08 600	105	0.91 400	9.99 680	3
58	9.08 383	103	9.08 705	105	0.91 295	9.99 678	2
59	9.08 486	103	9.08 810	104	0.91 190	9.99 677	1
60	9.08 589		9.08 914		0.91 086	9.99 675	**0**
′	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	121	120	119	118
2	24.2	24.0	23.8	23.6
3	36.3	36.0	35.7	35.4
4	48.4	48.0	47.6	47.2
5	60.5	60.0	59.5	59.0
6	72.6	72.0	71.4	70.8
7	84.7	84.0	83.3	82.6
8	96.8	96.0	95.2	94.4
9	108.9	108.0	107.1	106.2

	117	116	115	114
2	23.4	23.2	23.0	22.8
3	35.1	34.8	34.5	34.2
4	46.8	46.4	46.0	45.6
5	58.5	58.0	57.5	57.0
6	70.2	69.6	69.0	68.4
7	81.9	81.2	80.5	79.8
8	93.6	92.8	92.0	91.2
9	105.3	104.4	103.5	102.6

	113	112	111	110
2	22.6	22.4	22.2	22.0
3	33.9	33.6	33.3	33.0
4	45.2	44.8	44.4	44.0
5	56.5	56.0	55.5	55.0
6	67.8	67.2	66.6	66.0
7	79.1	78.4	77.7	77.0
8	90.4	89.6	88.8	88.0
9	101.7	100.8	99.9	99.0

	109	108	107	106
2	21.8	21.6	21.4	21.2
3	32.7	32.4	32.1	31.8
4	43.6	43.2	42.8	42.4
5	54.5	54.0	53.5	53.0
6	65.4	64.8	64.2	63.6
7	76.3	75.6	74.9	74.2
8	87.2	86.4	85.6	84.8
9	98.1	97.2	96.3	95.4

From the top:

For **6°+** or **186°+**, read as printed; for **96°+** or **276°+**, read co-function.

From the bottom:

For **83°+** or **263°+**, read as printed; for **173°+** or **353°+**, read co-function.

Prop. Pts.

'	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	9.08 589	103	9.08 914	105	0.91 086	9.99 675	**60**
1	9.08 692	103	9.09 019	104	0.90 981	9.99 674	59
2	9.08 795	102	9.09 123	104	0.90 877	9.99 672	58
3	9.08 897	102	9.09 227	103	0.90 773	9.99 670	57
4	9.08 999	102	9.09 330	104	0.90 670	9.99 669	56
5	9.09 101	101	9.09 434	103	0.90 566	9.99 667	**55**
6	9.09 202	102	9.09 537	103	0.90 463	9.99 666	54
7	9.09 304	101	9.09 640	102	0.90 360	9.99 664	53
8	9.09 405	101	9.09 742	103	0.90 258	9.99 663	52
9	9.09 506	100	9.09 845	102	0.90 155	9.99 661	51
10	9.09 606	101	9.09 947	102	0.90 053	9.99 659	**50**
11	9.09 707	100	9.10 049	101	0.89 951	9.99 658	49
12	9.09 807	100	9.10 150	102	0.89 850	9.99 656	48
13	9.09 907	99	9.10 252	101	0.89 748	9.99 655	47
14	9.10 006	100	9.10 353	101	0.89 647	9.99 653	46
15	9.10 106	99	9.10 454	101	0.89 546	9.99 651	**45**
16	9.10 205	99	9.10 555	101	0.89 445	9.99 650	44
17	9.10 304	98	9.10 656	100	0.89 344	9.99 648	43
18	9.10 402	99	9.10 756	100	0.89 244	9.99 647	42
19	9.10 501	98	9.10 856	100	0.89 144	9.99 645	41
20	9.10 599	98	9.10 956	100	0.89 044	9.99 643	**40**
21	9.10 697	98	9.11 056	99	0.88 944	9.99 642	39
22	9.10 795	98	9.11 155	99	0.88 845	9.99 640	38
23	9.10 893	97	9.11 254	99	0.88 746	9.99 638	37
24	9.10 990	97	9.11 353	99	0.88 647	9.99 637	36
25	9.11 087	97	9.11 452	99	0.88 548	9.99 635	**35**
26	9.11 184	97	9.11 551	98	0.88 449	9.99 633	34
27	9.11 281	96	9.11 649	98	0.88 351	9.99 632	33
28	9.11 377	97	9.11 747	98	0.88 253	9.99 630	32
29	9.11 474	96	9.11 845	98	0.88 155	9.99 629	31
30	9.11 570	96	9.11 943	97	0.88 057	9.99 627	**30**
31	9.11 666	95	9.12 040	98	0.87 960	9.99 625	29
32	9.11 761	96	9.12 138	97	0.87 862	9.99 624	28
33	9.11 857	95	9.12 235	97	0.87 765	9.99 622	27
34	9.11 952	95	9.12 332	96	0.87 668	9.99 620	26
35	9.12 047	95	9.12 428	97	0.87 572	9.99 618	**25**
36	9.12 142	94	9.12 525	96	0.87 475	9.99 617	24
37	9.12 236	95	9.12 621	96	0.87 379	9.99 615	23
38	9.12 331	94	9.12 717	96	0.87 283	9.99 613	22
39	9.12 425	94	9.12 813	96	0.87 187	9.99 612	21
40	9.12 519	93	9.12 909	95	0.87 091	9.99 610	**20**
41	9.12 612	94	9.13 004	95	0.86 996	9.99 608	19
42	9.12 706	93	9.13 099	95	0.86 901	9.99 607	18
43	9.12 799	93	9.13 194	95	0.86 806	9.99 605	17
44	9.12 892	93	9.13 289	95	0.86 711	9.99 603	16
45	9.12 985	93	9.13 384	94	0.86 616	9.99 601	**15**
46	9.13 078	93	9.13 478	95	0.86 522	9.99 600	14
47	9.13 171	92	9.13 573	94	0.86 427	9.99 598	13
48	9.13 263	92	9.13 667	94	0.86 333	9.99 596	12
49	9.13 355	92	9.13 761	93	0.86 239	9.99 595	11
50	9.13 447	92	9.13 854	94	0.86 146	9.99 593	**10**
51	9.13 539	91	9.13 948	93	0.86 052	9.99 591	9
52	9.13 630	92	9.14 041	93	0.85 959	9.99 589	8
53	9.13 722	91	9.14 134	93	0.85 866	9.99 588	7
54	9.13 813	91	9.14 227	93	0.85 773	9.99 586	6
55	9.13 904	90	9.14 320	92	0.85 680	9.99 584	**5**
56	9.13 994	91	9.14 412	92	0.85 588	9.99 582	4
57	9.14 085	90	9.14 504	93	0.85 496	9.99 581	3
58	9.14 175	91	9.14 597	91	0.85 403	9.99 579	2
59	9.14 266	90	9.14 688	92	0.85 312	9.99 577	1
60	9.14 356		9.14 780		0.85 220	9.99 575	**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	'

Prop. Pts.

	105	104	103	102
2	21.0	20.8	20.6	20.4
3	31.5	31.2	30.9	30.6
4	42.0	41.6	41.2	40.8
5	52.5	52.0	51.5	51.0
6	63.0	62.4	61.8	61.2
7	73.5	72.8	72.1	71.4
8	84.0	83.2	82.4	81.6
9	94.5	93.6	92.7	91.8

	101	99	98	97
2	20.2	19.8	19.6	19.4
3	30.3	29.7	29.4	29.1
4	40.4	39.6	39.2	38.8
5	50.5	49.5	49.0	48.5
6	60.6	59.4	58.8	58.2
7	70.7	69.3	68.6	67.9
8	80.8	79.2	78.4	77.6
9	90.9	89.1	88.2	87.3

	96	95	94	93
2	19.2	19.0	18.8	18.6
3	28.8	28.5	28.2	27.9
4	38.4	38.0	37.6	37.2
5	48.0	47.5	47.0	46.5
6	57.6	57.0	56.4	55.8
7	67.2	66.5	65.8	65.1
8	76.8	76.0	75.2	74.4
9	86.4	85.5	84.6	83.7

	92	91	90
2	18.4	18.2	18.0
3	27 6	27.3	27.0
4	36.8	36.4	36.0
5	46.0	45.5	45.0
6	55.2	54.6	54.0
7	64.4	63.7	63.0
8	73.6	72.8	72.0
9	82.8	81.9	81.0

From the top:

For **7°+** or **187°+**, read as printed; for **97°+** or **277°+**, read co-function.

From the bottom:

For **82°+** or **262°+**, read as printed; for **172°+** or **352°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	9.14 356	89	9.14 780	92	0.85 220	9.99 575	**60**
1	9.14 445	90	9.14 872	91	0.85 128	9.99 574	59
2	9.14 535	89	9.14 963	91	0.85 037	9.99 572	58
3	9.14 624	90	9.15 054	91	0.84 946	9.99 570	57
4	9.14 714	89	9.15 145	91	0.84 855	9.99 568	56
5	9.14 803	88	9.15 236	91	0.84 764	9.99 566	**55**
6	9.14 891	89	9.15 327	90	0.84 673	9.99 565	54
7	9.14 980	89	9.15 417	91	0.84 583	9.99 563	53
8	9.15 069	88	9.15 508	90	0.84 492	9.99 561	52
9	9.15 157	88	9.15 598	90	0.84 402	9.99 559	51
10	9.15 245	88	9.15 688	89	0.84 312	9.99 557	**50**
11	9.15 333	88	9.15 777	90	0.84 223	9.99 556	49
12	9.15 421	87	9.15 867	89	0.84 133	9.99 554	48
13	9.15 508	88	9.15 956	90	0.84 044	9.99 552	47
14	9.15 596	87	9.16 046	89	0.83 954	9.99 550	46
15	9.15 683	87	9.16 135	89	0.83 865	9.99 548	**45**
16	9.15 770	87	9.16 224	88	0.83 776	9.99 546	44
17	9.15 857	87	9.16 312	89	0.83 688	9.99 545	43
18	9.15 944	86	9.16 401	88	0.83 599	9.99 543	42
19	9.16 030	86	9.16 489	88	0.83 511	9.99 541	41
20	9.16 116	87	9.16 577	88	0.83 423	9.99 539	**40**
21	9.16 203	86	9.16 665	88	0.83 335	9.99 537	39
22	9.16 289	85	9.16 753	88	0.83 247	9.99 535	38
23	9.16 374	86	9.16 841	87	0.83 159	9.99 533	37
24	9.16 460	85	9.16 928	88	0.83 072	9.99 532	36
25	9.16 545	86	9.17 016	87	0.82 984	9.99 530	**35**
26	9.16 631	85	9.17 103	87	0.82 897	9.99 528	34
27	9.16 716	85	9.17 190	87	0.82 810	9.99 526	33
28	9.16 801	85	9.17 277	86	0.82 723	9.99 524	32
29	9.16 886	84	9.17 363	87	0.82 637	9.99 522	31
30	9.16 970	85	9.17 450	86	0.82 550	9.99 520	**30**
31	9.17 055	84	9.17 536	86	0.82 464	9.99 518	29
32	9.17 139	84	9.17 622	86	0.82 378	9.99 517	28
33	9.17 223	84	9.17 708	86	0.82 292	9.99 515	27
34	9.17 307	84	9.17 794	86	0.82 206	9.99 513	26
35	9.17 391	83	9.17 880	85	0.82 120	9.99 511	**25**
36	9.17 474	84	9.17 965	86	0.82 035	9.99 509	24
37	9.17 558	83	9.18 051	85	0.81 949	9.99 507	23
38	9.17 641	83	9.18 136	85	0.81 864	9.99 505	22
39	9.17 724	83	9.18 221	85	0.81 779	9.99 503	21
40	9.17 807	83	9.18 306	85	0.81 694	9.99 501	**20**
41	9.17 890	83	9.18 391	84	0.81 609	9.99 499	19
42	9.17 973	82	9.18 475	85	0.81 525	9.99 497	18
43	9.18 055	82	9.18 560	84	0.81 440	9.99 495	17
44	9.18 137	83	9.18 644	84	0.81 356	9.99 494	16
45	9.18 220	82	9.18 728	84	0.81 272	9.99 492	**15**
46	9.18 302	81	9.18 812	84	0.81 188	9.99 490	14
47	9.18 383	82	9.18 896	83	0.81 104	9.99 488	13
48	9.18 465	82	9.18 979	84	0.81 021	9.99 486	12
49	9.18 547	81	9.19 063	83	0.80 937	9.99 484	11
50	9.18 628	81	9.19 146	83	0.80 854	9.99 482	**10**
51	9.18 709	81	9.19 229	83	0.80 771	9.99 480	9
52	9.18 790	81	9.19 312	83	0.80 688	9.99 478	8
53	9.18 871	81	9.19 395	83	0.80 605	9.99 476	7
54	9.18 952	81	9.19 478	83	0.80 522	9.99 474	6
55	9.19 033	80	9.19 561	82	0.80 439	9.99 472	**5**
56	9.19 113	80	9.19 643	82	0.80 357	9.99 470	4
57	9.19 193	80	9.19 725	82	0.80 275	9.99 468	3
58	9.19 273	80	9.19 807	82	0.80 193	9.99 466	2
59	9.19 353	80	9.19 889	82	0.80 111	9.99 464	1
60	9.19 433		9.19 971		0.80 029	9.99 462	**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	92	91	90	89
2	18.4	18.2	18.0	17.8
3	27.6	27.3	27.0	26.7
4	36.8	36.4	36.0	35.6
5	46.0	45.5	45.0	44.5
6	55.2	54.6	54.0	53.4
7	64.4	63.7	63.0	62.3
8	73.6	72.8	72.0	71.2
9	82.8	81.9	81.0	80.1

	88	87	86
2	17.6	17.4	17.2
3	26.4	26.1	25.8
4	35.2	34.8	34.4
5	44.0	43.5	43.0
6	52.8	52.2	51.6
7	61.6	60.9	60.2
8	70.4	69.6	68.8
9	79.2	78.3	77.4

	85	84	83
2	17.0	16.8	16.6
3	25.5	25.2	24.9
4	34.0	33.6	33.2
5	42.5	42.0	41.5
6	51.0	50.4	49.8
7	59.5	58.8	58.1
8	68.0	67.2	66.4
9	76.5	75.6	74.7

	82	81	80
2	16.4	16.2	16.0
3	24.6	24.3	24.0
4	32.8	32.4	32.0
5	41.0	40.5	40.0
6	49.2	48.6	48.0
7	57.4	56.7	56.0
8	65.6	64.8	64.0
9	73.8	72.9	72.0

From the top:

For **8°+** or **188°+**, read as printed; for **98°+** or **278°+**, read co-function.

From the bottom:

For **81°+** or **261°+**, read as printed; for **171°+** or **351°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	9.19 433	80	9.19 971	82	0.80 029	9.99 462	**60**
1	9.19 513	79	9.20 053	81	0.79 947	9.99 460	59
2	9.19 592	80	9.20 134	82	0.79 866	9.99 458	58
3	9.19 672	79	9.20 216	81	0.79 784	9.99 456	57
4	9.19 751	79	9.20 297	81	0.79 703	9.99 454	56
5	9.19 830	79	9.20 378	81	0.79 622	9.99 452	**55**
6	9.19 909	79	9.20 459	81	0.79 541	9.99 450	54
7	9.19 988	79	9.20 540	81	0.79 460	9.99 448	53
8	9.20 067	78	9.20 621	80	0.79 379	9.99 446	52
9	9.20 145	78	9.20 701	81	0.79 299	9.99 444	51
10	9.20 223	79	9.20 782	80	0.79 218	9.99 442	**50**
11	9.20 302	78	9.20 862	80	0.79 138	9.99 440	49
12	9.20 380	78	9.20 942	80	0.79 058	9.99 438	48
13	9.20 458	77	9.21 022	80	0.78 978	9.99 436	47
14	9.20 535	78	9.21 102	80	0.78 898	9.99 434	46
15	9.20 613	78	9.21 182	79	0.78 818	9.99 432	**45**
16	9.20 691	77	9.21 261	80	0.78 739	9.99 429	44
17	9.20 768	77	9.21 341	79	0.78 659	9.99 427	43
18	9.20 845	77	9.21 420	79	0.78 580	9.99 425	42
19	9.20 922	77	9.21 499	79	0.78 501	9.99 423	41
20	9.20 999	77	9.21 578	79	0.78 422	9.99 421	**40**
21	9.21 076	77	9.21 657	79	0.78 343	9.99 419	39
22	9.21 153	76	9.21 736	78	0.78 264	9.99 417	38
23	9.21 229	77	9.21 814	79	0.78 186	9.99 415	37
24	9.21 306	76	9.21 893	78	0.78 107	9.99 413	36
25	9.21 382	76	9.21 971	78	0.78 029	9.99 411	**35**
26	9.21 458	76	9.22 049	78	0.77 951	9.99 409	34
27	9.21 534	76	9.22 127	78	0.77 873	9.99 407	33
28	9.21 610	75	9.22 205	78	0.77 795	9.99 404	32
29	9.21 685	76	9.22 283	78	0.77 717	9.99 402	31
30	9.21 761	75	9.22 361	77	0.77 639	9.99 400	**30**
31	9.21 836	76	9.22 438	78	0.77 562	9.99 398	29
32	9.21 912	75	9.22 516	77	0.77 484	9.99 396	28
33	9.21 987	75	9.22 593	77	0.77 407	9.99 394	27
34	9.22 062	75	9.22 670	77	0.77 330	9.99 392	26
35	9.22 137	74	9.22 747	77	0.77 253	9.99 390	**25**
36	9.22 211	75	9.22 824	77	0.77 176	9.99 388	24
37	9.22 286	75	9.22 901	76	0.77 099	9.99 385	23
38	9.22 361	74	9.22 977	77	0.77 023	9.99 383	22
39	9.22 435	74	9.23 054	76	0.76 946	9.99 381	21
40	9.22 509	74	9.23 130	76	0.76 870	9.99 379	**20**
41	9.22 583	74	9.23 206	77	0.76 794	9.99 377	19
42	9.22 657	74	9.23 283	76	0.76 717	9.99 375	18
43	9.22 731	74	9.23 359	76	0.76 641	9.99 372	17
44	9.22 805	73	9.23 435	75	0.76 565	9.99 370	16
45	9.22 878	74	9.23 510	76	0.76 490	9.99 368	**15**
46	9.22 952	73	9.23 586	75	0.76 414	9.99 366	14
47	9.23 025	73	9.23 661	76	0.76 339	9.99 364	13
48	9.23 098	73	9.23 737	75	0.76 263	9.99 362	12
49	9.23 171	73	9.23 812	75	0.76 188	9.99 359	11
50	9.23 244	73	9.23 887	75	0.76 113	9.99 357	**10**
51	9.23 317	73	9.23 962	75	0.76 038	9.99 355	9
52	9.23 390	72	9.24 037	75	0.75 963	9.99 353	8
53	9.23 462	73	9.24 112	74	0.75 888	9.99 351	7
54	9.23 535	72	9.24 186	75	0.75 814	9.99 348	6
55	9.23 607	72	9.24 261	74	0.75 739	9.99 346	**5**
56	9.23 679	73	9.24 335	75	0.75 665	9.99 344	4
57	9.23 752	71	9.24 410	74	0.75 590	9.99 342	3
58	9.23 823	72	9.24 484	74	0.75 516	9.99 340	2
59	9.23 895	72	9.24 558	74	0.75 442	9.99 337	1
60	9.23 967		9.24 632		0.75 368	9.99 335	**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	82	**81**	**80**	**79**
2	16.4	16.2	16.0	15.8
3	24.6	24.3	24.0	23.7
4	32.8	32.4	32.0	31.6
5	41.0	40.5	40.0	39.5
6	49.2	48.6	48.0	47.4
7	57.4	56.7	56.0	55.3
8	65.6	64.8	64.0	63.2
9	73.8	72.9	72.0	71.1

	78	**77**	**76**	**75**
2	15.6	15.4	15.2	15.0
3	23.4	23.1	22.8	22.5
4	31.2	30.8	30.4	30.0
5	39.0	38.5	38.0	37.5
6	46.8	46.2	45.6	45.0
7	54.6	53.9	53.2	52.5
8	62.4	61.6	60.8	60.0
9	70.2	69.3	68.4	67.5

	74	**73**	**72**	**71**
2	14.8	14.6	14.4	14.2
3	22.2	21.9	21.6	21.3
4	29.6	29.2	28.8	28.4
5	37.0	36.5	36.0	35.5
6	44.4	43.8	43.2	42.6
7	51.8	51.1	50.4	49.7
8	59.2	58.4	57.6	56.8
9	66.6	65.7	64.8	63.9

From the top:

For **9°+**, or **189°+**, read as printed; for **99°+** or **279°+**, read co-function.

From the bottom:

For **80°+** or **260°+**, read as printed; for **170°+** or **350°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.23 967	72	9.24 632	74	0.75 368	9.99 335	2	**60**
1	9.24 039	71	9.24 706	73	0.75 294	9.99 333	2	59
2	9.24 110	71	9.24 779	74	0.75 221	9.99 331	3	58
3	9.24 181	72	9.24 853	73	0.75 147	9.99 328	2	57
4	9.24 253	71	9.24 926	74	0.75 074	9.99 326	2	56
5	9.24 324	71	9.25 000	73	0.75 000	9.99 324	2	**55**
6	9.24 395	71	9.25 073	73	0.74 927	9.99 322	3	54
7	9.24 466	70	9.25 146	73	0.74 854	9.99 319	2	53
8	9.24 536	71	9.25 219	73	0.74 781	9.99 317	2	52
9	9.24 607	70	9.25 292	73	0.74 708	9.99 315	2	51
10	9.24 677	71	9.25 365	72	0.74 635	9.99 313	3	**50**
11	9.24 748	70	9.25 437	73	0.74 563	9.99 310	2	49
12	9.24 818	70	9.25 510	72	0.74 490	9.99 308	2	48
13	9.24 888	70	9.25 582	73	0.74 418	9.99 306	2	47
14	9.24 958	70	9.25 655	72	0.74 345	9.99 304	3	46
15	9.25 028	70	9.25 727	72	0.74 273	9.99 301	2	**45**
16	9.25 098	70	9.25 799	72	0.74 201	9.99 299	2	44
17	9.25 168	69	9 25 871	72	0.74 129	9.99 297	3	43
18	9.25 237	70	9.25 943	72	0.74 057	9.99 294	2	42
19	9.25 307	69	9.26 015	71	0.73 985	9.99 292	2	41
20	9.25 376	69	9.26 086	72	0.73 914	9.99 290	2	**40**
21	9.25 445	69	9.26 158	71	0.73 842	9.99 288	3	39
22	9.25 514	69	9.26 229	72	0.73 771	9.99 285	2	38
23	9.25 583	69	9.26 301	71	0.73 699	9.99 283	2	37
24	9.25 652	69	6.26 372	71	0.73 628	9.99 281	3	36
25	9.25 721	69	9.26 443	71	0.73 557	9.99 278	2	**35**
26	9.25 790	68	9.26 514	71	0.73 486	9.99 276	2	34
27	9.25 858	69	9.26 585	70	0.73 415	9.99 274	3	33
28	9.25 927	68	9.26 655	71	0.73 345	9.99 271	2	32
29	9.25 995	68	9.26 726	71	0.73 274	9.99 269	2	31
30	9.26 063	68	9.26 797	70	0.73 203	9.99 267	3	**30**
31	9.26 131	68	9.26 867	70	0.73 133	9.99 264	2	29
32	9.26 199	68	9.26 937	71	0.73 063	9.99 262	2	28
33	9.26 267	68	9.27 008	70	0.72 992	9.99 260	3	27
34	9.26 335	68	9.27 078	70	0.72 922	9.99 257	2	26
35	9.26 403	67	9.27 148	70	0.72 852	9.99 255	3	**25**
36	9.26 470	68	9.27 218	70	0.72 782	9.99 252	2	24
37	9.26 538	67	9.27 288	69	0.72 712	9.99 250	2	23
38	9.26 605	67	9.27 357	70	0.72 643	9.99 248	3	22
39	9.26 672	67	9.27 427	69	0.72 573	9.99 245	2	21
40	9.26 739	67	9.27 496	70	0.72 504	9.99 243	2	**20**
41	9.26 806	67	9.27 566	69	0.72 434	9.99 241	3	19
42	9.26 873	67	9.27 635	69	0.72 365	9.99 238	2	18
43	9 26 940	67	9.27 704	69	0.72 296	9.99 236	3	17
44	9.27 007	66	9.27 773	69	0.72 227	9.99 233	2	16
45	9.27 073	67	9.27 842	69	0.72 158	9.99 231	2	**15**
46	9.27 140	66	9.27 911	69	0.72 089	9.99 229	3	14
47	9.27 206	67	9.27 980	69	0.72 020	9.99 226	2	13
48	9.27 273	66	9.28 049	68	0.71 951	9.99 224	3	12
49	9.27 339	66	9.28 117	69	0.71 883	9.99 221	2	11
50	9.27 405	66	9.28 186	68	0.71 814	9.99 219	2	**10**
51	9.27 471	66	9.28 254	69	0.71 746	9.99 217	3	9
52	9.27 537	65	9.28 323	68	0.71 677	9.99 214	2	8
53	9.27 602	66	9.28 391	68	0.71 609	9.99 212	3	7
54	9.27 668	66	9.28 459	68	0.71 541	9.99 209	2	6
55	9.27 734	65	9.28 527	68	0.71 473	9.99 207	3	**5**
56	9.27 799	65	9.28 595	67	0.71 405	9.99 204	2	4
57	9.27 864	66	9.28 662	68	0.71 338	9.99 202	2	3
58	9.27 930	65	9.28 730	68	0.71 270	9.99 200	3	2
59	9.27 995	65	9.28 798	67	0.71 202	9.99 197	2	1
60	9.28 060		9.28 865		0.71 135	9.99 195		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	74	73	72
2	14.8	14.6	14.4
3	22.2	21.9	21.6
4	29.6	29.2	28.8
5	37.0	36.5	36.0
6	44.4	43.8	43.2
7	51.8	51.1	50.4
8	59.2	58.4	57.6
9	66.6	65.7	64.8

	71	70	69
2	14.2	14.0	13.8
3	21.3	21.0	20.7
4	28.4	28.0	27.6
5	35.5	35.0	34.5
6	42.6	42.0	41.4
7	49.7	49.0	48.3
8	56.8	56.0	55.2
9	63.9	63.0	62.1

	68	67	66
2	13.6	13.4	13.2
3	20.4	20.1	19.8
4	27.2	26.8	26.4
5	34.0	33.5	33.0
6	40.8	40.2	39.6
7	47.6	46.9	46.2
8	54.4	53.6	52.8
9	61.2	60.3	59.4

	65	3
2	13.0	0.6
3	19.5	0.9
4	26.0	1.2
5	32.5	1.5
6	39.0	1.8
7	45.5	2.1
8	52.0	2.4
9	58.5	2.7

From the top:

For **10°**+ or **190°**+, read as printed; for **100°**+ or **280°**+, read co-function.

From the bottom:

For **79°**+ or **259°**+, read as printed; for **139°**+ or **349°**+, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.28 060	65	9.28 865	68	0.71 135	9.99 195	3	60
1	9.28 125	65	9.28 933	67	0.71 067	9.99 192	2	59
2	9.28 190	64	9.29 000	67	0.71 000	9.99 190	3	58
3	9.28 254	65	9.29 067	67	0.70 933	9.99 187	2	57
4	9.28 319	65	9.29 134	67	0.70 866	9.99 185	3	56
5	9.28 384	64	9.29 201	67	0.70 799	9.99 182	2	55
6	9.28 448	64	9.29 268	67	0.70 732	9.99 180	3	54
7	9.28 512	65	9.29 335	67	0.70 665	9.99 177	2	53
8	9.28 577	64	9.29 402	66	0.70 598	9.99 175	3	52
9	9.28 641	64	9.29 468	67	0.70 532	9.99 172	2	51
10	9.28 705	64	9.29 535	66	0.70 465	9.99 170	3	50
11	9.28 769	64	9.29 601	67	0.70 399	9.99 167	2	49
12	9.28 833	63	9.29 668	66	0.70 332	9.99 165	3	48
13	9.28 896	64	9.29 734	66	0.70 266	9.99 162	2	47
14	9.28 960	64	9.29 800	66	0.70 200	9.99 160	3	46
15	9.29 024	63	9.29 866	66	0.70 134	9.99 157	2	45
16	9.29 087	63	9.29 932	66	0.70 068	9.99 155	3	44
17	9.29 150	64	9.29 998	66	0.70 002	9.99 152	2	43
18	9.29 214	63	9.30 064	66	0.69 936	9.99 150	3	42
19	9.29 277	63	9.30 130	65	0.69 870	9.99 147	2	41
20	9.29 340	63	9.30 195	66	0.69 805	9.99 145	3	40
21	9.29 403	63	9.30 261	65	0.69 739	9.99 142	2	39
22	9.29 466	63	9.30 326	65	0.69 674	9.99 140	3	38
23	9.29 529	62	9.30 391	66	0.69 609	9.99 137	2	37
24	9.29 591	63	9.30 457	65	0.69 543	9.99 135	3	36
25	9.29 654	62	9.30 522	65	0.69 478	9.99 132	2	35
26	9.29 716	63	9.30 587	65	0.69 413	9.99 130	3	34
27	9.29 779	62	9.30 652	65	0.69 348	9.99 127	3	33
28	9.29 841	62	9.30 717	65	0.69 283	9.99 124	2	32
29	9.29 903	63	9.30 782	64	0.69 218	9.99 122	3	31
30	9.29 966	62	9.30 846	65	0.69 154	9.99 119	2	30
31	9.30 028	62	9.30 911	64	0.69 089	9.99 117	3	29
32	9.30 090	61	9.30 975	65	0.69 025	9.99 114	2	28
33	9.30 151	62	9.31 040	64	0.68 960	9.99 112	3	27
34	9.30 213	62	9.31 104	64	0.68 896	9.99 109	3	26
35	9.30 275	61	9.31 168	65	0.68 832	9.99 106	2	25
36	9.30 336	62	9.31 233	64	0.68 767	9.99 104	3	24
37	9.30 398	61	9.31 297	64	0.68 703	9.99 101	2	23
38	9.30 459	62	9.31 361	64	0.68 639	9.99 099	3	22
39	9.30 521	61	9.31 425	64	0.68 575	9.99 096	3	21
40	9.30 582	61	9.31 489	63	0.68 511	9.99 093	2	20
41	9.30 643	61	9.31 552	64	0.68 448	9.99 091	3	19
42	9.30 704	61	9.31 616	63	0.68 384	9.99 088	2	18
43	9.30 765	61	9.31 679	64	0.68 321	9.99 086	3	17
44	9.30 826	61	9.31 743	63	0.68 257	9.99 083	3	16
45	9.30 887	60	9.31 806	64	0.68 194	9.99 080	2	15
46	9.30 947	61	9.31 870	63	0.68 130	9.99 078	3	14
47	9.31 008	60	9.31 933	63	0.68 067	9.99 075	3	13
48	9.31 068	61	9.31 996	63	0.68 004	9.99 072	2	12
49	9.31 129	60	9.32 059	63	0.67 941	9.99 070	3	11
50	9.31 189	61	9.32 122	63	0.67 878	9.99 067	3	10
51	9.31 250	60	9.32 185	63	0.67 815	9.99 064	2	9
52	9.31 310	60	9.32 248	63	0.67 752	9.99 062	3	8
53	9.31 370	60	9.32 311	62	0.67 689	9.99 059	3	7
54	9.31 430	60	9.32 373	63	0.67 627	9.99 056	2	6
55	9.31 490	59	9.32 436	62	0.67 564	9.99 054	3	5
56	9.31 549	60	9.32 498	63	0.67 502	9.99 051	3	4
57	9.31 609	60	9.32 561	62	0.67 439	9.99 048	2	3
58	9.31 669	59	9.32 623	62	0.67 377	9.99 046	3	2
59	9.31 728	60	9.32 685	62	0.67 315	9.99 043	3	1
60	9.31 788		9.32 747		0.67 253	9.99 040		0
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	68	67	66
2	13.6	13.4	13.2
3	20.4	20.1	19.8
4	27.2	26.8	26.4
5	34.0	33.5	33.0
6	40.8	40.2	39.6
7	47.6	46.9	46.2
8	54.4	53.6	52.8
9	61.2	60.3	59.4

	65	64	63
2	13.0	12.8	12.6
3	19.5	19.2	18.9
4	26.0	25.6	25.2
5	32.5	32.0	31.5
6	39.0	38.4	37.8
7	45.5	44.8	44.1
8	52.0	51.2	50.4
9	58.5	57.6	56.7

	62	61	60
2	12.4	12.2	12.0
3	18.6	18.3	18.0
4	24.8	24.4	24.0
5	31.0	30.5	30.0
6	37.2	36.6	36.0
7	43.4	42.7	42.0
8	49.6	48.8	48.0
9	55.8	54.9	54.0

	59	3
2	11.8	0.6
3	17.7	0.9
4	23.6	1.2
5	29.5	1.5
6	35.4	1.8
7	41.3	2.1
8	47.2	2.4
9	53.1	2.7

From the top:

For **11°+** or **191°+**, read as printed; for **101°+** or **281°+**, read co-function.

From the bottom:

For **78°+** or **258°+**, read as printed; for **168°+** or **348°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.31 788	59	9.32 747	63	0.67 253	9.99 040	2	**60**
1	9.31 847	60	9.32 810	62	0.67 190	9.99 038	3	59
2	9.31 907	59	9.32 872	61	0.67 128	9.99 035	3	58
3	9.31 966	59	9.32 933	62	0.67 067	9.99 032	2	57
4	9.32 025	59	9.32 995	62	0.67 005	9.99 030	3	56
5	9.32 084	59	9.33 057	62	0.66 943	9.99 027	3	**55**
6	9.32 143	59	9.33 119	61	0.66 881	9.99 024	2	54
7	9.32 202	59	9.33 180	62	0.66 820	9.99 022	3	53
8	9.32 261	58	9.33 242	61	0.66 758	9.99 019	3	52
9	9.32 319	59	9.33 303	62	0.66 697	9.99 016	3	51
10	9.32 378	59	9.33 365	61	0.66 635	9.99 013	2	**50**
11	9.32 437	58	9.33 426	61	0.66 574	9.99 011	3	49
12	9.32 495	58	9.33 487	61	0.66 513	9.99 008	3	48
13	9.32 553	59	9.33 548	61	0.66 452	9.99 005	3	47
14	9.32 612	58	9.33 609	61	0.66 391	9.99 002	2	46
15	9.32 670	58	9.33 670	61	0.66 330	9.99 000	3	**45**
16	9.32 728	58	9.33 731	61	0.66 269	9.98 997	3	44
17	9.32 786	58	9.33 792	61	0.66 208	9.98 994	3	43
18	9.32 844	58	9.33 853	60	0.66 147	9.98 991	2	42
19	9.32 902	58	9.33 913	61	0.66 087	9.98 989	3	41
20	9.32 960	58	9.33 974	60	0.66 026	9.98 986	3	**40**
21	9.33 018	57	9.34 034	61	0.65 966	9.98 983	3	39
22	9.33 075	58	9.34 095	60	0.65 905	9.98 980	2	38
23	9.33 133	57	9.34 155	60	0.65 845	9.98 978	3	37
24	9.33 190	58	9.34 215	61	0.65 785	9.98 975	3	36
25	9.33 248	57	9.34 276	60	0.65 724	9.98 972	3	**35**
26	9.33 305	57	9.34 336	60	0.65 664	9.98 969	2	34
27	9.33 362	58	9.34 396	60	0.65 604	9.98 967	3	33
28	9.33 420	57	9.34 456	60	0.65 544	9.98 964	3	32
29	9.33 477	57	9.34 516	60	0.65 484	9.98 961	3	31
30	9.33 534	57	9.34 576	59	0.65 424	9.98 958	3	**30**
31	9.33 591	56	9.34 635	60	0.65 365	9.98 955	2	29
32	9.33 647	57	9.34 695	60	0.65 305	9.98 953	3	28
33	9.33 704	57	9.34 755	59	0.65 245	9.98 950	3	27
34	9.33 761	57	9.34 814	60	0.65 186	9.98 947	3	26
35	9.33 818	56	9.34 874	59	0.65 126	9.98 944	3	**25**
36	9.33 874	57	9.34 933	59	0.65 067	9.98 941	3	24
37	9.33 931	56	9.34 992	59	0.65 008	9.98 938	2	23
38	9.33 987	56	9.35 051	60	0.64 949	9.98 936	3	22
39	9.34 043	57	9.35 111	59	0.64 889	9.98 933	3	21
40	9.34 100	56	9.35 170	59	0.64 830	9.98 930	3	**20**
41	9.34 156	56	9.35 229	59	0.64 771	9.98 927	3	19
42	9.34 212	56	9.35 288	59	0.64 712	9.98 924	3	18
43	9.34 268	56	9.35 347	58	0.64 653	9.98 921	2	17
44	9.34 324	56	9.35 405	59	0.64 595	9.98 919	3	16
45	9.34 380	56	9.35 464	59	0.64 536	9.98 916	3	**15**
46	9.34 436	55	9.35 523	58	0.64 477	9.98 913	3	14
47	9.34 491	56	9.35 581	59	0.64 419	9.98 910	3	13
48	9.34 547	55	9.35 640	58	0.64 360	9.98 907	3	12
49	9.34 602	56	9.35 698	59	0.64 302	9.98 904	3	11
50	9.34 658	55	9.35 757	58	0.64 243	9.98 901	3	**10**
51	9.34 713	56	9.35 815	58	0.64 185	9.98 898	2	9
52	9.34 769	55	9.35 873	58	0.64 127	9.98 896	3	8
53	9.34 824	55	9.35 931	58	0.64 069	9.98 893	3	7
54	9.34 879	55	9.35 989	58	0.64 011	9.98 890	3	6
55	9.34 934	55	9.36 047	58	0.63 953	9.98 887	3	**5**
56	9.34 989	55	9.36 105	58	0.63 895	9.98 884	3	4
57	9.35 044	55	9.36 163	58	0.63 837	9.98 881	3	3
58	9.35 099	55	9.36 221	58	0.63 779	9.98 878	3	2
59	9.35 154	55	9.36 279	57	0.63 721	9.98 875	3	1
60	9.35 209		9.36 336		0.63 664	9.98 872		**0**
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	63	62	61
2	12.6	12.4	12.2
3	18.9	18.6	18.3
4	25.2	24.8	24.4
5	31.5	31.0	30.5
6	37.8	37.2	36.6
7	44.1	43.4	42.7
8	50.4	49.6	48.8
9	56.7	55.8	54.9

	60	59	58
2	12.0	11.8	11.6
3	18.0	17.7	17.4
4	24.0	23.6	23.2
5	30.0	29.5	29.0
6	36.0	35.4	34.8
7	42.0	41.3	40.6
8	48.0	47.2	46.4
9	54.0	53.1	52.2

	57	56
2	11.4	11.2
3	17.1	16.8
4	22.8	22.4
5	28.5	28.0
6	34.2	33.6
7	39.9	39.2
8	45.6	44.8
9	51.3	50.4

	55	3
2	11.0	0.6
3	16.5	0.9
4	22.0	1.2
5	27.5	1.5
6	33.0	1.8
7	38.5	2.1
8	44.0	2.4
9	49.5	2.7

From the top :

For **12°**+ or **192°**+, read as printed; for **102°**+ or **282°**+, read co-function.

From the bottom :

For **77°** or **257°**, read as printed ; for **167°** or **347°**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.35 209	54	9.36 336	58	0.63 664	9.98 872	3	**60**
1	9.35 263	55	9.36 394	58	0.63 606	9.98 869	2	59
2	9.35 318	55	9.36 452	57	0.63 548	9.98 867	3	58
3	9.35 373	54	9.36 509	57	0.63 491	9.98 864	3	57
4	9.35 427	54	9.36 566	58	0.63 434	9.98 861	3	56
5	9.35 481	55	9.36 624	57	0.63 376	9.98 858	3	**55**
6	9.35 536	54	9.36 681	57	0.63 319	9.98 855	3	54
7	9.35 590	54	9.36 738	57	0.63 262	9.98 852	3	53
8	9.35 644	54	9.36 795	57	0.63 205	9.98 849	3	52
9	9.35 698	54	9.36 852	57	0.63 148	9.98 846	3	51
10	9.35 752	54	9.36 909	57	0.63 091	9.98 843	3	**50**
11	9.35 806	54	9.36 966	57	0.63 034	9.98 840	3	49
12	9.35 860	54	9.37 023	57	0.62 977	9.98 837	3	48
13	9.35 914	54	9.37 080	57	0.62 920	9.98 834	3	47
14	9.35 968	54	9.37 137	56	0.62 863	9.98 831	3	46
15	9.36 022	53	9.37 193	57	0.62 807	9.98 828	3	**45**
16	9.36 075	54	9.37 250	56	0.62 750	9.98 825	3	44
17	9.36 129	53	9.37 306	57	0.62 694	9.98 822	3	43
18	9.36 182	54	9.37 363	56	0.62 637	9.98 819	3	42
19	9.36 236	53	9.37 419	57	0.62 581	9.98 816	3	41
20	9.36 289	53	9.37 476	56	0.62 524	9.98 813	3	**40**
21	9.36 342	53	9.37 532	56	0.62 468	9.98 810	3	39
22	9.36 395	54	9.37 588	56	0.62 412	9.98 807	3	38
23	9.36 449	53	9.37 644	56	0.62 356	9.98 804	3	37
24	9.36 502	53	9.37 700	56	0.62 300	9.98 801	3	36
25	9.36 555	53	9.37 756	56	0.62 244	9.98 798	3	**35**
26	9.36 608	52	9.37 812	56	0.62 188	9.98 795	3	34
27	9.36 660	53	9.37 868	56	0.62 132	9.98 792	3	33
28	9.36 713	53	9.37 924	56	0.62 076	9.98 789	3	32
29	9.36 766	53	9.37 980	55	0.62 020	9.98 786	3	31
30	9.36 819	52	9.38 035	56	0.61 965	9.98 783	3	**30**
31	9.36 871	53	9.38 091	56	0.61 909	9.98 780	3	29
32	9.36 924	52	9.38 147	55	0.61 853	9.98 777	3	28
33	9.36 976	52	9.38 202	55	0.61 798	9.98 774	3	27
34	9.37 028	53	9.38 257	56	0.61 743	9.98 771	3	26
35	9.37 081	52	9.38 313	55	0.61 687	9.98 768	3	**25**
36	9.37 133	52	9.38 368	55	0.61 632	9.98 765	3	24
37	9.37 185	52	9.38 423	56	0.61 577	9.98 762	3	23
38	9.37 237	52	9.38 479	55	0.61 521	9.98 759	3	22
39	9.37 289	52	9.38 534	55	0.61 466	9.98 756	3	21
40	9.37 341	52	9.38 589	55	0.61 411	9.98 753	3	**20**
41	9.37 393	52	9.38 644	55	0.61 356	9.98 750	4	19
42	9.37 445	52	9.38 699	55	0.61 301	9.98 746	3	18
43	9.37 497	52	9.38 754	54	0.61 246	9.98 743	3	17
44	9.37 549	51	9.38 808	55	0.61 192	9.98 740	3	16
45	9.37 600	52	9.38 863	55	0.61 137	9.98 737	3	**15**
46	9.37 652	51	9.38 918	54	0.61 082	9.98 734	3	14
47	9.37 703	52	9.38 972	55	0.61 028	9.98 731	3	13
48	9.37 755	51	9.39 027	55	0.60 973	9.98 728	3	12
49	9.37 806	52	9.39 082	54	0.60 918	9.98 725	3	11
50	9.37 858	51	9.39 136	54	0.60 864	9.98 722	3	**10**
51	9.37 909	51	9.39 190	55	0.60 810	9.98 719	4	9
52	9.37 960	51	9.39 245	54	0.60 755	9.98 715	3	8
53	9.38 011	51	9.39 299	54	0.60 701	9.98 712	3	7
54	9.38 062	51	9.39 353	54	0.60 647	9.98 709	3	6
55	9.38 113	51	9.39 407	54	0.60 593	9.98 706	3	**5**
56	9.38 164	51	9.39 461	54	0.60 539	9.98 703	3	4
57	9.38 215	51	9.39 515	54	0.60 485	9.98 700	3	3
58	9.38 266	51	9.39 569	54	0.60 431	9.98 697	3	2
59	9.38 317	51	9.39 623	54	0.60 377	9.98 694	4	1
60	9.38 368		9.39 677		0.60 323	9.98 690		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	58	57	56
2	11.6	11.4	11.2
3	17.4	17.1	16.8
4	23.2	22.8	22.4
5	29.0	28.5	28.0
6	34.8	34.2	33.6
7	40.6	39.9	39.2
8	46.4	45.6	44.8
9	52.2	51.3	50.4

	55	54	53
2	11.0	10.8	10.6
3	16.5	16.2	15.9
4	22.0	21.6	21.2
5	27.5	27.0	26.5
6	33.0	32.4	31.8
7	38.5	37.8	37.1
8	44.0	43.2	42.4
9	49.5	48.6	47.7

	52	51
2	10.4	10.2
3	15.6	15.3
4	20.8	20.4
5	26.0	25.5
6	31.2	30.6
7	36.4	35.7
8	41.6	40.8
9	46.8	45.9

	4	3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

From the top:

For **13°+** or **193°+**, read as printed; for **103°+** or **283°+**, read co-function.

From the bottom:

For **76°** or **256°**, read as printed; for **166°+** or **346°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.38 368	50	9.39 677	54	0.60 323	9.98 690	3	**60**
1	9.38 418	51	9.39 731	54	0.60 269	9.98 687	3	59
2	9.38 469	50	9.39 785	53	0.60 215	9.98 684	3	58
3	9.38 519	51	9.39 838	54	0.60 162	9.98 681	3	57
4	9.38 570	50	9.39 892	53	0.60 108	9.98 678	3	56
5	9.38 620	50	9.39 945	54	0.60 055	9.98 675	4	**55**
6	9.38 670	51	9.39 999	53	0.60 001	9.98 671	3	54
7	9.38 721	50	9.40 052	54	0.59 948	9.98 668	3	53
8	9.38 771	50	9.40 106	53	0.59 894	9.98 665	3	52
9	9.38 821	50	9.40 159	53	0.59 841	9.98 662	3	51
10	9.38 871	50	9.40 212	54	0.59 788	9.98 659	3	**50**
11	9.38 921	50	9.40 266	53	0.59 734	9.98 656	4	49
12	9.38 971	50	9.40 319	53	0.59 681	9.98 652	3	48
13	9.39 021	50	9.40 372	53	0.59 628	9.98 649	3	47
14	9.39 071	50	9.40 425	53	0.59 575	9.98 646	3	46
15	9.39 121	49	9.40 478	53	0.59 522	9.98 643	3	**45**
16	9.39 170	50	9.40 531	53	0.59 469	9.98 640	4	44
17	9.39 220	50	9.40 584	52	0.59 416	9.98 636	3	43
18	9.39 270	49	9.40 636	53	0.59 364	9.98 633	3	42
19	9.39 319	50	9.40 689	53	0.59 311	9.98 630	3	41
20	9.39 369	49	9.40 742	53	0.59 258	9.98 627	4	**40**
21	9.39 418	49	9.40 795	52	0.59 205	9.98 623	3	39
22	9.39 467	50	9.40 847	53	0.59 153	9.98 620	3	38
23	9.39 517	49	9.40 900	52	0.59 100	9.98 617	3	37
24	9.39 566	49	9.40 952	53	0.59 048	9.98 614	4	36
25	9.39 615	49	9.41 005	52	0.58 995	9.98 610	3	**35**
26	9.39 664	49	9.41 057	52	0.58 943	9.98 607	3	34
27	9.39 713	49	9.41 109	52	0.58 891	9.98 604	3	33
28	9.39 762	49	9.41 161	53	0.58 839	9.98 601	4	32
29	9.39 811	49	9.41 214	52	0.58 786	9.98 597	3	31
30	9.39 860	49	9.41 266	52	0.58 734	9.98 594	3	**30**
31	9.39 909	49	9.41 318	52	0.58 682	9.98 591	3	29
32	9.39 958	48	9.41 370	52	0.58 630	9.98 588	4	28
33	9.40 006	49	9.41 422	52	0.58 578	9.98 584	3	27
34	9.40 055	48	9.41 474	52	0.58 526	9.98 581	3	26
35	9.40 103	49	9.41 526	52	0.58 474	9.98 578	4	**25**
36	9.40 152	48	9.41 578	51	0.58 422	9.98 574	3	24
37	9.40 200	49	9.41 629	52	0.58 371	9.98 571	3	23
38	9.40 249	48	9.41 681	52	0.58 319	9.98 568	3	22
39	9.40 297	49	9.41 733	51	0.58 267	9.98 565	4	21
40	9.40 346	48	9.41 784	52	0.58 216	9.98 561	3	**20**
41	9.40 394	48	9.41 836	51	0.58 164	9.98 558	3	19
42	9.40 442	48	9.41 887	52	0.58 113	9.98 555	4	18
43	9.40 490	48	9.41 939	51	0.58 061	9.98 551	3	17
44	9.40 538	48	9.41 990	51	0.58 010	9.98 548	3	16
45	9.40 586	48	9.42 041	52	0.57 959	9.98 545	4	**15**
46	9.40 634	48	9.42 093	51	0.57 907	9.98 541	3	14
47	9.40 682	48	9.42 144	51	0.57 856	9.98 538	3	13
48	9.40 730	48	9.42 195	51	0.57 805	9.98 535	4	12
49	9.40 778	47	9.42 246	51	0.57 754	9.98 531	3	11
50	9.40 825	48	9.42 297	51	0.57 703	9.98 528	3	**10**
51	9.40 873	48	9.42 348	51	0.57 652	9.98 525	4	9
52	9.40 921	47	9.42 399	51	0.57 601	9.98 521	3	8
53	9.40 968	48	9.42 450	51	0.57 550	9.98 518	3	7
54	9.41 016	47	9.42 501	51	0.57 499	9.98 515	4	6
55	9.41 063	48	9.42 552	51	0.57 448	9.98 511	3	**5**
56	9.41 111	47	9.42 603	50	0.57 397	9.98 508	3	4
57	9.41 158	47	9.42 653	51	0.57 347	9.98 505	4	3
58	9.41 205	47	9.42 704	51	0.57 296	9.98 501	3	2
59	9.41 252	48	9.42 755	50	0.57 245	9.98 498	4	1
60	9.41 300		9.42 805		0.57 195	9.98 494		**0**
	L Cos	**d**	**L Ctn**	**c d**	**L Tan**	**L Sin**	**d**	**′**

Prop. Pts.

	54	**53**	**52**
2	10.8	10.6	10.4
3	16.2	15.9	15.6
4	21.6	21.2	20.8
5	27.0	26.5	26.0
6	32.4	31.8	31.2
7	37.8	37.1	36.4
8	43.2	42.4	41.6
9	48.6	47.7	46.8

	51	**50**	**49**
2	10.2	10.0	9.8
3	15.3	15.0	14.7
4	20.4	20.0	19.6
5	25.5	25.0	24.5
6	30.6	30.0	29.4
7	35.7	35.0	34.3
8	40.8	40.0	39.2
9	45.9	45.0	44.1

	48	**47**
2	9.6	9.4
3	14.4	14.1
4	19.2	18.8
5	24.0	23.5
6	28.8	28.2
7	33.6	32.9
8	38.4	37.6
9	43.2	42.3

	4	**3**
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

From the top:

For **14°+** or **194°+**, read as printed; for **104°+** or **284°+**, read co-function.

From the bottom:

For **75°+** or **255°+**, read as printed; for **165°+** or **345°+**, read co-function.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.41 300	47	9.42 805	51	0.57 195	9.98 494	3	**60**
1	9.41 347	47	9.42 856	50	0.57 144	9.98 491	3	59
2	9.41 394	47	9.42 906	51	0.57 094	9.98 488	4	58
3	9.41 441	47	9.42 957	50	0.57 043	9.98 484	3	57
4	9.41 488	47	9.43 007	50	0.56 993	9.98 481	4	56
5	9.41 535	47	9.43 057	51	0.56 943	9.98 477	3	**55**
6	9.41 582	46	9.43 108	50	0.56 892	9.98 474	3	54
7	9.41 628	47	9.43 158	50	0.56 842	9.98 471	4	53
8	9.41 675	47	9.43 208	50	0.56 792	9.98 467	3	52
9	9.41 722	46	9.43 258	50	0.56 742	9.98 464	4	51
10	9.41 768	47	9.43 308	50	0.56 692	9.98 460	3	**50**
11	9.41 815	46	9.43 358	50	0.56 642	9.98 457	4	49
12	9.41 861	47	9.43 408	50	0.56 592	9.98 453	3	48
13	9.41 908	46	9.43 458	50	0.56 542	9.98 450	3	47
14	9.41 954	47	9.43 508	50	0.56 492	9.98 447	4	46
15	9.42 001	46	9.43 558	49	0.56 442	9.98 443	3	**45**
16	9.42 047	46	9.43 607	50	0.56 393	9.98 440	4	44
17	9.42 093	47	9.43 657	50	0.56 343	9.98 436	3	43
18	9.42 140	46	9.43 707	49	0.56 293	9.98 433	4	42
19	9.42 186	46	9.43 756	50	0.56 244	9.98 429	3	41
20	9.42 232	46	9.43 806	49	0.56 194	9.98 426	4	**40**
21	9.42 278	46	9.43 855	50	0.56 145	9.98 422	3	39
22	9.42 324	46	9.43 905	49	0.56 095	9.98 419	4	38
23	9.42 370	46	9.43 954	50	0.56 046	9.98 415	3	37
24	9.42 416	45	9.44 004	49	0.55 996	9.98 412	3	36
25	9.42 461	46	9.44 053	49	0.55 947	9.98 409	4	**35**
26	9.42 507	46	9.44 102	49	0.55 898	9.98 405	3	34
27	9.42 553	46	9.44 151	50	0.55 849	9.98 402	4	33
28	9.42 599	45	9.44 201	49	0.55 799	9.98 398	3	32
29	9.42 644	46	9.44 250	49	0.55 750	9.98 395	4	31
30	9.42 690	45	9.44 299	49	0.55 701	9.98 391	3	**30**
31	9.42 735	46	9.44 348	49	0.55 652	9.98 388	4	29
32	9.42 781	45	9.44 397	49	0.55 603	9.98 384	3	28
33	9.42 826	46	9.44 446	49	0.55 554	9.98 381	4	27
34	9.42 872	45	9.44 495	49	0.55 505	9.98 377	4	26
35	9.42 917	45	9.44 544	48	0.55 456	9.98 373	3	**25**
36	9.42 962	46	9.44 592	49	0.55 408	9.98 370	4	24
37	9.43 008	45	9.44 641	49	0.55 359	9.98 366	3	23
38	9.43 053	45	9.44 690	48	0.55 310	9.98 363	4	22
39	9.43 098	45	9.44 738	49	0.55 262	9.98 359	3	21
40	9.43 143	45	9.44 787	49	0.55 213	9.98 356	4	**20**
41	9.43 188	45	9.44 836	48	0.55 164	9.98 352	3	19
42	9.43 233	45	9.44 884	49	0.55 116	9.98 349	4	18
43	9.43 278	45	9.44 933	48	0.55 067	9.98 345	3	17
44	9.43 323	44	9.44 981	48	0.55 019	9.98 342	4	16
45	9.43 367	45	9.45 029	49	0.54 971	9.98 338	4	**15**
46	9.43 412	45	9.45 078	48	0.54 922	9.98 334	3	14
47	9.43 457	45	9.45 126	48	0.54 874	9.98 331	4	13
48	9.43 502	44	9.45 174	48	0.54 826	9.98 327	3	12
49	9.43 546	45	9.45 222	49	0.54 778	9.98 324	4	11
50	9.43 591	44	9.45 271	48	0.54 729	9.98 320	3	**10**
51	9.43 635	45	9.45 319	48	0.54 681	9.98 317	4	9
52	9.43 680	44	9.45 367	48	0.54 633	9.98 313	4	8
53	9.43 724	45	9.45 415	48	0.54 585	9.98 309	3	7
54	9.43 769	44	9.45 463	48	0.54 537	9.98 306	4	6
55	9.43 813	44	9.45 511	48	0.54 489	9.98 302	3	**5**
56	9.43 857	44	9.45 559	47	0.54 441	9.98 299	4	4
57	9.43 901	45	9.45 606	48	0.54 394	9.98 295	4	3
58	9.43 946	44	9.45 654	48	0.54 346	9.98 291	3	2
59	9.43 990	44	9.45 702	48	0.54 298	9.98 288	4	1
60	9.44 034		9.45 750		0.54 250	9.98 284		**0**
	L Cos	**d**	**L Ctn**	**c d**	**L Tan**	**L Sin**	**d**	′

Prop. Pts.

	51	50	49
2	10.2	10.0	9.8
3	15.3	15.0	14.7
4	20.4	20.0	19.6
5	25.5	25.0	24.5
6	30.6	30.0	29.4
7	35.7	35.0	34.3
8	40.8	40.0	39.2
9	45.9	45.0	44.1

	48	47	46
2	9.6	9.4	9.2
3	14.4	14.1	13.8
4	19.2	18.8	18.4
5	24.0	23.5	23.0
6	28.8	28.2	27.6
7	33.6	32.9	32.2
8	38.4	37.6	36.8
9	43.2	42.3	41.4

	45	44
2	9.0	8.8
3	13.5	13.2
4	18.0	17.6
5	22.5	22.0
6	27.0	26.4
7	31.5	30.8
8	36.0	35.2
9	40.5	39.6

	4	3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

From the top:

For **15°+** or **195°+**, read as printed; for **105°+** or **285°+**, read co-function.

From the bottom:

For **74°+** or **254°+**, read as printed; for **164°+** or **344°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.44 034	44	9.45 750	47	0.54 250	9.98 284	3	**60**
1	9.44 078	44	9.45 797	48	0.54 203	9.98 281	4	59
2	9.44 122	44	9.45 845	47	0.54 155	9.98 277	4	58
3	9.44 166	44	9.45 892	48	0.54 108	9.98 273	3	57
4	9.44 210	43	9.45 940	47	0.54 060	9.98 270	4	56
5	9.44 253	44	9.45 987	48	0.54 013	9.98 266	4	**55**
6	9.44 297	44	9.46 035	47	0.53 965	9.98 262	3	54
7	9.44 341	44	9.46 082	48	0.53 918	9.98 259	4	53
8	9.44 385	43	9.46 130	47	0.53 870	9.98 255	4	52
9	9.44 428	44	9.46 177	47	0.53 823	9.98 251	3	51
10	9.44 472	44	9.46 224	47	0.53 776	9.98 248	4	**50**
11	9.44 516	43	9.46 271	48	0.53 729	9.98 244	4	49
12	9.44 559	43	9.46 319	47	0.53 681	9.98 240	3	48
13	9.44 602	44	9.46 366	47	0.53 634	9.98 237	4	47
14	9.44 646	43	9.46 413	47	0.53 587	9.98 233	4	46
15	9.44 689	44	9.46 460	47	0.53 540	9.98 229	3	**45**
16	9.44 733	43	9.46 507	47	0.53 493	9.98 226	4	44
17	9.44 776	43	9.46 554	47	0.53 446	9.98 222	4	43
18	9.44 819	43	9.46 601	47	0.53 399	9.98 218	3	42
19	9.44 862	43	9.46 648	46	0.53 352	9.98 215	4	41
20	9.44 905	43	9.46 694	47	0.53 306	9.98 211	4	**40**
21	9.44 948	44	9.46 741	47	0.53 259	9.98 207	3	39
22	9.44 992	43	9.46 788	47	0.53 212	9.98 204	4	38
23	9.45 035	42	9.46 835	46	0.53 165	9.98 200	4	37
24	9.45 077	43	9.46 881	47	0.53 119	9.98 196	4	36
25	9.45 120	43	9.46 928	47	0.53 072	9.98 192	3	**35**
26	9.45 163	43	9.46 975	46	0.53 025	9.98 189	4	34
27	9.45 206	43	9.47 021	47	0.52 979	9.98 185	4	33
28	9.45 249	43	9.47 068	46	0.52 932	9.98 181	4	32
29	9.45 292	42	9.47 114	46	0.52 886	9.98 177	3	31
30	9.45 334	43	9.47 160	47	0.52 840	9.98 174	4	**30**
31	9.45 377	42	9.47 207	46	0.52 793	9.98 170	4	29
32	9.45 419	43	9.47 253	46	0.52 747	9.98 166	4	28
33	9.45 462	42	9.47 299	47	0.52 701	9.98 162	3	27
34	9.45 504	43	9.47 346	46	0.52 654	9.98 159	4	26
35	9.45 547	42	9.47 392	46	0.52 608	9.98 155	4	**25**
36	9.45 589	43	9.47 438	46	0.52 562	9.98 151	4	24
37	9.45 632	42	9.47 484	46	0.52 516	9.98 147	3	23
38	9.45 674	42	9.47 530	46	0.52 470	9.98 144	4	22
39	9.45 716	42	9.47 576	46	0.52 424	9.98 140	4	21
40	9.45 758	43	9.47 622	46	0.52 378	9.98 136	4	**20**
41	9.45 801	42	9.47 668	46	0.52 332	9.98 132	3	19
42	9.45 843	42	9.47 714	46	0.52 286	9.98 129	4	18
43	9.45 885	42	9.47 760	46	0.52 240	9.98 125	4	17
44	9.45 927	42	9.47 806	46	0.52 194	9.98 121	4	16
45	9.45 969	42	9.47 852	45	0.52 148	9.98 117	4	**15**
46	9.46 011	42	9.47 897	46	0.52 103	9.98 113	3	14
47	9.46 053	42	9.47 943	46	0.52 057	9.98 110	4	13
48	9.46 095	41	9.47 989	46	0.52 011	9.98 106	4	12
49	9.46 136	42	9.48 035	45	0.51 965	9.98 102	4	11
50	9.46 178	42	9.48 080	46	0.51 920	9.98 098	4	**10**
51	9.46 220	42	9.48 126	45	0.51 874	9.98 094	4	9
52	9.46 262	41	9.48 171	46	0.51 829	9.98 090	3	8
53	9.46 303	42	9.48 217	45	0.51 783	9.98 087	4	7
54	9.46 345	41	9.48 262	45	0.51 738	9.98 083	4	6
55	9.46 386	42	9.48 307	46	0.51 693	9.98 079	4	**5**
56	9.46 428	41	9.48 353	45	0.51 647	9.98 075	4	4
57	9.46 469	42	9.48 398	45	0.51 602	9.98 071	4	3
58	9.46 511	41	9.48 443	46	0.51 557	9.98 067	4	2
59	9.46 552	42	9.48 489	45	0.51 511	9.98 063	3	1
60	9.46 594		9.48 534		0.51 466	9.98 060		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	48	47	46
2	9.6	9.4	9.2
3	14.4	14.1	13.8
4	19.2	18.8	18.4
5	24.0	23.5	23.0
6	28.8	28.2	27.6
7	33.6	32.9	32.2
8	38.4	37.6	36.8
9	43.2	42.3	41.4

	45	44	43
2	9.0	8.8	8.6
3	13.5	13.2	12.9
4	18.0	17.6	17.2
5	22.5	22.0	21.5
6	27.0	26.4	25.8
7	31.5	30.8	30.1
8	36.0	35.2	34.4
9	40.5	39.6	38.7

	42	41
2	8.4	8.2
3	12.6	12.3
4	16.8	16.4
5	21.0	20.5
6	25.2	24.6
7	29.4	28.7
8	33.6	32.8
9	37.8	36.9

	4	3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

From the top :

For **16°**+ or **196°**+, read as printed; for **106°**+ or **286°**+, read co-function.

From the bottom

For **73°**+ or **253°**+, read as printed ; for **163°**+ or **343°**+, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.46 594	41	9.48 534	45	0.51 466	9.98 060	4	**60**
1	9.46 635	41	9.48 579	45	0.51 421	9.98 056	4	59
2	9.46 676	41	9.48 624	45	0.51 376	9.98 052	4	58
3	9.46 717	41	9.48 669	45	0.51 331	9.98 048	4	57
4	9.46 758	42	9.48 714	45	0.51 286	9.98 044	4	56
5	9.46 800	41	9.48 759	45	0.51 241	9.98 040	4	**55**
6	9.46 841	41	9.48 804	45	0.51 196	9.98 036	4	54
7	9.46 882	41	9.48 849	45	0.51 151	9.98 032	3	53
8	9.46 923	41	9.48 894	45	0.51 106	9.98 029	4	52
9	9.46 964	41	9.48 939	45	0.51 061	9.98 025	4	51
10	9.47 005	40	9.48 984	45	0.51 016	9.98 021	4	**50**
11	9.47 045	41	9.49 029	44	0.50 971	9.98 017	4	49
12	9.47 086	41	9.49 073	45	0.50 927	9.98 013	4	48
13	9.47 127	41	9.49 118	45	0.50 882	9.98 009	4	47
14	9.47 168	41	9.49 163	44	0.50 837	9.98 005	4	46
15	9.47 209	40	9.49 207	45	0.50 793	9.98 001	4	**45**
16	9.47 249	41	9.49 252	44	0.50 748	9.97 997	4	44
17	9.47 290	40	9.49 296	45	0.50 704	9.97 993	4	43
18	9.47 330	41	9.49 341	44	0.50 659	9.97 989	3	42
19	9.47 371	40	9.49 385	45	0.50 615	9.97 986	4	41
20	9.47 411	41	9.49 430	44	0.50 570	9.97 982	4	**40**
21	9.47 452	40	9.49 474	45	0.50 526	9.97 978	4	39
22	9.47 492	41	9.49 519	44	0.50 481	9.97 974	4	38
23	9.47 533	40	9.49 563	44	0.50 437	9.97 970	4	37
24	9.47 573	40	9.49 607	45	0.50 393	9.97 966	4	36
25	9.47 613	41	9.49 652	44	0.50 348	9.97 962	4	**35**
26	9.47 654	40	9.49 696	44	0.50 304	9.97 958	4	34
27	9.47 694	40	9.49 740	44	0.50 260	9.97 954	4	33
28	9.47 734	40	9.49 784	44	0.50 216	9.97 950	4	32
29	9.47 774	40	9.49 828	44	0.50 172	9.97 946	4	31
30	9.47 814	40	9.49 872	44	0.50 128	9.97 942	4	**30**
31	9.47 854	40	9.49 916	44	0.50 084	9.97 938	4	29
32	9.47 894	40	9.49 960	44	0.50 040	9.97 934	4	28
33	9.47 934	40	9.50 004	44	0.49 996	9.97 930	4	27
34	9.47 974	40	9.50 048	44	0.49 952	9.97 926	4	26
35	9.48 014	40	9.50 092	44	0.49 908	9.97 922	4	**25**
36	9.48 054	40	9.50 136	44	0.49 864	9.97 918	4	24
37	9.48 094	39	9.50 180	43	0.49 820	9.97 914	4	23
38	9.48 133	40	9.50 223	44	0.49 777	9.97 910	4	22
39	9.48 173	40	9.50 267	44	0.49 733	9.97 906	4	21
40	9.48 213	39	9.50 311	44	0.49 689	9.97 902	4	**20**
41	9.48 252	40	9.50 355	43	0.49 645	9.97 898	4	19
42	9.48 292	40	9.50 398	44	0.49 602	9.97 894	4	18
43	9.48 332	39	9.50 442	43	0.49 558	9.97 890	4	17
44	9.48 371	40	9.50 485	44	0.49 515	9.97 886	4	16
45	9.48 411	39	9.50 529	43	0.49 471	9.97 882	4	**15**
46	9.48 450	40	9.50 572	44	0.49 428	9.97 878	4	14
47	9.48 490	39	9.50 616	43	0.49 384	9.97 874	4	13
48	9.48 529	39	9.50 659	44	0.49 341	9.97 870	4	12
49	9.48 568	39	9.50 703	43	0.49 297	9.97 866	5	11
50	9.48 607	40	9.50 746	43	0.49 254	9.97 861	4	**10**
51	9.48 647	39	9.50 789	44	0.49 211	9.97 857	4	9
52	9.48 686	39	9.50 833	43	0.49 167	9.97 853	4	8
53	9.48 725	39	9.50 876	43	0.49 124	9.97 849	4	7
54	9.48 764	39	9.50 919	43	0.49 081	9.97 845	4	6
55	9.48 803	39	9.50 962	43	0.49 038	9.97 841	4	**5**
56	9.48 842	39	9.51 005	43	0.48 995	9.97 837	4	4
57	9.48 881	39	9.51 048	44	0.48 952	9.97 833	4	3
58	9.48 920	39	9.51 092	43	0.48 908	9.97 829	4	2
59	9.48 959	39	9.51 135	43	0.48 865	9.97 825	4	1
60	9.48 998		9.51 178		0.48 822	9.97 821		**0**
	L Cos	**d**	**L Ctn**	**c d**	**L Tan**	**L Sin**	**d**	**′**

Prop. Pts.

	45	44	43
2	9.0	8.8	8.6
3	13.5	13.2	12.9
4	18.0	17.6	17.2
5	22.5	22.0	21.5
6	27.0	26.4	25.8
7	31.5	30.8	30.1
8	36.0	35.2	34.4
9	40.5	39.6	38.7

	42	41	40
2	8.4	8.2	8.0
3	12.6	12.3	12.0
4	16.8	16.4	16.0
5	21.0	20.5	20.0
6	25.2	24.6	24.0
7	29.4	28.7	28.0
8	33.6	32.8	32.0
9	37.8	36.9	36.0

	39	5
2	7.8	1.0
3	11.7	1.5
4	15.6	2.0
5	19.5	2.5
6	23.4	3.0
7	27.3	3.5
8	31.2	4.0
9	35.1	4.5

	4	3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

From the top:

For **17°+** or **197°+**, read as printed; for **107°+** or **287°+**, read co-function.

From the bottom:

For **72°+** or **252°+**, read as printed; for **162°+** or **342°+**, read co-function.

Prop. Pts.

'	L Sin	d	L Tan	c d	L Ctn	L Cos	d	'
0	9.48 998	39	9.51 178	43	0.48 822	9.97 821	4	**60**
1	9.49 037	39	9.51 221	43	0.48 779	9.97 817	5	59
2	9.49 076	39	9.51 264	42	0.48 736	9.97 812	4	58
3	9.49 115	38	9.51 306	43	0.48 694	9.97 808	4	57
4	9.49 153	39	9.51 349	43	0.48 651	9.97 804	4	56
5	9.49 192	39	9.51 392	43	0.48 608	9.97 800	4	**55**
6	9.49 231	38	9.51 435	43	0.48 565	9.97 796	4	54
7	9.49 269	39	9.51 478	42	0.48 522	9.97 792	4	53
8	9.49 308	39	9.51 520	43	0.48 480	9.97 788	4	52
9	9.49 347	38	9.51 563	43	0.48 437	9.97 784	5	51
10	9.49 385	39	9.51 606	42	0.48 394	9.97 779	4	**50**
11	9.49 424	38	9.51 648	43	0.48 352	9.97 775	4	49
12	9.49 462	38	9.51 691	43	0.48 309	9.97 771	4	48
13	9.49 500	39	9.51 734	42	0.48 266	9.97 767	4	47
14	9.49 539	38	9.51 776	43	0.48 224	9.97 763	4	46
15	9.49 577	38	9.51 819	42	0.48 181	9.97 759	5	**45**
16	9.49 615	39	9.51 861	42	0.48 139	9.97 754	4	44
17	9.49 654	38	9.51 903	43	0.48 097	9.97 750	4	43
18	9.49 692	38	9.51 946	42	0.48 054	9.97 746	4	42
19	9.49 730	38	9.51 988	43	0.48 012	9.97 742	4	41
20	9.49 768	38	9.52 031	42	0.47 969	9.97 738	4	**40**
21	9.49 806	38	9.52 073	42	0.47 927	9.97 734	5	39
22	9.49 844	38	9.52 115	42	0.47 885	9.97 729	4	38
23	9.49 882	38	9.52 157	43	0.47 843	9.97 725	4	37
24	9.49 920	38	9.52 200	42	0.47 800	9.97 721	4	36
25	9.49 958	38	9.52 242	42	0.47 758	9.97 717	4	**35**
26	9.49 996	38	9.52 284	42	0.47 716	9.97 713	5	34
27	9.50 034	38	9.52 326	42	0.47 674	9.97 708	4	33
28	9.50 072	38	9.52 368	42	0.47 632	9.97 704	4	32
29	9.50 110	38	9.52 410	42	0.47 590	9.97 700	4	31
30	9.50 148	37	9.52 452	42	0.47 548	9.97 696	5	**30**
31	9.50 185	38	9.52 494	42	0.47 506	9.97 691	4	29
32	9.50 223	38	9.52 536	42	0.47 464	9.97 687	4	28
33	9.50 261	37	9.52 578	42	0.47 422	9.97 683	4	27
34	9.50 298	38	9.52 620	41	0.47 380	9.97 679	5	26
35	9.50 336	38	9.52 661	42	0.47 339	9.97 674	4	**25**
36	9.50 374	37	9.52 703	42	0.47 297	9.97 670	4	24
37	9.50 411	38	9.52 745	42	0.47 255	9.97 666	4	23
38	9.50 449	37	9.52 787	42	0.47 213	9.97 662	5	22
39	9.50 486	37	9.52 829	41	0.47 171	9.97 657	4	21
40	9.50 523	38	9.52 870	42	0.47 130	9.97 653	4	**20**
41	9.50 561	37	9.52 912	41	0.47 088	9.97 649	4	19
42	9.50 598	37	9.52 953	42	0.47 047	9.97 645	5	18
43	9.50 635	38	9.52 995	42	0.47 005	9.97 640	4	17
44	9.50 673	37	9.53 037	41	0.46 963	9.97 636	4	16
45	9.50 710	37	9.53 078	42	0.46 922	9.97 632	4	**15**
46	9.50 747	37	9.53 120	41	0.46 880	9.97 628	5	14
47	9.50 784	37	9.53 161	41	0.46 839	9.97 623	4	13
48	9.50 821	37	9.53 202	42	0.46 798	9.97 619	4	12
49	9.50 858	38	9.53 244	41	0.46 756	9.97 615	5	11
50	9.50 896	37	9.53 285	42	0.46 715	9.97 610	4	**10**
51	9.50 933	37	9.53 327	41	0.46 673	9.97 606	4	9
52	9.50 970	37	9.53 368	41	0.46 632	9.97 602	5	8
53	9.51 007	36	9.53 409	41	0.46 591	9.97 597	4	7
54	9.51 043	37	9.53 450	42	0.46 550	9.97 593	4	6
55	9.51 080	37	9.53 492	41	0.46 508	9.97 589	5	**5**
56	9.51 117	37	9.53 533	41	0.46 467	9.97 584	4	4
57	9.51 154	37	9.53 574	41	0.46 426	9.97 580	4	3
58	9.51 191	36	9.53 615	41	0.46 385	9.97 576	5	2
59	9.51 227	37	9.53 656	41	0.46 344	9.97 571	4	1
60	9.51 264		9.53 697		0.46 303	9.97 567		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	'

Prop. Pts.

	43	**42**	**41**
2	8.6	8.4	8.2
3	12.9	12.6	12.3
4	17.2	16.8	16.4
5	21.5	21.0	20.5
6	25.8	25.2	24.6
7	30.1	29.4	28.7
8	34.4	33.6	32.8
9	38.7	37.8	36.9

	39	**38**	**37**
2	7.8	7.6	7.4
3	11.7	11.4	11.1
4	15.6	15.2	14.8
5	19.5	19.0	18.5
6	23.4	22.8	22.2
7	27.3	26.6	25.9
8	31.2	30.4	29.6
9	35.1	34.2	33.3

	36	**5**	**4**
2	7.2	1.0	0.8
3	10.8	1.5	1.2
4	14.4	2.0	1.6
5	18.0	2.5	2.0
6	21.6	3.0	2.4
7	25.2	3.5	2.8
8	28.8	4.0	3.2
9	32.4	4.5	3.6

From the top:

For **18°**+ or **198°**+, read as printed; for **108°**+ or **288°**+, read co-function.

From the bottom:

For **71°**+ or **251°**+, read as printed; for **161°**+ or **341°**+, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.51 264	37	9.53 697	41	0.46 303	9.97 567	4	**60**
1	9.51 301	37	9.53 738	41	0.46 262	9.97 563	5	59
2	9.51 338	36	9.53 779	41	0.46 221	9.97 558	4	58
3	9.51 374	37	9.53 820	41	0.46 180	9.97 554	4	57
4	9.51 411	36	9.53 861	41	0.46 139	9.97 550	5	56
5	9.51 447	37	9.53 902	41	0.46 098	9.97 545	4	**55**
6	9.51 484	36	9.53 943	41	0.46 057	9.97 541	5	54
7	9.51 520	37	9.53 984	41	0.46 016	9.97 536	4	53
8	9.51 557	36	9.54 025	40	0.45 975	9.97 532	4	52
9	9.51 593	36	9.54 065	41	0.45 935	9.97 528	5	51
10	9.51 629	37	9.54 106	41	0.45 894	9.97 523	4	**50**
11	9.51 666	36	9.54 147	40	0.45 853	9.97 519	4	49
12	9.51 702	36	9.54 187	41	0.45 813	9.97 515	5	48
13	9.51 738	36	9.54 228	41	0.45 772	9.97 510	4	47
14	9.51 774	37	9.54 269	40	0.45 731	9.97 506	5	46
15	9.51 811	36	9.54 309	41	0.45 691	9.97 501	4	**45**
16	9.51 847	36	9.54 350	40	0.45 650	9.97 497	5	44
17	9.51 883	36	9.54 390	41	0.45 610	9.97 492	4	43
18	9.51 919	36	9.54 431	40	0.45 569	9.97 488	4	42
19	9.51 955	36	9.54 471	41	0.45 529	9.97 484	5	41
20	9.51 991	36	9.54 512	40	0.45 488	9.97 479	4	**40**
21	9.52 027	36	9.54 552	41	0.45 448	9.97 475	5	39
22	9.52 063	36	9.54 593	40	0.45 407	9.97 470	4	38
23	9.52 099	36	9.54 633	40	0.45 367	9.97 466	5	37
24	9.52 135	36	9.54 673	41	0.45 327	9.97 461	4	36
25	9.52 171	36	9.54 714	40	0.45 286	9.97 457	4	**35**
26	9.52 207	35	9.54 754	40	0.45 246	9.97 453	5	34
27	9.52 242	36	9.54 794	41	0.45 206	9.97 448	4	33
28	9.52 278	36	9.54 835	40	0.45 165	9.97 444	5	32
29	9.52 314	36	9.54 875	40	0.45 125	9.97 439	4	31
30	9.52 350	35	9.54 915	40	0.45 085	9.97 435	5	**30**
31	9.52 385	36	9.54 955	40	0.45 045	9.97 430	4	29
32	9.52 421	35	9.54 995	40	0.45 005	9.97 426	5	28
33	9.52 456	36	9.55 035	40	0.44 965	9.97 421	4	27
34	9.52 492	35	9.55 075	40	0.44 925	9.97 417	5	26
35	9.52 527	36	9.55 115	40	0.44 885	9.97 412	4	**25**
36	9.52 563	35	9.55 155	40	0.44 845	9.97 408	5	24
37	9.52 598	36	9.55 195	40	0.44 805	9.97 403	4	23
38	9.52 634	35	9.55 235	40	0.44 765	9.97 399	5	22
39	9.52 669	36	9.55 275	40	0.44 725	9.97 394	4	21
40	9.52 705	35	9.55 315	40	0.44 685	9.97 390	5	**20**
41	9.52 740	35	9.55 355	40	0.44 645	9.97 385	4	19
42	9.52 775	36	9.55 395	39	0.44 605	9.97 381	5	18
43	9.52 811	35	9.55 434	40	0.44 566	9.97 376	4	17
44	9.52 846	35	9.55 474	40	0.44 526	9.97 372	5	16
45	9.52 881	35	9.55 514	40	0.44 486	9.97 367	4	**15**
46	9.52 916	35	9.55 554	39	0.44 446	9.97 363	5	14
47	9.52 951	35	9.55 593	40	0.44 407	9.97 358	5	13
48	9.52 986	35	9.55 633	40	0.44 367	9.97 353	4	12
49	9.53 021	35	9.55 673	39	0.44 327	9.97 349	5	11
50	9.53 056	36	9.55 712	40	0.44 288	9.97 344	4	**10**
51	9.53 092	34	9.55 752	39	0.44 248	9.97 340	5	9
52	9.53 126	35	9.55 791	40	0.44 209	9.97 335	4	8
53	9.53 161	35	9.55 831	39	0.44 169	9.97 331	5	7
54	9.53 196	35	9.55 870	40	0.44 130	9.97 326	4	6
55	9.53 231	35	9.55 910	39	0.44 090	9.97 322	5	**5**
56	9.53 266	35	9.55 949	40	0.44 051	9.97 317	5	4
57	9.53 301	35	9.55 989	39	0.44 011	9.97 312	4	3
58	9.53 336	34	9.56 028	39	0.43 972	9.97 308	5	2
59	9.53 370	35	9.56 067	40	0.43 933	9.97 303	4	1
60	9.53 405		9.56 107		0.43 893	9.97 299		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	41	40	39
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1

	37	36	35
2	7.4	7.2	7.0
3	11.1	10.8	10.5
4	14.8	14.4	14.0
5	18.5	18.0	17.5
6	22.2	21.6	21.0
7	25.9	25.2	24.5
8	29.6	28.8	28.0
9	33.3	32.4	31.5

	34	5	4
2	6.8	1.0	0.8
3	10.2	1.5	1.2
4	13.6	2.0	1.6
5	17.0	2.5	2.0
6	20.4	3.0	2.4
7	23.8	3.5	2.8
8	27.2	4.0	3.2
9	30.6	4.5	3.6

From the top:

For **19°+** or **199°+**, read as printed; for **109°+** or **289°+**, read co-function.

From the bottom:

For **70°+** or **250°+**, read as printed; for **160°+** or **340°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.53 405	35	9.56 107	39	0.43 893	9.97 299	5	**60**
1	9.53 440	35	9.56 146	39	0.43 854	9.97 294	5	59
2	9.53 475	34	9.56 185	39	0.43 815	9.97 289	4	58
3	9.53 509	35	9.56 224	40	0.43 776	9.97 285	5	57
4	9.53 544	34	9.56 264	39	0.43 736	9.97 280	4	56
5	9.53 578	35	9.56 303	39	0.43 697	9.97 276	5	**55**
6	9.53 613	34	9.56 342	39	0.43 658	9.97 271	5	54
7	9.53 647	35	9.56 381	39	0.43 619	9.97 266	4	53
8	9.53 682	34	9.56 420	39	0.43 580	9.97 262	5	52
9	9.53 716	35	9.56 459	39	0.43 541	9.97 257	5	51
10	9.53 751	34	9.56 498	39	0.43 502	9.97 252	4	**50**
11	9.53 785	34	9.56 537	39	0.43 463	9.97 248	5	49
12	9.53 819	35	9.56 576	39	0.43 424	9.97 243	5	48
13	9.53 854	34	9.56 615	39	0.43 385	9.97 238	4	47
14	9.53 888	34	9.56 654	39	0.43 346	9.97 234	5	46
15	9.53 922	35	9.56 693	39	0.43 307	9.97 229	5	**45**
16	9.53 957	34	9.56 732	39	0.43 268	9.97 224	4	44
17	9.53 991	34	9.56 771	39	0.43 229	9.97 220	5	43
18	9.54 025	34	9.56 810	39	0.43 190	9.97 215	5	42
19	9.54 059	34	9.56 849	38	0.43 151	9.97 210	4	41
20	9.54 093	34	9.56 887	39	0.43 113	9.97 206	5	**40**
21	9.54 127	34	9.56 926	39	0.43 074	9.97 201	5	39
22	9.54 161	34	9.56 965	39	0.43 035	9.97 196	4	38
23	9.54 195	34	9.57 004	38	0.42 996	9.97 192	5	37
24	9.54 229	34	9.57 042	39	0.42 958	9.97 187	5	36
25	9.54 263	34	9.57 081	39	0.42 919	9.97 182	4	**35**
26	9.54 297	34	9.57 120	38	0.42 880	9.97 178	5	34
27	9.54 331	34	9.57 158	39	0.42 842	9.97 173	5	33
28	9.54 365	34	9.57 197	38	0.42 803	9.97 168	5	32
29	9.54 399	34	9.57 235	39	0.42 765	9.97 163	4	31
30	9.54 433	33	9.57 274	38	0.42 726	9.97 159	5	**30**
31	9.54 466	34	9.57 312	39	0.42 688	9.97 154	5	29
32	9.54 500	34	9.57 351	38	0.42 649	9.97 149	4	28
33	9.54 534	33	9.57 389	39	0.42 611	9.97 145	5	27
34	9.54 567	34	9.57 428	38	0.42 572	9.97 140	5	26
35	9.54 601	34	9.57 466	38	0.42 534	9.97 135	5	**25**
36	9.54 635	33	9.57 504	39	0.42 496	9.97 130	4	24
37	9.54 668	34	9.57 543	38	0.42 457	9.97 126	5	23
38	9.54 702	33	9.57 581	38	0.42 419	9.97 121	5	22
39	9.54 735	34	9.57 619	39	0.42 381	9.97 116	5	21
40	9.54 769	33	9.57 658	38	0.42 342	9.97 111	4	**20**
41	9.54 802	34	9.57 696	38	0.42 304	9.97 107	5	19
42	9.54 836	33	9.57 734	38	0.42 266	9.97 102	5	18
43	9.54 869	34	9.57 772	38	0.42 228	9.97 097	5	17
44	9.54 903	33	9.57 810	39	0.42 190	9.97 092	5	16
45	9.54 936	33	9.57 849	38	0.42 151	9.97 087	4	**15**
46	9.54 969	34	9.57 887	38	0.42 113	9.97 083	5	14
47	9.55 003	33	9.57 925	38	0.42 075	9.97 078	5	13
48	9.55 036	33	9.57 963	38	0.42 037	9.97 073	5	12
49	9.55 069	33	9.58 001	38	0.41 999	9.97 068	5	11
50	9.55 102	34	9.58 039	38	0.41 961	9.97 063	4	**10**
51	9.55 136	33	9.58 077	38	0.41 923	9.97 059	5	9
52	9.55 169	33	9.58 115	38	0.41 885	9.97 054	5	8
53	9.55 202	33	9.58 153	38	0.41 847	9.97 049	5	7
54	9.55 235	33	9.58 191	38	0.41 809	9.97 044	5	6
55	9.55 268	33	9.58 229	38	0.41 771	9.97 039	4	**5**
56	9.55 301	33	9.58 267	37	0.41 733	9.97 035	5	4
57	9.55 334	33	9.58 304	38	0.41 696	9.97 030	5	3
58	9.55 367	33	9.58 342	38	0.41 658	9.97 025	5	2
59	9.55 400	33	9.58 380	38	0.41 620	9.97 020	5	1
60	9.55 433		9.58 418		0.41 582	9.97 015		**0**
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	40	**39**	**38**
2	8.0	7.8	7.6
3	12.0	11.7	11.4
4	16.0	15.6	15.2
5	20.0	19.5	19.0
6	24.0	23.4	22.8
7	28.0	27.3	26.6
8	32.0	31.2	30.4
9	36.0	35.1	34.2

	37	**35**	**34**
2	7.4	7.0	6.8
3	11.1	10.5	10.2
4	14.8	14.0	13.6
5	18.5	17.5	17.0
6	22.2	21.0	20.4
7	25.9	24.5	23.8
8	29.6	28.0	27.2
9	33.3	31.5	30.6

	33	**5**	**4**
2	6.6	1.0	0.8
3	9.9	1.5	1.2
4	13.2	2.0	1.6
5	16.5	2.5	2.0
6	19.8	3.0	2.4
7	23.1	3.5	2.8
8	26.4	4.0	3.2
9	29.7	4.5	3.6

From the top:

For 20°+ or 200°+, read as printed; for **110°+ or 290°+**, read co-function.

From the bottom:

For 69°+ or 249°+, read as printed; for **159°+ or 339°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.55 433	33	9.58 418	37	0.41 582	9.97 015	5	**60**
1	9.55 466	33	9.58 455	38	0.41 545	9.97 010	5	59
2	9.55 499	33	9.58 493	38	0.41 507	9.97 005	4	58
3	9.55 532	32	9.58 531	38	0.41 469	9.97 001	5	57
4	9.55 564	33	9.58 569	37	0.41 431	9.96 996	5	56
5	9.55 597	33	9.58 606	38	0.41 394	9.96 991	5	**55**
6	9.55 630	33	9.58 644	37	0.41 356	9.96 986	5	54
7	9.55 663	32	9.58 681	38	0.41 319	9.96 981	5	53
8	9.55 695	33	9.58 719	38	0.41 281	9.96 976	5	52
9	9.55 728	33	9.58 757	37	0.41 243	9.96 971	5	51
10	9.55 761	32	9.58 794	38	0.41 206	9.96 966	4	**50**
11	9.55 793	33	9.58 832	37	0.41 168	9.96 962	5	49
12	9.55 826	32	9.58 869	38	0.41 131	9.96 957	5	48
13	9.55 858	33	9.58 907	37	0.41 093	9.96 952	5	47
14	9.55 891	32	9.58 944	37	0.41 056	9.96 947	5	46
15	9.55 923	33	9.58 981	38	0.41 019	9.96 942	5	**45**
16	9.55 956	32	9.59 019	37	0.40 981	9.96 937	5	44
17	9.55 988	33	9.59 056	38	0.40 944	9.96 932	5	43
18	9.56 021	32	9.59 094	37	0.40 906	9.96 927	5	42
19	9.56 053	32	9.59 131	37	0.40 869	9.96 922	5	41
20	9.56 085	33	9.59 168	37	0.40 832	9.96 917	5	**40**
21	9.56 118	32	9.59 205	38	0.40 795	9.96 912	5	39
22	9.56 150	32	9.59 243	37	0.40 757	9.96 907	4	38
23	9.56 182	33	9.59 280	37	0.40 720	9.96 903	5	37
24	9.56 215	32	9.59 317	37	0.40 683	9.96 898	5	36
25	9.56 247	32	9.59 354	37	0.40 646	9.96 893	5	**35**
26	9.56 279	32	9.59 391	38	0.40 609	9.96 888	5	34
27	9.56 311	32	9.59 429	37	0.40 571	9.96 883	5	33
28	9.56 343	32	9.59 466	37	0.40 534	9.96 878	5	32
29	9.56 375	33	9.59 503	37	0.40 497	9.96 873	5	31
30	9.56 408	32	9.59 540	37	0.40 460	9.96 868	5	**30**
31	9.56 440	32	9.59 577	37	0.40 423	9.96 863	5	29
32	9.56 472	32	9.59 614	37	0.40 386	9.96 858	5	28
33	9.56 504	32	9.59 651	37	0.40 349	9.96 853	5	27
34	9.56 536	32	9.59 688	37	0.40 312	9.96 848	5	26
35	9.56 568	31	9.59 725	37	0.40 275	9.96 843	5	**25**
36	9.56 599	32	9.59 762	37	0.40 238	9.96 838	5	24
37	9.56 631	32	9.59 799	36	0.40 201	9.96 833	5	23
38	9.56 663	32	9.59 835	37	0.40 165	9.96 828	5	22
39	9.56 695	32	9.59 872	37	0.40 128	9.96 823	5	21
40	9.56 727	32	9.59 909	37	0.40 091	9.96 818	5	**20**
41	9.56 759	31	9.59 946	37	0.40 054	9.96 813	5	19
42	9.56 790	32	9.59 983	36	0.40 017	9.96 808	5	18
43	9.56 822	32	9.60 019	37	0.39 981	9.96 803	5	17
44	9.56 854	32	9.60 056	37	0.39 944	9.96 798	5	16
45	9.56 886	31	9.60 093	37	0.39 907	9.96 793	5	**15**
46	9.56 917	32	9.60 130	36	0.39 870	9.96 788	5	14
47	9.56 949	31	9.60 166	37	0.39 834	9.96 783	5	13
48	9.56 980	32	9.60 203	37	0.39 797	9.96 778	6	12
49	9.57 012	32	9.60 240	36	0.39 760	9.96 772	5	11
50	9.57 044	31	9.60 276	37	0.39 724	9.96 767	5	**10**
51	9.57 075	32	9.60 313	36	0.39 687	9.96 762	5	9
52	9.57 107	31	9.60 349	37	0.39 651	9.96 757	5	8
53	9.57 138	31	9.60 386	36	0.39 614	9.96 752	5	7
54	9.57 169	32	9.60 422	37	0.39 578	9.96 747	5	6
55	9.57 201	31	9.60 459	36	0.39 541	9.96 742	5	**5**
56	9.57 232	32	9.60 495	37	0.39 505	9.96 737	5	4
57	9.57 264	31	9.60 532	36	0.39 468	9.96 732	5	3
58	9.57 295	31	9.60 568	37	0.39 432	9.96 727	5	2
59	9.57 326	32	9.60 605	36	0.39 395	9.96 722	5	1
60	9.57 358		9.60 641		0.39 359	9.96 717		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	38	37	36
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4

	33	32	31
2	6.6	6.4	6.2
3	9.9	9.6	9.3
4	13.2	12.8	12.4
5	16.5	16.0	15.5
6	19.8	19.2	18.6
7	23.1	22.4	21.7
8	26.4	25.6	24.8
9	29.7	28.8	27.9

	6	5	4
2	1.2	1.0	0.8
3	1.8	1.5	1.2
4	2.4	2.0	1.6
5	3.0	2.5	2.0
6	3.6	3.0	2.4
7	4.2	3.5	2.8
8	4.8	4.0	3.2
9	5.4	4.5	3.6

From the top:

For **21°**+ or **201°**+, read as printed; for **111°**+ or **291°**+, read co-function.

From the bottom:

For **68°**+ or **248°**+, read as printed; for **158°**+ or **338°**+, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.57 358	31	9.60 641	36	0.39 359	9.96 717	6	**60**
1	9.57 389	31	9.60 677	37	0.39 323	9.96 711	5	59
2	9.57 420	31	9.60 714	36	0.39 286	9.96 706	5	58
3	9.57 451	31	9.60 750	36	0.39 250	9.96 701	5	57
4	9.57 482	32	9.60 786	37	0.39 214	9.96 696	5	56
5	9.57 514	31	9.60 823	36	0.39 177	9.96 691	5	**55**
6	9.57 545	31	9.60 859	36	0.39 141	9.96 686	5	54
7	9.57 576	31	9.60 895	36	0.39 105	9.96 681	5	53
8	9.57 607	31	9.60 931	36	0.39 069	9.96 676	6	52
9	9.57 638	31	9.60 967	37	0.39 033	9.96 670	5	51
10	9.57 669	31	9.61 004	36	0.38 996	9.96 665	5	**50**
11	9.57 700	31	9.61 040	36	0.38 960	9.96 660	5	49
12	9.57 731	31	9.61 076	36	0.38 924	9.96 655	5	48
13	9.57 762	31	9.61 112	36	0.38 888	9.96 650	5	47
14	9.57 793	31	9.61 148	36	0.38 852	9.96 645	5	46
15	9.57 824	31	9.61 184	36	0.38 816	9.96 640	6	**45**
16	9.57 855	30	9.61 220	36	0.38 780	9.96 634	5	44
17	9.57 885	31	9.61 256	36	0.38 744	9.96 629	5	43
18	9.57 916	31	9.61 292	36	0.38 708	9.96 624	5	42
19	9.57 947	31	9.61 328	36	0.38 672	9.96 619	5	41
20	9.57 978	30	9.61 364	36	0.38 636	9.96 614	6	**40**
21	9.58 008	31	9.61 400	36	0.38 600	9.96 608	5	39
22	9.58 039	31	9.61 436	36	0.38 564	9.96 603	5	38
23	9.58 070	31	9.61 472	36	0.38 528	9.96 598	5	37
24	9.58 101	30	9.61 508	36	0.38 492	9.96 593	5	36
25	9.58 131	31	9.61 544	35	0.38 456	9.96 588	6	**35**
26	9.58 162	30	9.61 579	36	0.38 421	9.96 582	5	34
27	9.58 192	31	9.61 615	36	0.38 385	9.96 577	5	33
28	9.58 223	30	9.61 651	36	0.38 349	9.96 572	5	32
29	9.58 253	31	9.61 687	35	0.38 313	9.96 567	5	31
30	9.58 284	30	9.61 722	36	0.38 278	9.96 562	6	**30**
31	9.58 314	31	9.61 758	36	0.38 242	9.96 556	5	29
32	9.58 345	30	9.61 794	36	0.38 206	9.96 551	5	28
33	9.58 375	31	9.61 830	35	0.38 170	9.96 546	5	27
34	9.58 406	30	9.61 865	36	0.38 135	9.96 541	6	26
35	9.58 436	31	9.61 901	35	0.38 099	9.96 535	5	**25**
36	9.58 467	30	9.61 936	36	0.38 064	9.96 530	5	24
37	9.58 497	30	9.61 972	36	0.38 028	9.96 525	5	23
38	9.58 527	30	9.62 008	35	0.37 992	9.96 520	6	22
39	9.58 557	31	9.62 043	36	0.37 957	9.96 514	5	21
40	9.58 588	30	9.62 079	35	0.37 921	9.96 509	5	**20**
41	9.58 618	30	9.62 114	36	0.37 886	9.96 504	6	19
42	9.58 648	30	9.62 150	35	0.37 850	9.96 498	5	18
43	9.58 678	31	9.62 185	36	0.37 815	9.96 493	5	17
44	9.58 709	30	9.62 221	35	0.37 779	9.96 488	5	16
45	9.58 739	30	9.62 256	36	0.37 744	9.96 483	6	**15**
46	9.58 769	30	9.62 292	35	0.37 708	9.96 477	5	14
47	9.58 799	30	9.62 327	35	0.37 673	9.96 472	5	13
48	9.58 829	30	9.62 362	36	0.37 638	9.96 467	6	12
49	9.58 859	30	9.62 398	35	0.37 602	9.96 461	5	11
50	9.58 889	30	9.62 433	35	0.37 567	9.96 456	5	**10**
51	9.58 919	30	9.62 468	36	0.37 532	9.96 451	6	9
52	9.58 949	30	9.62 504	35	0.37 496	9.96 445	5	8
53	9.58 979	30	9.62 539	35	0.37 461	9.96 440	5	7
54	9.59 009	30	9.62 574	35	0.37 426	9.96 435	6	6
55	9.59 039	30	9.62 609	36	0.37 391	9.96 429	5	**5**
56	9.59 069	29	9.62 645	35	0.37 355	9.96 424	5	4
57	9.59 098	30	9.62 680	35	0.37 320	9.96 419	6	3
58	9.59 128	30	9.62 715	35	0.37 285	9.96 413	5	2
59	9.59 156	30	9.62 750	35	0.37 250	9.96 408	5	1
60	9.59 188		9.62 785		0.37 215	9.96 403		**0**
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	37	36	35
2	7.4	7.2	7.0
3	11.1	10.8	10.5
4	14.8	14.4	14.0
5	18.5	18.0	17.5
6	22.2	21.6	21.0
7	25.9	25.2	24.5
8	29.6	28.8	28.0
9	33.3	32.4	31.5

	32	31	30
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

	29	6	5
2	5.8	1.2	1.0
3	8.7	1.8	1.5
4	11.6	2.4	2.0
5	14.5	3.0	2.5
6	17.4	3.6	3.0
7	20.3	4.2	3.5
8	23.2	4.8	4.0
9	26.1	5.4	4.5

From the top:

For **22°+** or **202°+**, read as printed; for **112°+** or **292°+**, read co-function.

From the bottom:

For **67°+** or **247°+**, read as printed; for **157°+** or **337°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.59 188	30	9.62 785	35	0.37 215	9.96 403	6	**60**
1	9.59 218	29	9.62 820	35	0.37 180	9.96 397	5	59
2	9.59 247	30	9.62 855	35	0.37 145	9.96 392	5	58
3	9.59 277	30	9.62 890	36	0.37 110	9.96 387	6	57
4	9.59 307	29	9.62 926	35	0.37 074	9.96 381	5	56
5	9.59 336	30	9.62 961	35	0.37 039	9.96 376	6	**55**
6	9.59 366	30	9.62 996	35	0.37 004	9.96 370	5	54
7	9.59 396	29	9.63 031	35	0.36 969	9.96 365	5	53
8	9.59 425	30	9.63 066	35	0.36 934	9.96 360	6	52
9	9.59 455	29	9.63 101	34	0.36 899	9.96 354	5	51
10	9.59 484	30	9.63 135	35	0.36 865	9.96 349	6	**50**
11	9.59 514	29	9.63 170	35	0.36 830	9.96 343	5	49
12	9.59 543	30	9.63 205	35	0.36 795	9.96 338	5	48
13	9.59 573	29	9.63 240	35	0.36 760	9.96 333	6	47
14	9.59 602	30	9.63 275	35	0.36 725	9.96 327	5	46
15	9.59 632	29	9.63 310	35	0.36 690	9.96 322	6	**45**
16	9.59 661	29	9.63 345	34	0.36 655	9.96 316	5	44
17	9.59 690	30	9.63 379	35	0.36 621	9.96 311	6	43
18	9.59 720	29	9.63 414	35	0.36 586	9.96 305	5	42
19	9.59 749	29	9.63 449	35	0.36 551	9.96 300	6	41
20	9.59 778	30	9.63 484	35	0.36 516	9.96 294	5	**40**
21	9.59 808	29	9.63 519	34	0.36 481	9.96 289	5	39
22	9.59 837	29	9.63 553	35	0.36 447	9.96 284	6	38
23	9.59 866	29	9.63 588	35	0.36 412	9.96 278	5	37
24	9.59 895	29	9.63 623	34	0.36 377	9.96 273	6	36
25	9.59 924	30	9.63 657	35	0.36 343	9.96 267	5	**35**
26	9.59 954	29	9.63 692	34	0.36 308	9.96 262	6	34
27	9.59 983	29	9.63 726	35	0.36 274	9.96 256	5	33
28	9.60 012	29	9.63 761	35	0.36 239	9.96 251	6	32
29	9.60 041	29	9.63 796	34	0.36 204	9.96 245	5	31
30	9.60 070	29	9.63 830	35	0.36 170	9.96 240	6	**30**
31	9.60 099	29	9.63 865	34	0.36 135	9.96 234	5	29
32	9.60 128	29	9.63 899	35	0.36 101	9.96 229	6	28
33	9.60 157	29	9.63 934	34	0.36 066	9.96 223	5	27
34	9.60 186	29	9.63 968	35	0.36 032	9.96 218	6	26
35	9.60 215	29	9.64 003	34	0.35 997	9.96 212	5	**25**
36	9.60 244	29	9.64 037	35	0.35 963	9.96 207	6	24
37	9.60 273	29	9.64 072	34	0.35 928	9.96 201	5	23
38	9.60 302	29	9.64 106	34	0.35 894	9.96 196	6	22
39	9.60 331	28	9.64 140	35	0.35 860	9.96 190	5	21
40	9.60 359	29	9.64 175	34	0.35 825	9.96 185	6	**20**
41	9.60 388	29	9.64 209	34	0.35 791	9.96 179	5	19
42	9.60 417	29	9.64 243	35	0.35 757	9.96 174	6	18
43	9.60 446	28	9.64 278	34	0.35 722	9.96 168	6	17
44	9.60 474	29	9.64 312	34	0.35 688	9.96 162	5	16
45	9.60 503	29	9.64 346	35	0.35 654	9.96 157	6	**15**
46	9.60 532	29	9.64 381	34	0.35 619	9.96 151	5	14
47	9.60 561	28	9.64 415	34	0.35 585	9.96 146	6	13
48	9.60 589	29	9.64 449	34	0.35 551	9.96 140	5	12
49	9.60 618	28	9.64 483	34	0.35 517	9.96 135	6	11
50	9.60 646	29	9.64 517	35	0.35 483	9.96 129	6	**10**
51	9.60 675	29	9.64 552	34	0.35 448	9.96 123	5	9
52	9.60 704	28	9.64 586	34	0.35 414	9.96 118	6	8
53	9.60 732	29	9.64 620	34	0.35 380	9.96 112	5	7
54	9.60 761	28	9.64 654	34	0.35 346	9.96 107	6	6
55	9.60 789	29	9.64 688	34	0.35 312	9.96 101	6	**5**
56	9.60 818	28	9.64 722	34	0.35 278	9.96 095	5	4
57	9.60 846	29	9.64 756	34	0.35 244	9.96 090	6	3
58	9.60 875	28	9.64 790	34	0.35 210	9.96 084	5	2
59	9.60 903	28	9.64 824	34	0.35 176	9.96 079	6	1
60	9.60 931		9.64 858		0.35 142	9.96 073		**0**
	L Cos	**d**	**L Ctn**	**c d**	**L Tan**	**L Sin**	**d**	**′**

Prop. Pts.

	36	35	34
2	7.2	7.0	6.8
3	10.8	10.5	10.2
4	14.4	14.0	13.6
5	18.0	17.5	17.0
6	21.6	21.0	20.4
7	25.2	24.5	23.8
8	28.8	28.0	27.2
9	32.4	31.5	30.6

	30	29	28
2	6.0	5.8	5.6
3	9.0	8.7	8.4
4	12.0	11.6	11.2
5	15.0	14.5	14.0
6	18.0	17.4	16.8
7	21.0	20.3	19.6
8	24.0	23.2	22.4
9	27.0	26.1	25.2

	6	5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

From the top:

For **23°+** or **203°+**, read as printed; for **113°+** or **293°+**, read co-function.

From the bottom:

For **66°+** or **246°+**, read as printed; for **156°+** or **336°+**, read co-function.

Prop. Pts.

	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.60 931	29	9.64 858	34	0.35 142	9.96 073	6	**60**
1	9.60 960	28	9.64 892	34	0.35 108	9.96 067	5	59
2	9.60 988	28	9.64 926	34	0.35 074	9.96 062	6	58
3	9.61 016	29	9.64 960	34	0.35 040	9.96 056	6	57
4	9.61 045	28	9.64 994	34	0.35 006	9.96 050	5	56
5	9.61 073	28	9.65 028	34	0.34 972	9.96 045	6	**55**
6	9.61 101	28	9.65 062	34	0.34 938	9.96 039	5	54
7	9.61 129	29	9.65 096	34	0.34 904	9.96 034	6	53
8	9.61 158	28	9.65 130	34	0.34 870	9.96 028	6	52
9	9.61 186	28	9.65 164	33	0.34 836	9.96 022	5	51
10	9.61 214	28	9.65 197	34	0.34 803	9.96 017	6	**50**
11	9.61 242	28	9.65 231	34	0.34 769	9.96 011	6	49
12	9.61 270	28	9.65 265	34	0.34 735	9.96 005	5	48
13	9.61 298	28	9.65 299	34	0.34 701	9.96 000	6	47
14	9.61 326	28	9.65 333	33	0.34 667	9.95 994	6	46
15	9.61 354	28	9.65 366	34	0.34 634	9.95 988	6	**45**
16	9.61 382	29	9.65 400	34	0.34 600	9.95 982	5	44
17	9.61 411	27	9.65 434	33	0.34 566	9.95 977	6	43
18	9.61 438	28	9.65 467	34	0.34 533	9.95 971	6	42
19	9.61 466	28	9.65 501	34	0.34 499	9.95 965	5	41
20	9.61 494	28	9.65 535	33	0.34 465	9.95 960	6	**40**
21	9.61 522	28	9.65 568	34	0.34 432	9.95 954	6	39
22	9.61 550	28	9.65 602	34	0.34 398	9.95 948	6	38
23	9.61 578	28	9.65 636	33	0.34 364	9.95 942	5	37
24	9.61 606	28	9.65 669	34	0.34 331	9.95 937	6	36
25	9.61 634	28	9.65 703	33	0.34 297	9.95 931	6	**35**
26	9.61 662	27	9.65 736	34	0.34 264	9.95 925	5	34
27	9.61 689	28	9.65 770	33	0.34 230	9.95 920	6	33
28	9.61 717	28	9.65 803	34	0.34 197	9.95 914	6	32
29	9.61 745	28	9.65 837	33	0.34 163	9.95 908	6	31
30	9.61 773	27	9.65 870	34	0.34 130	9.95 902	5	**30**
31	9.61 800	28	9.65 904	33	0.34 096	9.95 897	6	29
32	9.61 828	28	9.65 937	34	0.34 063	9.95 891	6	28
33	9.61 856	27	9.65 971	33	0.34 029	9.95 885	6	27
34	9.61 883	28	9.66 004	34	0.33 996	9.95 879	6	26
35	9.61 911	28	9.66 038	33	0.33 962	9.95 873	5	**25**
36	9.61 939	27	9.66 071	33	0.33 929	9.95 868	6	24
37	9.61 966	28	9.66 104	34	0.33 896	9.95 862	6	23
38	9.61 994	27	9.66 138	33	0.33 862	9.95 856	6	22
39	9.62 021	28	9.66 171	33	0.33 829	9.95 850	6	21
40	9.62 049	27	9.66 204	34	0.33 796	9.95 844	5	**20**
41	9.62 076	28	9.66 238	33	0.33 762	9.95 839	6	19
42	9.62 104	27	9.66 271	33	0.33 729	9.95 833	6	18
43	9.62 131	28	9.66 304	33	0.33 696	9.95 827	6	17
44	9.62 159	27	9.66 337	34	0.33 663	9.95 821	6	16
45	9.62 186	28	9.66 371	33	0.33 629	9.95 815	5	**15**
46	9.62 214	27	9.66 404	33	0.33 596	9.95 810	6	14
47	9.62 241	27	9.66 437	33	0.33 563	9.95 804	6	13
48	9.62 268	28	9.66 470	33	0.33 530	9.95 798	6	12
49	9.62 296	27	9.66 503	34	0.33 497	9.95 792	6	11
50	9.62 323	27	9.66 537	33	0.33 463	9.95 786	6	**10**
51	9.62 350	27	9.66 570	33	0.33 430	9.95 780	5	9
52	9.62 377	28	9.66 603	33	0.33 397	9.95 775	6	8
53	9.62 405	27	9.66 636	33	0.33 364	9.95 769	6	7
54	9.62 432	27	9.66 669	33	0.33 331	9.95 763	6	6
55	9.62 459	27	9.66 702	33	0.33 298	9.95 757	6	**5**
56	9.62 486	27	9.66 735	33	0.33 265	9.95 751	6	4
57	9.62 513	28	9.66 768	33	0.33 232	9.95 745	6	3
58	9.62 541	27	9.66 801	33	0.33 199	9.95 739	6	2
59	9.62 568	27	9.66 834	33	0.33 166	9.95 733	5	1
60	9.62 595		9.66 867		0.33 133	9.95 728		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	'

Prop. Pts.

	34	33	29
2	6.8	6.6	5.8
3	10.2	9.9	8.7
4	13.6	13.2	11.6
5	17.0	16.5	14.5
6	20.4	19.8	17.4
7	23.8	23.1	20.3
8	27.2	26.4	23.2
9	30.6	29.7	26.1

	28	27
2	5.6	5.4
3	8.4	8.1
4	11.2	10.8
5	14.0	13.5
6	16.8	16.2
7	19.6	18.9
8	22.4	21.6
9	25.2	24.3

	6	5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

From the top:

For **24°+** or **204°+**, read as printed; for **114°+** or **294°+**, read co-function.

From the bottom:

For **65°+** or **245°+**, read as printed; for **155°+** or **335°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.62 595	27	9.66 867	33	0.33 133	9.95 728	6	**60**
1	9.62 622	27	9.66 900	33	0.33 100	9.95 722	6	59
2	9.62 649	27	9.66 933	33	0.33 067	9.95 716	6	58
3	9.62 676	27	9.66 966	33	0.33 034	9.95 710	6	57
4	9.62 703	27	9.66 999	33	0.33 001	9.95 704	6	56
5	9.62 730	27	9.67 032	33	0.32 968	9.95 698	6	**55**
6	9.62 757	27	9.67 065	33	0.32 935	9.95 692	6	54
7	9.62 784	27	9.67 098	33	0.32 902	9.95 686	6	53
8	9.62 811	27	9.67 131	32	0.32 869	9.95 680	6	52
9	9.62 838	27	9.67 163	33	0.32 837	9.95 674	6	51
10	9.62 865	27	9.67 196	33	0.32 804	9.95 668	5	**50**
11	9.62 892	26	9.67 229	33	0.32 771	9.95 663	6	49
12	9.62 918	27	9.67 262	33	0.32 738	9.95 657	6	48
13	9.62 945	27	9.67 295	32	0.32 705	9.95 651	6	47
14	9.62 972	27	9.67 327	33	0.32 673	9.95 645	6	46
15	9.62 999	27	9.67 360	33	0.32 640	9.95 639	6	**45**
16	9.63 026	26	9.67 393	33	0.32 607	9.95 633	6	44
17	9.63 052	27	9.67 426	32	0.32 574	9.95 627	6	43
18	9.63 079	27	9.67 458	33	0.32 542	9.95 621	6	42
19	9.63 106	27	9.67 491	33	0.32 509	9.95 615	6	41
20	9.63 133	26	9.67 524	32	0.32 476	9.95 609	6	**40**
21	9.63 159	27	9.67 556	33	0.32 444	9.95 603	6	39
22	9.63 186	27	9.67 589	33	0.32 411	9.95 597	6	38
23	9.63 213	26	9.67 622	32	0.32 378	9.95 591	6	37
24	9.63 239	27	9.67 654	33	0.32 346	9.95 585	6	36
25	9.63 266	26	9.67 687	32	0.32 313	9.95 579	6	**35**
26	9.63 292	27	9.67 719	33	0.32 281	9.95 573	6	34
27	9.63 319	26	9.67 752	33	0.32 248	9.95 567	6	33
28	9.63 345	27	9.67 785	32	0.32 215	9.95 561	6	32
29	9.63 372	26	9.67 817	33	0.32 183	9.95 555	6	31
30	9.63 398	27	9.67 850	32	0.32 150	9.95 549	6	**30**
31	9.63 425	26	9.67 882	33	0.32 118	9.95 543	6	29
32	9.63 451	27	9.67 915	32	0.32 085	9.95 537	6	28
33	9.63 478	26	9.67 947	33	0.32 053	9.95 531	6	27
34	9.63 504	27	9.67 980	32	0.32 020	9.95 525	6	26
35	9.63 531	26	9.68 012	32	0.31 988	9.95 519	6	**25**
36	9.63 557	26	9.68 044	33	0.31 956	9.95 513	6	24
37	9.63 583	27	9.68 077	32	0.31 923	9.95 507	7	23
38	9.63 610	26	9.68 109	33	0.31 891	9.95 500	6	22
39	9.63 636	26	9.68 142	32	0.31 858	9.95 494	6	21
40	9.63 662	27	9.68 174	32	0.31 826	9.95 488	6	**20**
41	9.63 689	26	9.68 206	33	0.31 794	9.95 482	6	19
42	9.63 715	26	9.68 239	32	0.31 761	9.95 476	6	18
43	9.63 741	26	9.68 271	32	0.31 729	9.95 470	6	17
44	9.63 767	27	9.68 303	33	0.31 697	9.95 464	6	16
45	9.63 794	26	9.68 336	32	0.31 664	9.95 458	6	**15**
46	9.63 820	26	9.68 368	32	0.31 632	9.95 452	6	14
47	9.63 846	26	9.68 400	32	0.31 600	9.95 446	6	13
48	9.63 872	26	9.68 432	33	0.31 568	9.95 440	6	12
49	9.63 898	26	9.68 465	32	0.31 535	9.95 434	7	11
50	9.63 924	26	9.68 497	32	0.31 503	9.95 427	6	**10**
51	9.63 950	26	9.68 529	32	0.31 471	9.95 421	6	9
52	9.63 976	26	9.68 561	32	0.31 439	9.95 415	6	8
53	9.64 002	26	9.68 593	33	0.31 407	9.95 409	6	7
54	9.64 028	26	9.68 626	32	0.31 374	9.95 403	6	6
55	9.64 054	26	9.68 658	32	0.31 342	9.95 397	6	**5**
56	9.64 080	26	9.68 690	32	0.31 310	9.95 391	7	4
57	9.64 106	26	9.68 722	32	0.31 278	9.95 384	6	3
58	9.64 132	26	9.68 754	32	0.31 246	9.95 378	6	2
59	9.64 158	26	9.68 786	32	0.31 214	9.95 372	6	1
60	9.64 184		9.68 818		0.31 182	9.95 366		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	33	32	27
2	6.6	6.4	5.4
3	9.9	9.6	8.1
4	13.2	12.8	10.8
5	16.5	16.0	13.5
6	19.8	19.2	16.2
7	23.1	22.4	18.9
8	26.4	25.6	21.6
9	29.7	28.8	24.3

	26	7
2	5.2	1.4
3	7.8	2.1
4	10.4	2.8
5	13.0	3.5
6	15.6	4.2
7	18.2	4.9
8	20.8	5.6
9	23.4	6.3

	6	5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

From the top:

For **25°+** or **205°+**, read as printed; for **115°+** or **295°+**, read co-function.

From the bottom:

For **64°+** or **244°+**, read as printed; for **154°+** or **334°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.64 184	26	9.68 818	32	0.31 182	9.95 366	6	60
1	9.64 210	26	9.68 850	32	0.31 150	9.95 360	6	59
2	9.64 236	26	9.68 882	32	0.31 118	9.95 354	6	58
3	9.64 262	26	9.68 914	32	0.31 086	9.95 348	7	57
4	9.64 288	25	9.68 946	32	0.31 054	9.95 341	6	56
5	9.64 313	26	9.68 978	82	0.31 022	9.95 335	6	55
6	9.64 339	26	9.69 010	32	0.30 990	9.95 329	6	54
7	9.64 365	26	9.69 042	32	0.30 958	9.95 323	6	53
8	9.64 391	26	9.69 074	32	0.30 926	9.95 317	7	52
9	9.64 417	25	9.69 106	32	0.30 894	9.95 310	6	51
10	9.64 442	26	9.69 138	32	0.30 862	9.95 304	6	50
11	9.64 468	26	9.69 170	32	0.30 830	9.95 298	6	49
12	9.64 494	25	9.69 202	32	0.30 798	9.95 292	6	48
13	9.64 519	26	9.69 234	32	0.30 766	9.95 286	7	47
14	9.64 545	26	9.69 266	32	0.30 734	9.95 279	6	46
15	9.64 571	25	9.69 298	31	0.30 702	9.95 273	6	45
16	9.64 596	26	9.69 329	32	0.30 671	9.95 267	6	44
17	9.64 622	25	9.69 361	32	0.30 639	9.95 261	7	43
18	9.64 647	26	9.69 393	32	0.30 607	9.95 254	6	42
19	9.64 673	25	9.69 425	32	0.30 575	9.95 248	6	41
20	9.64 698	26	9.69 457	31	0.30 543	9.95 242	6	40
21	9.64 724	25	9.69 488	32	0.30 512	9.95 236	7	39
22	9.64 749	26	9.69 520	32	0.30 480	9.95 229	6	38
23	9.64 775	25	9.69 552	32	0.30 448	9.95 223	6	37
24	9.64 800	26	9.69 584	31	0.30 416	9.95 217	6	36
25	9.64 826	25	9.69 615	32	0.30 385	9.95 211	7	35
26	9.64 851	26	9.69 647	32	0.30 353	9.95 204	6	34
27	9.64 877	25	9.69 679	31	0.30 321	9.95 198	6	33
28	9.64 902	25	9.69 710	32	0.30 290	9.95 192	7	32
29	9.64 927	26	9.69 742	32	0.30 258	9.95 185	6	31
30	9.64 953	25	9.69 774	31	0.30 226	9.95 179	6	30
31	9.64 978	25	9.69 805	32	0.30 195	9.95 173	6	29
32	9.65 003	26	9.69 837	31	0.30 163	9.95 167	7	28
33	9.65 029	25	9.69 868	32	0.30 132	9.95 160	6	27
34	9.65 054	25	9.69 900	32	0.30 100	9.95 154	6	26
35	9.65 079	25	9.69 932	31	0.30 068	9.95 148	7	25
36	9.65 104	26	9.69 963	32	0.30 037	9.95 141	6	24
37	9.65 130	25	9.69 995	31	0.30 005	9.95 135	6	23
38	9.65 155	25	9.70 026	32	0.29 974	9.95 129	7	22
39	9.65 180	25	9.70 058	31	0.29 942	9.95 122	6	21
40	9.65 205	25	9.70 089	32	0.29 911	9.95 116	6	20
41	9.65 230	25	9.70 121	31	0.29 879	9.95 110	7	19
42	9.65 255	26	9.70 152	32	0.29 848	9.95 103	6	18
43	9.65 281	25	9.70 184	31	0.29 816	9.95 097	7	17
44	9.65 306	25	9.70 215	32	0.29 785	9.95 090	6	16
45	9.65 331	25	9.70 247	31	0.29 753	9.95 084	6	15
46	9.65 356	25	9.70 278	31	0.29 722	9.95 078	7	14
47	9.65 381	25	9.70 309	32	0.29 691	9.95 071	6	13
48	9.65 406	25	9.70 341	31	0.29 659	9.95 065	6	12
49	9.65 431	25	9.70 372	32	0.29 628	9.95 059	7	11
50	9.65 456	25	9.70 404	31	0.29 596	9.95 052	6	10
51	9.65 481	25	9.70 435	31	0.29 565	9.95 046	7	9
52	9.65 506	25	9.70 466	32	0.29 534	9.95 039	6	8
53	9.65 531	25	9.70 498	31	0.29 502	9.95 033	6	7
54	9.65 556	24	9.70 529	31	0.29 471	9.95 027	7	6
55	9.65 580	25	9.70 560	32	0.29 440	9.95 020	6	5
56	9.65 605	25	9.70 592	31	0.29 408	9.95 014	7	4
57	9.65 630	25	9.70 623	31	0.29 377	9.95 007	6	3
58	9.65 655	25	9.70 654	31	0.29 346	9.95 001	6	2
59	9.65 680	25	9.70 685	32	0.29 315	9.94 995	7	1
60	9.65 705		9.70 717		0.29 283	9.94 988		0
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	32	31	26
2	6.4	6.2	5.2
3	9.6	9.3	7.8
4	12.8	12.4	10.4
5	16.0	15.5	13.0
6	19.2	18.6	15.6
7	22.4	21.7	18.2
8	25.6	24.8	20.8
9	28.8	27.9	23.4

	25	24
2	5.0	4.8
3	7.5	7.2
4	10.0	9.6
5	12.5	12.0
6	15.0	14.4
7	17.5	16.8
8	20.0	19.2
9	22.5	21.6

	7	6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

From the top:

For 26°+ or 206°+, read as printed; for **116°+ or 296°+**, read co-function.

From the bottom:

For 63°+ or 243°+, read as printed; for **153°+ or 333°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.65 705	24	9.70 717	31	0.29 283	9.94 988	6	**60**
1	9.65 729	25	9.70 748	31	0.29 252	9.94 982	7	59
2	9.65 754	25	9.70 779	31	0.29 221	9.94 975	6	58
3	9.65 779	25	9.70 810	31	0.29 190	9.94 969	7	57
4	9.65 804	24	9.70 841	32	0.29 159	9.94 962	6	56
5	9.65 828	25	9.70 873	31	0.29 127	9.94 956	7	**55**
6	9.65 853	25	9.70 904	31	0.29 096	9.94 949	6	54
7	9.65 878	24	9.70 935	31	0.29 065	9.94 943	7	53
8	9.65 902	25	9.70 966	31	0.29 034	9.94 936	6	52
9	9.65 927	25	9.70 997	31	0.29 003	9.94 930	7	51
10	9.65 952	24	9.71 028	31	0.28 972	9.94 923	6	**50**
11	9.65 976	25	9.71 059	31	0.28 941	9.94 917	6	49
12	9.66 001	24	9.71 090	31	0.28 910	9.94 911	7	48
13	9.66 025	25	9.71 121	32	0.28 879	9.94 904	6	47
14	9.66 050	25	9.71 153	31	0.28 847	9.94 898	7	46
15	9.66 075	24	9.71 184	31	0.28 816	9.94 891	6	**45**
16	9.66 099	25	9.71 215	31	0.28 785	9.94 885	7	44
17	9.66 124	24	9.71 246	31	0.28 754	9.94 878	7	43
18	9.66 148	25	9.71 277	31	0.28 723	9.94 871	6	42
19	9.66 173	24	9.71 308	31	0.28 692	9.94 865	7	41
20	9.66 197	24	9.71 339	31	0.28 661	9.94 858	6	**40**
21	9.66 221	25	9.71 370	31	0.28 630	9.94 852	7	39
22	9.66 246	24	9.71 401	30	0.28 599	9.94 845	6	38
23	9.66 270	25	9.71 431	31	0.28 569	9.94 839	7	37
24	9.66 295	24	9.71 462	31	0.28 538	9.94 832	6	36
25	9.66 319	24	9.71 493	31	0.28 507	9.94 826	7	**35**
26	9.66 343	25	9.71 524	31	0.28 476	9.94 819	6	34
27	9.66 368	24	9.71 555	31	0.28 445	9.94 813	7	33
28	9.66 392	24	9.71 586	31	0.28 414	9.94 806	7	32
29	9.66 416	25	9.71 617	31	0.28 383	9.94 799	6	31
30	9.66 441	24	9.71 648	31	0.28 352	9.94 793	7	**30**
31	9.66 465	24	9.71 679	30	0.28 321	9.94 786	6	29
32	9.66 489	24	9.71 709	31	0.28 291	9.94 780	7	28
33	9.66 513	24	9.71 740	31	0.28 260	9.94 773	6	27
34	9.66 537	25	9.71 771	31	0.28 229	9.94 767	7	26
35	9.66 562	24	9.71 802	31	0.28 198	9.94 760	7	**25**
36	9.66 586	24	9.71 833	30	0.28 167	9.94 753	6	24
37	9.66 610	24	9.71 863	31	0.28 137	9.94 747	7	23
38	9.66 634	24	9.71 894	31	0.28 106	9.94 740	6	22
39	9.66 658	24	9.71 925	30	0.28 075	9.94 734	7	21
40	9.66 682	24	9.71 955	31	0.28 045	9.94 727	7	**20**
41	9.66 706	25	9.71 986	31	0.28 014	9.94 720	6	19
42	9.66 731	24	9.72 017	31	0.27 983	9.94 714	7	18
43	9.66 755	24	9.72 048	30	0.27 952	9.94 707	7	17
44	9.66 779	24	9.72 078	31	0.27 922	9.94 700	6	16
45	9.66 803	24	9.72 109	31	0.27 891	9.94 694	7	**15**
46	9.66 827	24	9.72 140	30	0.27 860	9.94 687	7	14
47	9.66 851	24	9.72 170	31	0.27 830	9.94 680	6	13
48	9.66 875	24	9.72 201	30	0.27 799	9.94 674	7	12
49	9.66 899	23	9.72 231	31	0.27 769	9.94 667	7	11
50	9.66 922	24	9.72 262	31	0.27 738	9.94 660	6	**10**
51	9.66 946	24	9.72 293	30	0.27 707	9.94 654	7	9
52	9.66 970	24	9.72 323	31	0.27 677	9.94 647	7	8
53	9.66 994	24	9.72 354	30	0.27 646	9.94 640	6	7
54	9.67 018	24	9.72 384	31	0.27 616	9.94 634	7	6
55	9.67 042	24	9.72 415	30	0.27 585	9.94 627	7	**5**
56	9.67 066	24	9.72 445	31	0.27 555	9.94 620	6	4
57	9.67 090	23	9.72 476	30	0.27 524	9.94 614	7	3
58	9.67 113	24	9.72 506	31	0.27 494	9.94 607	7	2
59	9.67 137	24	9.72 537	30	0.27 463	9.94 600	7	1
60	9.67 161		9.72 567		0.27 433	9.94 593		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	32	**31**	**30**
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

	25	**24**	**23**
2	5.0	4.8	4.6
3	7.5	7.2	6.9
4	10.0	9.6	9.2
5	12.5	12.0	11.5
6	15.0	14.4	13.8
7	17.5	16.8	16.1
8	20.0	19.2	18.4
9	22.5	21.6	20.7

	7	**6**
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

From the top :

For **27°+** or **207°+**, read as printed; for **117°+** or **297°+**, read co-function.

From the bottom :

For **62°+** or **242°+**, read as printed; for **152°+** or **332°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.67 161	24	9.72 567	31	0.27 433	9.94 593	6	**60**
1	9.67 185	23	9.72 598	30	0.27 402	9.94 587	7	59
2	9.67 208	24	9.72 628	31	0.27 372	9.94 580	7	58
3	9.67 232	24	9.72 659	30	0.27 341	9.94 573	6	57
4	9.67 256	24	9.72 689	31	0.27 311	9.94 567	7	56
5	9.67 280	23	9.72 720	30	0.27 280	9.94 560	7	**55**
6	9.67 303	24	9.72 750	30	0.27 250	9.94 553	7	54
7	9.67 327	23	9.72 780	31	0.27 220	9.94 546	6	53
8	9.67 350	24	9.72 811	30	0.27 189	9.94 540	7	52
9	9.67 374	24	9.72 841	31	0.27 159	9.94 533	7	51
10	*9.67 398*	23	*9.72 872*	30	*0.27 128*	*9.94 526*	7	**50**
11	9.67 421	24	9.72 902	30	0.27 098	9.94 519	6	49
12	9.67 445	23	9.72 932	31	0.27 068	9.94 513	7	48
13	9.67 468	24	9.72 963	30	0.27 037	9.94 506	7	47
14	9.67 492	23	9.72 993	30	0.27 007	9.94 499	7	46
15	9.67 515	24	9.73 023	31	0.26 977	9.94 492	7	**45**
16	9.67 539	23	9.73 054	30	0.26 946	9.94 485	6	44
17	9.67 562	24	9.73 084	30	0.26 916	9.94 479	7	43
18	9.67 586	23	9.73 114	30	0.26 886	9.94 472	7	42
19	9.67 609	24	9.73 144	31	0.26 856	9.94 465	7	41
20	9.67 633	23	9.73 175	30	0.26 825	9.94 458	7	**40**
21	9.67 656	24	9.73 205	30	0.26 795	9.94 451	6	39
22	9.67 680	23	9.73 235	30	0.26 765	9.94 445	7	38
23	9.67 703	23	9.73 265	30	0.26 735	9.94 438	7	37
24	9.67 726	24	9.73 295	31	0.26 705	9.94 431	7	36
25	9.67 750	23	9.73 326	30	0.26 674	9.94 424	7	**35**
26	9.67 773	23	9.73 356	30	0.26 644	9.94 417	7	34
27	9.67 796	24	9.73 386	30	0.26 614	9.94 410	6	33
28	9.67 820	23	9.73 416	30	0.26 584	9.94 404	7	32
29	9.67 843	23	9.73 446	30	0.26 554	9.94 397	7	31
30	9.67 866	24	9.73 476	31	0.26 524	9.94 390	7	**30**
31	9.67 890	23	9.73 507	30	0.26 493	9.94 383	7	29
32	9.67 913	23	9.73 537	30	0.26 463	9.94 376	7	28
33	9.67 936	23	9.73 567	30	0.26 433	9.94 369	7	27
34	9.67 959	23	9.73 597	30	0.26 403	9.94 362	7	26
35	9.67 982	24	9.73 627	30	0.26 373	9.94 355	6	**25**
36	9.68 006	23	9.73 657	30	0.26 343	9.94 349	7	24
37	9.68 029	23	9.73 687	30	0.26 313	9.94 342	7	23
38	9.68 052	23	9.73 717	30	0.26 283	9.94 335	7	22
39	9.68 075	23	9.73 747	30	0.26 253	9.94 328	7	21
40	9.68 098	23	9.73 777	30	0.26 223	9.94 321	7	**20**
41	9.68 121	23	9.73 807	30	0.26 193	9.94 314	7	19
42	9.68 144	23	9.73 837	30	0.26 163	9.94 307	7	18
43	9.68 167	23	9.73 867	30	0.26 133	9.94 300	7	17
44	9.68 190	23	9.73 897	30	0.26 103	9.94 293	7	16
45	9.68 213	24	9.73 927	30	0.26 073	9.94 286	7	**15**
46	9.68 237	23	9.73 957	30	0.26 043	9.94 279	6	14
47	9.68 260	23	9.73 987	30	0.26 013	9.94 273	7	13
48	9.68 283	22	9.74 017	30	0.25 983	9.94 266	7	12
49	9.68 305	23	9.74 047	30	0.25 953	9.94 259	7	11
50	9.68 328	23	9.74 077	30	0.25 923	9.94 252	7	**10**
51	9.68 351	23	9.74 107	30	0.25 893	9.94 245	7	9
52	9.68 374	23	9.74 137	29	0.25 863	9.94 238	7	8
53	9.68 397	23	9.74 166	30	0.25 834	9.94 231	7	7
54	9.68 420	23	9.74 196	30	0.25 804	9.94 224	7	6
55	9.68 443	23	9.74 226	30	0.25 774	9.94 217	7	**5**
56	9.68 466	23	9.74 256	30	0.25 744	9.94 210	7	4
57	9.68 489	23	9.74 286	30	0.25 714	9.94 203	7	3
58	9.68 512	22	9.74 316	29	0.25 684	9.94 196	7	2
59	9.68 534	23	9.74 345	30	0.25 655	9.94 189	7	1
60	9.68 557		9.74 375		0.25 625	9.94 182		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	31	30	29
2	6.2	6.0	5.8
3	9.3	9.0	8.7
4	12.4	12.0	11.6
5	15.5	15.0	14.5
6	*18.6*	*18.0*	*17.4*
7	21.7	21.0	20.3
8	24.8	24.0	23.2
9	27.9	27.0	26.1

	24	23	22
2	4.8	4.6	4.4
3	7.2	6.9	6.6
4	9.6	9.2	8.8
5	12.0	11.5	11.0
6	14.4	13.8	13.2
7	16.8	16.1	15.4
8	19.2	18.4	17.6
9	21.6	20.7	19.8

	7	6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

From the top:

For **28°+** or **208°+**, read as printed; for **118°+** or **298°+**, read co-function.

From the bottom:

For **61°+** or **241°+**, read as printed; for **151°+** or **331°+**, read co-function.

Prop. Pts.

'	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.68 557	23	9.74 375	30	0.25 625	9.94 182	7	**60**
1	9.68 580	23	9.74 405	30	0.25 595	9.94 175	7	59
2	9.68 603	22	9.74 435	30	0.25 565	9.94 168	7	58
3	9.68 625	23	9.74 465	29	0.25 535	9.94 161	7	57
4	9.68 648	23	9.74 494	30	0.25 506	9.94 154	7	56
5	9.68 671	23	9.74 524	30	0.25 476	9.94 147	7	**55**
6	9.68 694	22	9.74 554	29	0.25 446	9.94 140	7	54
7	9.68 716	23	9.74 583	30	0.25 417	9.94 133	7	53
8	9.68 739	23	9.74 613	30	0.25 387	9.94 126	7	52
9	9.68 762	22	9.74 643	30	0.25 357	9.94 119	7	51
10	9.68 784	23	9.74 673	29	0.25 327	9.94 112	7	**50**
11	9.68 807	22	9.74 702	30	0.25 298	9.94 105	7	49
12	9.68 829	23	9.74 732	30	0.25 268	9.94 098	8	48
13	9.68 852	23	9.74 762	29	0.25 238	9.94 090	7	47
14	9.68 875	22	9.74 791	30	0.25 209	9.94 083	7	46
15	9.68 897	23	9.74 821	30	0.25 179	9.94 076	7	**45**
16	9.68 920	22	9.74 851	29	0.25 149	9.94 069	7	44
17	9.68 942	23	9.74 880	30	0.25 120	9.94 062	7	43
18	9.68 965	22	9.74 910	29	0.25 090	9.94 055	7	42
19	9.68 987	23	9.74 939	30	0.25 061	9.94 048	7	41
20	9.69 010	22	9.74 969	29	0.25 031	9.94 041	7	**40**
21	9.69 032	23	9.74 998	30	0.25 002	9.94 034	7	39
22	9.69 055	22	9.75 028	30	0.24 972	9.94 027	7	38
23	9.69 077	23	9.75 058	29	0.24 942	9.94 020	8	37
24	9.69 100	22	9.75 087	30	0.24 913	9.94 012	7	36
25	9.69 122	22	9.75 117	29	0.24 883	9.94 005	7	**35**
26	9.69 144	23	9.75 146	30	0.24 854	9.93 998	7	34
27	9.69 167	22	9.75 176	29	0 24 824	9.93 991	7	33
28	9.69 189	23	9.75 205	30	0.24 795	9.93 984	7	32
29	9.69 212	22	9.75 235	29	0.24 765	9.93 977	7	31
30	9.69 234	22	9.75 264	30	0.24 736	9.93 970	7	**30**
31	9.69 256	23	9.75 294	29	0.24 706	9.93 963	8	29
32	9.69 279	22	9.75 323	30	0.24 677	9.93 955	7	28
33	9.69 301	22	9.75 353	29	0.24 647	9.93 948	7	27
34	9.69 323	22	9.75 382	29	0.24 618	9.93 941	7	26
35	9.69 345	23	9.75 411	30	0.24 589	9.93 934	7	**25**
36	9.69 368	22	9.75 441	29	0.24 559	9.93 927	7	24
37	9.69 390	22	9.75 470	30	0.24 530	9.93 920	8	23
38	9.69 412	22	9.75 500	29	0.24 500	9.93 912	7	22
39	9.69 434	22	9.75 529	29	0.24 471	9.93 905	7	21
40	9.69 456	23	9.75 558	30	0.24 442	9.93 898	7	**20**
41	9.69 479	22	9.75 588	29	0.24 412	9.93 891	7	19
42	9.69 501	22	9.75 617	30	0.24 383	9.93 884	8	18
43	9.69 523	22	9.75 647	29	0.24 353	9.93 876	7	17
44	9.69 545	22	9.75 676	29	0.24 324	9.93 869	7	16
45	9.69 567	22	9.75 705	30	0.24 295	9.93 862	7	**15**
46	9.69 589	22	9.75 735	29	0.24 265	9.93 855	8	14
47	9.69 611	22	9.75 764	29	0.24 236	9.93 847	7	13
48	9.69 633	22	9.75 793	29	0.24 207	9.93 840	7	12
49	9.69 655	22	9.75 822	30	0.24 178	9.93 833	7	11
50	9.69 677	22	9.75 852	29	0.24 148	9.93 826	7	**10**
51	9.69 699	22	9.75 881	29	0.24 119	9.93 819	8	9
52	9.69 721	22	9.75 910	29	0.24 090	9.93 811	7	8
53	9.69 743	22	9.75 939	30	0.24 061	9.93 804	7	7
54	9.69 765	22	9.75 969	29	0.24 031	9.93 797	8	6
55	9.69 787	22	9.75 998	29	0.24 002	9.93 789	7	**5**
56	9.69 809	22	9.76 027	29	0.23 973	9.93 782	7	4
57	9.69 831	22	9.76 056	30	0.23 944	9.93 775	7	3
58	9.69 853	22	9.76 086	29	0.23 914	9.93 768	8	2
59	9.69 875	22	9.76 115	29	0.23 885	9.93 760	7	1
60	9.69 897		9.76 144		0.23 856	9.93 753		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	'

Prop. Pts.

	30	29	23
2	6.0	5.8	4.6
3	9.0	8.7	6.9
4	12.0	11.6	9.2
5	15.0	14.5	11.5
6	18.0	17.4	13.8
7	21.0	20.3	16.1
8	24.0	23.2	18.4
9	27.0	26.1	20.7

	22	8	7
2	4.4	1.6	1.4
3	6.6	2.4	2.1
4	8.8	3.2	2.8
5	11.0	4.0	3.5
6	13.2	4.8	4.2
7	15.4	5.6	4.9
8	17.6	6.4	5.6
9	19.8	7.2	6.3

From the top:

For **29°+** or **209°+**, read as printed; for **119°+** or **299°+**, read co-function.

From the bottom:

For **60°+** or **240°+**, read as printed; for **150°+** or **330°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.69 897	22	9.76 144	29	0.23 856	9.93 753	7	**60**
1	9.69 919	22	9.76 173	29	0.23 827	9.93 746	8	59
2	9.69 941	22	9.76 202	29	0.23 798	9.93 738	7	58
3	9.69 963	21	9.76 231	30	0.23 769	9.93 731	7	57
4	9.69 984	22	9.76 261	29	0.23 739	9.93 724	7	56
5	9.70 006	22	9.76 290	29	0.23 710	9.93 717	8	**55**
6	9.70 028	22	9.76 319	29	0.23 681	9.93 709	7	54
7	9.70 050	22	9.76 348	29	0.23 652	9.93 702	7	53
8	9.70 072	21	9.76 377	29	0.23 623	9.93 695	8	52
9	9.70 093	22	9.76 406	29	0.23 594	9.93 687	7	51
10	9.70 115	22	9.76 435	29	0.23 565	9.93 680	7	**50**
11	9.70 137	22	9.76 464	29	9.23 536	9.93 673	8	49
12	9.70 159	21	9.76 493	29	0.23 507	9.93 665	7	48
13	9.70 180	22	9.76 522	29	0.23 478	9.93 658	8	47
14	9.70 202	22	9.76 551	29	0.23 449	9.93 650	7	46
15	9.70 224	21	9.76 580	29	0.23 420	9.93 643	7	**45**
16	9.70 245	22	9.76 609	30	0.23 391	9.93 636	8	44
17	9.70 267	21	9.76 639	29	0.23 361	9.93 628	7	43
18	9.70 288	22	9.76 668	29	0.23 332	9.93 621	7	42
19	9.70 310	22	9.76 697	28	0.23 303	9.93 614	8	41
20	9.70 332	21	9.76 725	29	0.23 275	9.93 606	7	**40**
21	9.70 353	22	9.76 754	29	0.23 246	9.93 599	8	39
22	9.70 375	21	9.76 783	29	0.23 217	9.93 591	7	38
23	9.70 396	22	9.76 812	29	0.23 188	9.93 584	7	37
24	9.70 418	21	9.76 841	29	0.23 159	9.93 577	8	36
25	9.70 439	22	9.76 870	29	0.23 130	9.93 569	7	**35**
26	9.70 461	21	9.76 899	29	0.23 101	9.93 562	8	34
27	9.70 482	22	9.76 928	29	0.23 072	9.93 554	7	33
28	9.70 504	21	9.76 957	29	0.23 043	9.93 547	8	32
29	9.70 525	22	9.76 986	29	0.23 014	9.93 539	7	31
30	9.70 547	21	9.77 015	29	0.22 985	9.93 532	7	**30**
31	9.70 568	22	9.77 044	29	0.22 956	9.93 525	8	29
32	9.70 590	21	9.77 073	28	0.22 927	9.93 517	7	28
33	9.70 611	22	9.77 101	29	0.22 899	9.93 510	8	27
34	9.70 633	21	9.77 130	29	0.22 870	9.93 502	7	26
35	9.70 654	21	9.77 159	29	0.22 841	9.93 495	8	**25**
36	9.70 675	22	9.77 188	29	0.22 812	9.93 487	7	24
37	9.70 697	21	9.77 217	29	0.22 783	9.93 480	8	23
38	9.70 718	21	9.77 246	28	0.22 754	9.93 472	7	22
39	9.70 739	22	9.77 274	29	0.22 726	9.93 465	8	21
40	9.70 761	21	9.77 303	29	0.22 697	9.93 457	7	**20**
41	9.70 782	21	9.77 332	29	0.22 668	9.93 450	8	19
42	9.70 803	21	9.77 361	29	0.22 639	9.93 442	7	18
43	9.70 824	22	9.77 390	28	0.22 610	9.93 435	8	17
44	9.70 846	21	9.77 418	29	0.22 582	9.93 427	7	16
45	9.70 867	21	9.77 447	29	0.22 553	9.93 420	8	**15**
46	9.70 888	21	9.77 476	29	0.22 524	9.93 412	7	14
47	9.70 909	22	9.77 505	28	0.22 495	9.93 405	8	13
48	9.70 931	21	9.77 533	29	0.22 467	9.93 397	7	12
49	9.70 952	21	9.77 562	29	0.22 438	9.93 390	8	11
50	9.70 973	21	9.77 591	28	0.22 409	9.93 382	7	**10**
51	9.70 994	21	9.77 619	29	0.22 381	9.93 375	8	9
52	9.71 015	21	9.77 648	29	0.22 352	9.93 367	7	8
53	9.71 036	22	9.77 677	29	0.22 323	9.93 360	8	7
54	9.71 058	21	9.77 706	28	0.22 294	9.93 352	8	6
55	9.71 079	21	9.77 734	29	0.22 266	9.93 344	7	**5**
56	9.71 100	21	9.77 763	28	0.22 237	9.93 337	8	4
57	9.71 121	21	9.77 791	29	0.22 209	9.93 329	7	3
58	9.71 142	21	9.77 820	29	0.22 180	9.93 322	8	2
59	9.71 163	21	9.77 849	28	0.22 151	9.93 314	7	1
60	9.71 184		9.77 877		0.22 123	9.93 307		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	30	**29**	**28**
2	6.0	5.8	5.6
3	9.0	8.7	8.4
4	12.0	11.6	11.2
5	15.0	14.5	14.0
6	18.0	17.4	16.8
7	21.0	20.3	19.6
8	24.0	23.2	22.4
9	27.0	26.1	25.2

	22	**21**
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

	8	**7**
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

From the top:

For **30°+** or **210°+**, read as printed; for **120°+** or **300°+**, read co-function.

From the bottom:

For **59°+** or **239°+**, read as printed; for **149°+** or **329°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	′
0	9.71 184	21	9.77 877	29	0.22 123	9.93 307	8	**60**
1	9.71 205	21	9.77 906	29	0.22 094	9.93 299	8	59
2	9.71 226	21	9.77 935	28	0.22 065	9.93 291	7	58
3	9.71 247	21	9.77 963	29	0.22 037	9.93 284	8	57
4	9.71 268	21	9.77 992	28	0.22 008	9.93 276	7	56
5	9.71 289	21	9.78 020	29	0.21 980	9.93 269	8	**55**
6	9.71 310	21	9.78 049	28	0.21 951	9.93 261	8	54
7	9.71 331	21	9.78 077	29	0.21 923	9.93 253	7	53
8	9.71 352	21	9.78 106	29	0.21 894	9.93 246	8	52
9	9.71 373	20	9.78 135	28	0.21 865	9.93 238	8	51
10	9.71 393	21	9.78 163	29	0.21 837	9.93 230	7	**50**
11	9.71 414	21	9.78 192	28	0.21 808	9.93 223	8	49
12	9.71 435	21	9.78 220	29	0.21 780	9.93 215	8	48
13	9.71 456	21	9.78 249	28	0.21 751	9.93 207	7	47
14	9.71 477	21	9.78 277	29	0.21 723	9.93 200	8	46
15	9.71 498	21	9.78 306	28	0.21 694	9.93 192	8	**45**
16	9.71 519	20	9.78 334	29	0.21 666	9.93 184	7	44
17	9.71 539	21	9.78 363	28	0.21 637	9.93 177	8	43
18	9.71 560	21	9.78 391	28	0.21 609	9.93 169	8	42
19	9.71 581	21	9.78 419	29	0.21 581	9.93 161	7	41
20	9.71 602	20	9.78 448	28	0.21 552	9.93 154	8	**40**
21	9.71 622	21	9.78 476	29	0.21 524	9.93 146	8	39
22	9.71 643	21	9.78 505	28	0.21 495	9.93 138	7	38
23	9.71 664	21	9.78 533	29	0.21 467	9.93 131	8	37
24	9.71 685	20	9.78 562	28	0.21 438	9.93 123	8	36
25	9.71 705	21	9.78 590	28	0.21 410	9.93 115	7	**35**
26	9.71 726	21	9.78 618	29	0.21 382	9.93 108	8	34
27	9.71 747	20	9.78 647	28	0.21 353	9.93 100	8	33
28	9.71 767	21	9.78 675	29	0.21 325	9.93 092	8	32
29	9.71 788	21	9.78 704	28	0.21 296	9.93 084	7	31
30	9.71 809	20	9.78 732	28	0.21 268	9.93 077	8	**30**
31	9.71 829	21	9.78 760	29	0.21 240	9.93 069	8	29
32	9.71 850	20	9.78 789	28	0.21 211	9.93 061	8	28
33	9.71 870	21	9.78 817	28	0.21 183	9.93 053	7	27
34	9.71 891	20	9.78 845	29	0.21 155	9.93 046	8	26
35	9.71 911	21	9.78 874	28	0.21 126	9.93 038	8	**25**
36	9.71 932	20	9.78 902	28	0.21 098	9.93 030	8	24
37	9.71 952	21	9.78 930	29	0.21 070	9.93 022	8	23
38	9.71 973	21	9.78 959	28	0.21 041	9.93 014	7	22
39	9.71 994	20	9.78 987	28	0.21 013	9.93 007	8	21
40	9.72 014	20	9.79 015	28	0.20 985	9.92 999	8	**20**
41	9.72 034	21	9.79 043	29	0.20 957	9.92 991	8	19
42	9.72 055	20	9.79 072	28	0.20 928	9.92 983	7	18
43	9.72 075	21	9.79 100	28	0.20 900	9.92 976	8	17
44	9.72 096	20	9.79 128	28	0.20 872	9.92 968	8	16
45	9.72 116	21	9.79 156	29	0.20 844	9.92 960	8	**15**
46	9.72 137	20	9.79 185	28	0.20 815	9.92 952	8	14
47	9.72 157	20	9.79 213	28	0.20 787	9.92 944	8	13
48	9.72 177	21	9.79 241	28	0.20 759	9.92 936	7	12
49	9.72 198	20	9.79 269	28	0.20 731	9.92 929	8	11
50	9.72 218	20	9.79 297	29	0.20 703	9.92 921	8	**10**
51	9.72 238	21	9.79 326	28	0.20 674	9.92 913	8	9
52	9.72 259	20	9.79 354	28	0.20 646	9.92 905	8	8
53	9.72 279	20	9.79 382	28	0.20 618	9.92 897	8	7
54	9.72 299	21	9.79 410	28	0.20 590	9.92 889	8	6
55	9.72 320	20	9.79 438	28	0.20 562	9.92 881	7	**5**
56	9.72 340	20	9.79 466	29	0.20 534	9.92 874	8	4
57	9.72 360	21	9.79 495	28	0.20 505	9.92 866	8	3
58	9.72 381	20	9.79 523	28	0.20 477	9.92 858	8	2
59	9.72 401	20	9.79 551	28	0.20 449	9.92 850	8	1
60	9.72 421		9.79 579		0.20 421	9.92 842		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	29	**28**	**21**
2	5.8	5.6	4.2
3	8.7	8.4	6.3
4	11.6	11.2	8.4
5	14.5	14.0	10.5
6	17.4	16.8	12.6
7	20.3	19.6	14.7
8	23.2	22.4	16.8
9	26.1	25.2	18.9

	20	**8**	**7**
2	4.0	1.6	1.4
3	6.0	2.4	2.1
4	8.0	3.2	2.8
5	10.0	4.0	3.5
6	12.0	4.8	4.2
7	14.0	5.6	4.9
8	16.0	6.4	5.6
9	18.0	7.2	6.3

From the top:

For **31°+** or **211°+**, read as printed; for **121°+** or **301°+**, read co-function.

From the bottom:

For **58°+** or **238°+**, read as printed; for **148°+** or **328°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.72 421	20	9.79 579	28	0.20 421	9.92 842	8	**60**
1	9.72 441	20	9.79 607	28	0.20 393	9.92 834	8	59
2	9.72 461	21	9.79 635	28	0.20 365	9.92 826	8	58
3	9.72 482	20	9.79 663	28	0.20 337	9.92 818	8	57
4	9.72 502	20	9.79 691	28	0.20 309	9.92 810	7	56
5	9.72 522	20	9.79 719	28	0.20 281	9.92 803	8	**55**
6	9.72 542	20	9.79 747	29	0.20 253	9.92 795	8	54
7	9.72 562	20	9.79 776	28	0.20 224	9.92 787	8	53
8	9.72 582	20	9.79 804	28	0.20 196	9.92 779	8	52
9	9.72 602	20	9.79 832	28	0.20 168	9.92 771	8	51
10	9.72 622	21	9.79 860	28	0.20 140	9.92 763	8	**50**
11	9.72 643	20	9.79 888	28	0.20 112	9.92 755	8	49
12	9.72 663	20	9.79 916	28	0.20 084	9.92 747	8	48
13	9.72 683	20	9.79 944	28	0.20 056	9.92 739	8	47
14	9.72 703	20	9.79 972	28	0.20 028	9.92 731	8	46
15	9.72 723	20	9.80 000	28	0.20 000	9.92 723	8	**45**
16	9.72 743	20	9.80 028	28	0.19 972	9.92 715	8	44
17	9.72 763	20	9.80 056	28	0.19 944	9.92 707	8	43
18	9.72 783	20	9.80 084	28	0.19 916	9.92 699	8	42
19	9.72 803	20	9.80 112	28	0.19 888	9.92 691	8	41
20	9.72 823	20	9.80 140	28	0.19 860	9.92 683	8	**40**
21	9.72 843	20	9.80 168	27	0.19 832	9.92 675	8	39
22	9.72 863	20	9.80 195	28	0.19 805	9.92 667	8	38
23	9.72 883	19	9.80 223	28	0.19 777	9.92 659	8	37
24	9.72 902	20	9.80 251	28	0.19 749	9.92 651	8	36
25	9.72 922	20	9.80 279	28	0.19 721	9.92 643	8	**35**
26	9.72 942	20	9.80 307	28	0.19 693	9.92 635	8	34
27	9.72 962	20	9.80 335	28	0.19 665	9.92 627	8	33
28	9.72 982	20	9.80 363	28	0.19 637	9.92 619	8	32
29	9.73 002	20	9.80 391	28	0.19 609	9.92 611	8	31
30	9.73 022	19	9.80 419	28	0.19 581	9.92 603	8	**30**
31	9.73 041	20	9.80 447	27	0.19 553	9.92 595	8	29
32	9.73 061	20	9.80 474	28	0.19 526	9.92 587	8	28
33	9.73 081	20	9.80 502	28	0.19 498	9.92 579	8	27
34	9.73 101	20	9.80 530	28	0.19 470	9.92 571	8	26
35	9.73 121	19	9.80 558	28	0.19 442	9.92 563	8	**25**
36	9.73 140	20	9.80 586	28	0.19 414	9.92 555	9	24
37	9.73 160	20	9.80 614	28	0.19 386	9.92 546	8	23
38	9.73 180	20	9.80 642	27	0.19 358	9.92 538	8	22
39	9.73 200	19	9.80 669	28	0.19 331	9.92 530	8	21
40	9.73 219	20	9.80 697	28	0.19 303	9.92 522	8	**20**
41	9.73 239	20	9.80 725	28	0.19 275	9.92 514	8	19
42	9.73 259	19	9.80 753	28	0.19 247	9.92 506	8	18
43	9.73 278	20	9.80 781	27	0.19 219	9.92 498	8	17
44	9.73 298	20	9.80 808	28	0.19 192	9.92 490	8	16
45	9.73 318	19	9.80 836	28	0.19 164	9.92 482	9	**15**
46	9.73 337	20	9.80 864	28	0.19 136	9.92 473	8	14
47	9.73 357	20	9.80 892	27	0.19 108	9.92 465	8	13
48	9.73 377	19	9.80 919	28	0.19 081	9.92 457	8	12
49	9.73 396	20	9.80 947	28	0.19 053	9.92 449	8	11
50	9.73 416	19	9.80 975	28	0.19 025	9.92 441	8	**10**
51	9.73 435	20	9.81 003	27	0.18 997	9.92 433	8	9
52	9.73 455	19	9.81 030	28	0.18 970	9.92 425	9	8
53	9.73 474	20	9.81 058	28	0.18 942	9.92 416	8	7
54	9.73 494	19	9.81 086	27	0.18 914	9.92 408	8	6
55	9.73 513	20	9.81 113	28	0.18 887	9.92 400	8	**5**
56	9.73 533	19	9.81 141	28	0.18 859	9.92 392	8	4
57	9.73 552	20	9.81 169	27	0.18 831	9.92 384	8	3
58	9.73 572	19	9.81 196	28	0.18 804	9.92 376	9	2
59	9.73 591	20	9.81 224	28	0.18 776	9.92 367	8	1
60	9.73 611		9.81 252		0.18 748	9.92 359		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	29	28	27
2	5.8	5.6	5.4
3	8.7	8.4	8.1
4	11.6	11.2	10.8
5	14.5	14.0	13.5
6	17.4	16.8	16.2
7	20.3	19.6	18.9
8	23.2	22.4	21.6
9	26.1	25.2	24.3

	21	20	19
2	4.2	4.0	3.8
3	6.3	6.0	5.7
4	8.4	8.0	7.6
5	10.5	10.0	9.5
6	12.6	12.0	11.4
7	14.7	14.0	13.3
8	16.8	16.0	15.2
9	18.9	18.0	17.1

	9	8	7
2	1.8	1.6	1.4
3	2.7	2.4	2.1
4	3.6	3.2	2.8
5	4.5	4.0	3.5
6	5.4	4.8	4.2
7	6.3	5.6	4.9
8	7.2	6.4	5.6
9	8.1	7.2	6.3

From the top:

For **32°+** or **212°+**, read as printed; for **122°+** or **302°+**, read co-function.

From the bottom:

For **57°+** or **237°+**, read as printed; for **147°+** or **327°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.73 611	19	9.81 252	27	0.18 748	9.92 359	8	**60**
1	9.73 630	20	9.81 279	28	0.18 721	9.92 351	8	59
2	9.73 650	19	9.81 307	28	0.18 693	9.92 343	8	58
3	9.73 669	20	9.81 335	27	0.18 665	9.92 335	9	57
4	9.73 689	19	9.81 362	28	0.18 638	9.92 326	8	56
5	9.73 708	19	9.81 390	28	0.18 610	9.92 318	8	**55**
6	9.73 727	20	9.81 418	27	0.18 582	9.92 310	8	54
7	9.73 747	19	9.81 445	28	0.18 555	9.92 302	9	53
8	9.73 766	19	9.81 473	27	0.18 527	9.92 293	8	52
9	9.73 785	20	9.81 500	28	0.18 500	9.92 285	8	51
10	9.73 805	19	9.81 528	28	0.18 472	9.92 277	8	**50**
11	9.73 824	19	9.81 556	27	0.18 444	9.92 269	9	49
12	9.73 843	20	9.81 583	28	0.18 417	9.92 260	8	48
13	9.73 863	19	9.81 611	27	0.18 389	9.92 252	8	47
14	9.73 882	19	9.81 638	28	0.18 362	9.92 244	9	46
15	9.73 901	20	9.81 666	27	0.18 334	9.92 235	8	**45**
16	9.73 921	19	9.81 693	28	0.18 307	9.92 227	8	44
17	9.73 940	19	9.81 721	27	0.18 279	9.92 219	8	43
18	9.73 959	19	9.81 748	28	0.18 252	9.92 211	9	42
19	9.73 978	19	9.81 776	27	0.18 224	9.92 202	8	41
20	9.73 997	20	9.81 803	28	0.18 197	9.92 194	8	**40**
21	9.74 017	19	9.81 831	27	0.18 169	9.92 186	9	39
22	9.74 036	19	9.81 858	28	0.18 142	9.92 177	8	38
23	9.74 055	19	9.81 886	27	0.18 114	9.92 169	8	37
24	9.74 074	19	9.81 913	28	0.18 087	9.92 161	9	36
25	9.74 093	20	9.81 941	27	0.18 059	9.92 152	8	**35**
26	9.74 113	19	9.81 968	28	0.18 032	9.92 144	8	34
27	9.74 132	19	9.81 996	27	0.18 004	9.92 136	9	33
28	9.74 151	19	9.82 023	28	0.17 977	9.92 127	8	32
29	9.74 170	19	9.82 051	27	0.17 949	9.92 119	8	31
30	9.74 189	19	9.82 078	28	0.17 922	9.92 111	9	**30**
31	9.74 208	19	9.82 106	27	0.17 894	9.92 102	8	29
32	9.74 227	19	9.82 133	28	0.17 867	9.92 094	8	28
33	9.74 246	19	9.82 161	27	0.17 839	9.92 086	9	27
34	9.74 265	19	9.82 188	27	0.17 812	9.92 077	8	26
35	9.74 284	19	9.82 215	28	0.17 785	9.92 069	9	**25**
36	9.74 303	19	9.82 243	27	0.17 757	9.92 060	8	24
37	9.74 322	19	9.82 270	28	0.17 730	9.92 052	8	23
38	9.74 341	19	9.82 298	27	0.17 702	9.92 044	9	22
39	9.74 360	19	9.82 325	27	0.17 675	9.92 035	8	21
40	9.74 379	19	9.82 352	28	0.17 648	9.92 027	9	**20**
41	9.74 398	19	9.82 380	27	0.17 620	9.92 018	8	19
42	9.74 417	19	9.82 407	28	0.17 593	9.92 010	8	18
43	9.74 436	19	9.82 435	27	0.17 565	9.92 002	9	17
44	9.74 455	19	9.82 462	27	0.17 538	9.91 993	8	16
45	9.74 474	19	9.82 489	28	0.17 511	9.91 985	9	**15**
46	9.74 493	19	9.82 517	27	0 17 483	9.91 976	8	14
47	9.74 512	19	9.82 544	27	0.17 456	9.91 968	9	13
48	9.74 531	18	9.82 571	28	0.17 429	9.91 959	8	12
49	9.74 549	19	9.82 599	27	0.17 401	9.91 951	9	11
50	9.74 568	19	9.82 626	27	0.17 374	9.91 942	8	**10**
51	9.74 587	19	9.82 653	28	0.17 347	9.91 934	9	9
52	9.74 606	19	9.82 681	27	0.17 319	9.91 925	8	8
53	9.74 625	19	9.82 708	27	6.17 292	9.91 917	9	7
54	9.74 644	18	9.82 735	27	0.17 265	9.91 908	8	6
55	9.74 662	19	9.82 762	28	0.17 238	9.91 900	9	**5**
56	9.74 681	19	9.82 790	27	0.17 210	9.91 891	8	4
57	9.74 700	19	9.82 817	27	0.17 183	9.91 883	9	3
58	9.74 719	18	9.82 844	27	0.17 156	9.91 874	8	2
59	9.74 737	19	9.82 871	28	0.17 129	9.91 866	9	1
60	9.74 756		9.82 899		0.17 101	9.91 857		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	28	27	20
2	5.6	5.4	4.0
3	8.4	8.1	6.0
4	11.2	10.8	8.0
5	14.0	13.5	10.0
6	16.8	16.2	12.0
7	19.6	18.9	14.0
8	22.4	21.6	16.0
9	25.2	24.3	18.0

	19	18
2	3.8	3.6
3	5.7	5.4
4	7.6	7.2
5	9.5	9.0
6	11.4	10.8
7	13.3	12.6
8	15.2	14.4
9	17.1	16.2

	9	8
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

From the top:

For **33°+** or **213°+**, read as printed; for **123°+** or **303°+**, read co-function.

From the bottom:

For **56°+** or **236°+**, read as printed; for **146°+** or **326°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.74 756	19	9.82 899	27	0.17 101	9.91 857	8	**60**
1	9.74 775	19	9.82 926	27	0.17 074	9.91 849	9	59
2	9.74 794	18	9.82 953	27	0.17 047	9.91 840	8	58
3	9.74 812	19	9.82 980	28	0.17 020	9.91 832	9	57
4	9.74 831	19	9.83 008	27	0.16 992	9.91 823	8	56
5	9.74 850	18	9.83 035	27	0.16 965	9.91 815	9	**55**
6	9.74 868	19	9.83 062	27	0.16 938	9.91 806	8	54
7	9.74 887	19	9.83 089	28	0.16 911	9.91 798	9	53
8	9.74 906	18	9.83 117	27	0.16 883	9.91 789	8	52
9	9.74 924	19	9.83 144	27	0.16 856	9.91 781	9	51
10	9.74 943	18	9.83 171	27	0.16 829	9.91 772	9	**50**
11	9.74 961	19	9.83 198	27	0.16 802	9.91 763	8	49
12	9.74 980	19	9.83 225	27	0.16 775	9.91 755	9	48
13	9.74 999	18	9.83 252	28	0.16 748	9.91 746	8	47
14	9.75 017	19	9.83 280	27	0.16 720	9.91 738	9	46
15	9.75 036	18	9.83 307	27	0.16 693	9.91 729	9	**45**
16	9.75 054	19	9.83 334	27	0.16 666	9.91 720	8	44
17	9.75 073	18	9.83 361	27	0.16 639	9.91 712	9	43
18	9.75 091	19	9.83 388	27	0.16 612	9.91 703	8	42
19	9.75 110	18	9.83 415	27	0.16 585	9.91 695	9	41
20	9.75 128	19	9.83 442	28	0.16 558	9.91 686	9	**40**
21	9.75 147	18	9.83 470	27	0.16 530	9.91 677	8	39
22	9.75 165	19	9.83 497	27	0.16 503	9.91 669	9	38
23	9.75 184	18	9.83 524	27	0.16 476	9.91 660	9	37
24	9.75 202	19	9.83 551	27	0.16 449	9.91 651	8	36
25	9.75 221	18	9.83 578	27	0.16 422	9.91 643	9	**35**
26	9.75 239	19	9.83 605	27	0.16 395	9.91 634	9	34
27	9.75 258	18	9.83 632	27	0.16 368	9.91 625	8	33
28	9.75 276	18	9.83 659	27	0.16 341	9.91 617	9	32
29	9.75 294	19	9.83 686	27	0.16 314	9.91 608	9	31
30	9.75 313	18	9.83 713	27	0.16 287	9.91 599	8	**30**
31	9.75 331	19	9.83 740	28	0.16 260	9.91 591	9	29
32	9.75 350	18	9.83 768	27	0.16 232	9.91 582	9	28
33	9.75 368	18	9.83 795	27	0.16 205	9.91 573	8	27
34	9.75 386	19	9.83 822	27	0.16 178	9.91 565	9	26
35	9.75 405	18	9.83 849	27	0.16 151	9.91 556	9	**25**
36	9.75 423	18	9.83 876	27	0.16 124	9.91 547	9	24
37	9.75 441	18	9.83 903	27	0.16 097	9.91 538	8	23
38	9.75 459	19	9.83 930	27	0.16 070	9.91 530	9	22
39	9.75 478	18	9.83 957	27	0.16 043	9.91 521	9	21
40	9.75 496	18	9.83 984	27	0.16 016	9.91 512	8	**20**
41	9.75 514	19	9.84 011	27	0.15 989	9.91 504	9	19
42	9.75 533	18	9.84 038	27	0.15 962	9.91 495	9	18
43	9.75 551	18	9.84 065	27	0.15 935	9.91 486	9	17
44	9.75 569	18	9.84 092	27	0.15 908	9.91 477	8	16
45	9.75 587	18	9.84 119	27	0.15 881	9.91 469	9	**15**
46	9.75 605	19	9.84 146	27	0.15 854	9.91 460	9	14
47	9.75 624	18	9.84 173	27	0.15 827	9.91 451	9	13
48	9.75 642	18	9.84 200	27	0.15 800	9.91 442	9	12
49	9.75 660	18	9.84 227	27	0.15 773	9.91 433	8	11
50	9.75 678	18	9.84 254	26	0.15 746	9.91 425	9	**10**
51	9.75 696	18	9.84 280	27	0.15 720	9.91 416	9	9
52	9.75 714	19	9.84 307	27	0.15 693	9.91 407	9	8
53	9.75 733	18	9.84 334	27	0.15 666	9.91 398	9	7
54	9.75 751	18	9.84 361	27	0.15 639	9.91 389	8	6
55	9.75 769	18	9.84 388	27	0.15 612	9.91 381	9	**5**
56	9.75 787	18	9.84 415	27	0.15 585	9.91 372	9	4
57	9.75 805	18	9.84 442	27	0.15 558	9.91 363	9	3
58	9.75 823	18	9.84 469	27	0.15 531	9.91 354	9	2
59	9.75 841	18	9.84 496	27	0 15 504	9.91 345	9	1
60	9.75 859		9.84 523		0.15 477	9.91 336		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	28	**27**	**26**
2	5.6	5.4	5.2
3	8.4	8.1	7.8
4	11.2	10.8	10.4
5	14.0	13.5	13.0
6	16.8	16.2	15.6
7	19.6	18.9	18.2
8	22.4	21.6	20.8
9	25.2	24.3	23.4

	19	**18**
2	3.8	3.6
3	5.7	5.4
4	7.6	7.2
5	9.5	9.0
6	11.4	10.8
7	13.3	12.6
8	15.2	14.4
9	17.1	16.2

	9	**8**
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

From the top:

For **34°+** or **214°+**, read as printed; for **124°+** or **304°+**, read co-function.

From the bottom:

For **55°+** or **235°+**, read as printed; for **145°+** or **325°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.75 859	18	9.84 523	27	0.15 477	9.91 336	8	**60**
1	9.75 877	18	9.84 550	26	0.15 450	9.91 328	9	59
2	9.75 895	18	9.84 576	27	0.15 424	9.91 319	9	58
3	9.75 913	18	9.84 603	27	0.15 397	9.91 310	9	57
4	9.75 931	18	9.84 630	27	0.15 370	9.91 301	9	56
5	9.75 949	18	9.84 657	27	0.15 343	9.91 292	9	**55**
6	9.75 967	18	9.84 684	27	0.15 316	9.91 283	9	54
7	9.75 985	18	9.84 711	27	0.15 289	9.91 274	8	53
8	9.76 003	18	9.84 738	26	0.15 262	9.91 266	9	52
9	9.76 021	18	9.84 764	27	0.15 236	9.91 257	9	51
10	9.76 039	18	9.84 791	27	0.15 209	9.91 248	9	**50**
11	9.76 057	18	9.84 818	27	0.15 182	9.91 239	9	49
12	9.76 075	18	9.84 845	27	0.15 155	9.91 230	9	48
13	9.76 093	18	9.84 872	27	0.15 128	9.91 221	9	47
14	9.76 111	18	9.84 899	26	0.15 101	9.91 212	9	46
15	9.76 129	17	9.84 925	27	0.15 075	9.91 203	9	**45**
16	9.76 146	18	9.84 952	27	0.15 048	9.91 194	9	44
17	9.76 164	18	9.84 979	27	0.15 021	9.91 185	9	43
18	9.76 182	18	9.85 006	27	0.14 994	9.91 176	9	42
19	9.76 200	18	9.85 033	26	0.14 967	9.91 167	9	41
20	9.76 218	18	9.85 059	27	0.14 941	9.91 158	9	**40**
21	9.76 236	17	9.85 086	27	0.14 914	9.91 149	8	39
22	9.76 253	18	9.85 113	27	0.14 887	9.91 141	9	38
23	9.76 271	18	9.85 140	26	0.14 860	9.91 132	9	37
24	9.76 289	18	9.85 166	27	0.14 834	9.91 123	9	36
25	9.76 307	17	9.85 193	27	0.14 807	9.91 114	9	**35**
26	9.76 324	18	9.85 220	27	0.14 780	9.91 105	9	34
27	9.76 342	18	9.85 247	26	0.14 753	9.91 096	9	33
28	9.76 360	18	9.85 273	27	0.14 727	9.91 087	9	32
29	9.76 378	17	9.85 300	27	0.14 700	9.91 078	9	31
30	9.76 395	18	9.85 327	27	0.14 673	9.91 069	9	**30**
31	9.76 413	18	9.85 354	26	0.14 646	9.91 060	9	29
32	9.76 431	17	9.85 380	27	0.14 620	9.91 051	9	28
33	9.76 448	18	9.85 407	27	0.14 593	9.91 042	9	27
34	9.76 466	18	9.85 434	26	0.14 566	9.91 033	10	26
35	9.76 484	17	9.85 460	27	0.14 540	9.91 023	9	**25**
36	9.76 501	18	9.85 487	27	0.14 513	9.91 014	9	24
37	9.76 519	18	9.85 514	26	0.14 486	9.91 005	9	23
38	9.76 537	17	9.85 540	27	0.14 460	9.90 996	9	22
39	9.76 554	18	9.85 567	27	0.14 433	9.90 987	9	21
40	9.76 572	18	9.85 594	26	0.14 406	9.90 978	9	**20**
41	9.76 590	17	9.85 620	27	0.14 380	9.90 969	9	19
42	9.76 607	18	9.85 647	27	0.14 353	9.90 960	9	18
43	9.76 625	17	9.85 674	26	0.14 326	9.90 951	9	17
44	9.76 642	18	9.85 700	27	0.14 300	9.90 942	9	16
45	9.76 660	17	9.85 727	27	0.14 273	9.90 933	9	**15**
46	9.76 677	18	9.85 754	26	0.14 246	9.90 924	9	14
47	9.76 695	17	9.85 780	27	0.14 220	9.90 915	9	13
48	9.76 712	18	9.85 807	27	0.14 193	9.90 906	10	12
49	9.76 730	17	9.85 834	26	0.14 166	9.90 896	9	11
50	9.76 747	18	9.85 860	27	0.14 140	9.90 887	9	**10**
51	9.76 765	17	9.85 887	26	0.14 113	9.90 878	9	9
52	9.76 782	18	9.85 913	27	0.14 087	9.90 869	9	8
53	9.76 800	17	9.85 940	27	0.14 060	9.90 860	9	7
54	9.76 817	18	9.85 967	26	0.14 033	9.90 851	9	6
55	9.76 835	17	9.85 993	27	0.14 007	9.90 842	10	**5**
56	9.76 852	18	9.86 020	26	0.13 980	9.90 832	9	4
57	9.76 870	17	9.86 046	27	0.13 954	9.90 823	9	3
58	9.76 887	17	9.86 073	27	0.13 927	9.90 814	9	2
59	9.76 904	18	9.86 100	26	0.13 900	9.90 805	9	1
60	9.76 922		9.86 126		0.13 874	9.90 796		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	27	26	18
2	5.4	5.2	3.6
3	8.1	7.8	5.4
4	10.8	10.4	7.2
5	13.5	13.0	9.0
6	16.2	15.6	10.8
7	18.9	18.2	12.6
8	21.6	20.8	14.4
9	24.3	23.4	16.2

	17	10
2	3.4	2.0
3	5.1	3.0
4	6.8	4.0
5	8.5	5.0
6	10.2	6.0
7	11.9	7.0
8	13.6	8.0
9	15.3	9.0

	9	8
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

From the top:

For **35°+** or **215°+**, read as printed; for **125°+** or **305°+**, read co-function.

From the bottom:

For **54°+** or **234°+**, read as printed; for **144°+** or **324°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.76 922	17	9.86 126	27	0.13 874	9.90 796	9	**60**
1	9.76 939	18	9.86 153	26	0.13 847	9.90 787	10	59
2	9.76 957	17	9.86 179	27	0.13 821	9.90 777	9	58
3	9.76 974	17	9.86 206	26	0.13 794	9.90 768	9	57
4	9.76 991	18	9.86 232	27	0.13 768	9.90 759	9	56
5	9.77 009	17	9.86 259	26	0.13 741	9.90 750	9	**55**
6	9.77 026	17	9.86 285	27	0.13 715	9.90 741	10	54
7	9.77 043	18	9.86 312	26	0.13 688	9.90 731	9	53
8	9.77 061	17	9.86 338	27	0.13 662	9.90 722	9	52
9	9.77 078	17	9.86 365	27	0.13 635	9.90 713	9	51
10	9.77 095	17	9.86 392	26	0.13 608	9.90 704	10	**50**
11	9.77 112	18	9.86 418	27	0.13 582	9.90 694	9	49
12	9.77 130	17	9.86 445	26	0.13 555	9.90 685	9	48
13	9.77 147	17	9.86 471	27	0.13 529	9.90 676	9	47
14	9.77 164	17	9.86 498	26	0.13 502	9.90 667	10	46
15	9.77 181	18	9.86 524	27	0.13 476	9.90 657	9	**45**
16	9.77 199	17	9.86 551	26	0.13 449	9.90 648	9	44
17	9.77 216	17	9.86 577	26	0.13 423	9.90 639	9	43
18	9.77 233	17	9.86 603	27	0.13 397	9.90 630	10	42
19	9.77 250	18	9.86 630	26	0.13 370	9.90 620	9	41
20	9.77 268	17	9.86 656	27	0.13 344	9.90 611	9	**40**
21	9.77 285	17	9.86 683	26	0.13 317	9.90 602	10	39
22	9.77 302	17	9.86 709	27	0.13 291	9.90 592	9	38
23	9.77 319	17	9.86 736	26	0.13 264	9.90 583	9	37
24	9.77 336	17	9.86 762	27	0.13 238	9.90 574	9	36
25	9.77 353	17	9.86 789	26	0.13 211	9.90 565	10	**35**
26	9.77 370	17	9.86 815	27	0.13 185	9.90 555	9	34
27	9.77 387	18	9.86 842	26	0.13 158	9.90 546	9	33
28	9.77 405	17	9.86 868	26	0.13 132	9.90 537	10	32
29	9.77 422	17	9.86 894	27	0.13 106	9.90 527	9	31
30	9.77 439	17	9.86 921	26	0.13 079	9.90 518	9	**30**
31	9.77 456	17	9.86 947	27	0.13 053	9.90 509	10	29
32	9.77 473	17	9.86 974	26	0.13 026	9.90 499	9	28
33	9.77 490	17	9.87 000	27	0.13 000	9.90 490	10	27
34	9.77 507	17	9.87 027	26	0.12 973	9.90 480	9	26
35	9.77 524	17	9.87 053	26	0.12 947	9.90 471	9	**25**
36	9.77 541	17	9.87 079	27	0.12 921	9.90 462	10	24
37	9.77 558	17	9.87 106	26	0.12 894	9.90 452	9	23
38	9.77 575	17	9.87 132	26	0.12 868	9.90 443	9	22
39	9.77 592	17	9.87 158	27	0.12 842	9.90 434	10	21
40	9.77 609	17	9.87 185	26	0.12 815	9.90 424	9	**20**
41	9.77 626	17	9.87 211	27	0.12 789	9.90 415	10	19
42	9.77 643	17	9.87 238	26	0.12 762	9.90 405	9	18
43	9.77 660	17	9.87 264	26	0.12 736	9.90 396	10	17
44	9.77 677	17	9.87 290	27	0.12 710	9.90 386	9	16
45	9.77 694	17	9.87 317	26	0.12 683	9.90 377	9	**15**
46	9.77 711	17	9.87 343	26	0.12 657	9.90 368	10	14
47	9.77 728	16	9.87 369	27	0.12 631	9.90 358	9	13
48	9.77 744	17	9.87 396	26	0.12 604	9.90 349	10	12
49	9.77 761	17	9.87 422	26	0.12 578	9.90 339	9	11
50	9.77 778	17	9.87 448	27	0.12 552	9.90 330	10	**10**
51	9.77 795	17	9.87 475	26	0.12 525	9.90 320	9	9
52	9.77 812	17	9.87 501	26	0.12 499	9.90 311	10	8
53	9.77 829	17	9.87 527	27	0.12 473	9.90 301	9	7
54	9.77 846	16	9.87 554	26	0.12 446	9.90 292	10	6
55	9.77 862	17	9.87 580	26	0.12 420	9.90 282	9	**5**
56	9.77 879	17	9.87 606	27	0.12 394	9.90 273	10	4
57	9.77 896	17	9.87 633	26	0.12 367	9.90 263	9	3
58	9.77 913	17	9.87 659	26	0.12 341	9.90 254	10	2
59	9.77 930	16	9.87 685	26	0.12 315	9.90 244	9	1
60	9.77 946		9.87 711		0.12 289	9.90 235		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	27	**26**	**18**
2	5.4	5.2	3.6
3	8.1	7.8	5.4
4	10.8	10.4	7.2
5	13.5	13.0	9.0
6	16.2	15.6	10.8
7	18.9	18.2	12.6
8	21.6	20.8	14.4
9	24.3	23.4	16.2

	17	**16**
2	3.4	3.2
3	5.1	4.8
4	6.8	6.4
5	8.5	8.0
6	10.2	9.6
7	11.9	11.2
8	13.6	12.8
9	15.3	14.4

	10	**9**
2	2.0	1.8
3	3.0	2.7
4	4.0	3.6
5	5.0	4.5
6	6.0	5.4
7	7.0	6.3
8	8.0	7.2
9	9.0	8.1

From the top :

For **36°+** or **216°+**, read as printed; for **126°+** or **306°+**, read co-function.

From the bottom :

For **53°+** or **233°+**, read as printed; for **143°+** or **323°+**, read co-function.

Prop. Pts.

'	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.77 946	17	9.87 711	27	0.12 289	9.90 235	10	**60**
1	9.77 963	17	9.87 738	26	0.12 262	9.90 225	9	59
2	9.77 980	17	9.87 764	26	0.12 236	9.90 216	10	58
3	9.77 997	16	9.87 790	27	0.12 210	9.90 206	9	57
4	9.78 013	17	9.87 817	26	0.12 183	9.90 197	10	56
5	9.78 030	17	9.87 843	26	0.12 157	9.90 187	9	**55**
6	9.78 047	16	9.87 869	26	0.12 131	9.90 178	10	54
7	9.78 063	17	9.87 895	27	0.12 105	9.90 168	9	53
8	9.78 080	17	9.87 922	26	0.12 078	9.90 159	10	52
9	9.78 097	16	9.87 948	26	0.12 052	9.90 149	10	51
10	9.78 113	17	9.87 974	26	0.12 026	9.90 139	9	**50**
11	9.78 130	17	9.88 000	27	0.12 000	9.90 130	10	49
12	9.78 147	16	9.88 027	26	0.11 973	9.90 120	9	48
13	9.78 163	17	9.88 053	26	0.11 947	9.90 111	10	47
14	9.78 180	17	9.88 079	26	0.11 921	9.90 101	10	46
15	9.78 197	16	9.88 105	26	0.11 895	9.90 091	9	**45**
16	9.78 213	17	9.88 131	27	0.11 869	9.90 082	10	44
17	9.78 230	16	9.88 158	26	0.11 842	9.90 072	9	43
18	9.78 246	17	9.88 184	26	0.11 816	9.90 063	10	42
19	9.78 263	17	9.88 210	26	0.11 790	9.90 053	10	41
20	9.78 280	16	9.88 236	26	0.11 764	9.90 043	9	**40**
21	9.78 296	17	9.88 262	27	0.11 738	9.90 034	10	39
22	9.78 313	16	9.88 289	26	0.11 711	9.90 024	10	38
23	9.78 329	17	9.88 315	26	0.11 685	9.90 014	9	37
24	9.78 346	16	9.88 341	26	0.11 659	9.90 005	10	36
25	9.78 362	17	9.88 367	26	0.11 633	9.89 995	10	**35**
26	9.78 379	16	9.88 393	27	0.11 607	9.89 985	9	34
27	9.78 395	17	9.88 420	26	0.11 580	9.89 976	10	33
28	9.78 412	16	9.88 446	26	0.11 554	9.89 966	10	32
29	9.78 428	17	9.88 472	26	0.11 528	9.89 956	9	31
30	9.78 445	16	9.88 498	26	0.11 502	9.89 947	10	**30**
31	9.78 461	17	9.88 524	26	0.11 476	9.89 937	10	29
32	9.78 478	16	9.88 550	27	0.11 450	9.89 927	9	28
33	9.78 494	16	9.88 577	26	0.11 423	9.89 918	10	27
34	9.78 510	17	9.88 603	26	0.11 397	9.89 908	10	26
35	9.78 527	16	9.88 629	26	0.11 371	9.89 898	10	**25**
36	9.78 543	17	9.88 655	26	0.11 345	9.89 888	9	24
37	9.78 560	16	9.88 681	26	0.11 319	9.89 879	10	23
38	9.78 576	16	9.88 707	26	0.11 293	9.89 869	10	22
39	9.78 592	17	9.88 733	26	0.11 267	9.89 859	10	21
40	9.78 609	16	9.88 759	27	0.11 241	9.89 849	9	**20**
41	9.78 625	17	9.88 786	26	0.11 214	9.89 840	10	19
42	9.78 642	16	9.88 812	26	0.11 188	9.89 830	10	18
43	9.78 658	16	9.88 838	26	0.11 162	9.89 820	10	17
44	9.78 674	17	9.88 864	26	0.11 136	9.89 810	9	16
45	9.78 691	16	9.88 890	26	0.11 110	9.89 801	10	**15**
46	9.78 707	16	9.88 916	26	0.11 084	9.89 791	10	14
47	9.78 723	16	9.88 942	26	0.11 058	9.89 781	10	13
48	9.78 739	17	9.88 968	26	0.11 032	9.89 771	10	12
49	9.78 756	16	9.88 994	26	0.11 006	9.89 761	9	11
50	9.78 772	16	9.89 020	26	0.10 980	9.89 752	10	**10**
51	9.78 788	17	9.89 046	27	0.10 954	9.89 742	10	9
52	9.78 805	16	9.89 073	26	0.10 927	9.89 732	10	8
53	9.78 821	16	9.89 099	26	0.10 901	9.89 722	10	7
54	9.78 837	16	9.89 125	26	0.10 875	9.89 712	10	6
55	9.78 853	16	9.89 151	26	0.10 849	9.89 702	9	**5**
56	9.78 869	17	9.89 177	26	0.10 823	9.89 693	10	4
57	9.78 886	16	9.89 203	26	0.10 797	9.89 683	10	3
58	9.78 902	16	9.89 229	26	0.10 771	9.89 673	10	2
59	9.78 918	16	9.89 255	26	0.10 745	9.89 663	10	1
60	9.78 934		9.89 281		0.10 719	9.89 653		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	'

Prop. Pts.

	27	26	17
2	5.4	5.2	3.4
3	8.1	7.8	5.1
4	10.8	10.4	6.8
5	13.5	13.0	8.5
6	16.2	15.6	10.2
7	18.9	18.2	11.9
8	21.6	20.8	13.6
9	24.3	23.4	15.3

	16	10	9
2	3.2	2.0	1.8
3	4.8	3.0	2.7
4	6.4	4.0	3.6
5	8.0	5.0	4.5
6	9.6	6.0	5.4
7	11.2	7.0	6.3
8	12.8	8.0	7.2
9	14.4	9.0	8.1

From the top:

For **37°+** or **217°+**, read as printed; for **127°+** or **307°+**, read co-function.

From the bottom:

For **52°+** or **232°+**, read as printed; for **142°+** or **322°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.78 934	16	9.89 281	26	0.10 719	9.89 653	10	**60**
1	9.78 950	17	9.89 307	26	0.10 693	9.89 643	10	59
2	9.78 967	16	9.89 333	26	0.10 667	9.89 633	9	58
3	9.78 983	16	9.89 359	26	0.10 641	9.89 624	10	57
4	9.78 999	16	9.89 385	26	0.10 615	9.89 614	10	56
5	9.79 015	16	9.89 411	26	0.10 589	9.89 604	10	**55**
6	9.79 031	16	9.89 437	26	0.10 563	9.89 594	10	54
7	9.79 047	16	9.89 463	26	0.10 537	9.89 584	10	53
8	9.79 063	16	9.89 489	26	0.10 511	9.89 574	10	52
9	9.79 079	16	9.89 515	26	0.10 485	9.89 564	10	51
10	9.79 095	16	9.89 541	26	0.10 459	9.89 554	10	**50**
11	9.79 111	17	9.89 567	26	0.10 433	9.89 544	10	49
12	9.79 128	16	9.89 593	26	0.10 407	9.89 534	10	48
13	9.79 144	16	9.89 619	26	0.10 381	9.89 524	10	47
14	9.79 160	16	9.89 645	26	0.10 355	9.89 514	10	46
15	9.79 176	16	9.89 671	26	0.10 329	9.89 504	9	**45**
16	9.79 192	16	9.89 697	26	0.10 303	9.89 495	10	44
17	9.79 208	16	9.89 723	26	0.10 277	9.89 485	10	43
18	9.79 224	16	9.89 749	26	0.10 251	9.89 475	10	42
19	9.79 240	16	9.89 775	26	0.10 225	9.89 465	10	41
20	9.79 256	16	9.89 801	26	0.10 199	9.89 455	10	**40**
21	9.79 272	16	9.89 827	26	0.10 173	9.89 445	10	39
22	9.79 288	16	9.89 853	26	0.10 147	9.89 435	10	38
23	9.79 304	15	9.89 879	26	0.10 121	9.89 425	10	37
24	9.79 319	16	9.89 905	26	0.10 095	9.89 415	10	36
25	9.79 335	16	9.89 931	26	0.10 069	9.89 405	10	**35**
26	9.79 351	16	9.89 957	26	0.10 043	9.89 395	10	34
27	9.79 367	16	9.89 983	26	0.10 017	9.89 385	10	33
28	9.79 383	16	9.90 009	26	0.09 991	9.89 375	11	32
29	9.79 399	16	9.90 035	26	0.09 965	9.89 364	10	31
30	9.79 415	16	9.90 061	25	0.09 939	9.89 354	10	**30**
31	9.79 431	16	9.90 086	26	0.09 914	9.89 344	10	29
32	9.79 447	16	9.90 112	26	0.09 888	9.89 334	10	28
33	9.79 463	15	9.90 138	26	0.09 862	9.89 324	10	27
34	9.79 478	16	9.90 164	26	0.09 836	9.89 314	10	26
35	9.79 494	16	9.90 190	26	0.09 810	9.89 304	10	**25**
36	9.79 510	16	9.90 216	26	0.09 784	9.89 294	10	24
37	9.79 526	16	9.90 242	26	0.09 758	9.89 284	10	23
38	9.79 542	16	9.90 268	26	0.09 732	9.89 274	10	22
39	9.79 558	15	9.90 294	26	0.09 706	9.89 264	10	21
40	9.79 573	16	9.90 320	26	0.09 680	9.89 254	10	**20**
41	9.79 589	16	9.90 346	25	0.09 654	9.89 244	11	19
42	9.79 605	16	9.90 371	26	0.09 629	9.89 233	10	18
43	9.79 621	15	9.90 397	26	0.09 603	9.89 223	10	17
44	9.79 636	16	9.90 423	26	0.09 577	9.89 213	10	16
45	9.79 652	16	9.90 449	26	0.09 551	9.89 203	10	**15**
46	9.79 668	16	9.90 475	26	0.09 525	9.89 193	10	14
47	9.79 684	15	9.90 501	26	0.09 499	9.89 183	10	13
48	9.79 699	16	9.90 527	26	0.09 473	9.89 173	11	12
49	9.79 715	16	9.90 553	25	0.09 447	9.89 162	10	11
50	9.79 731	15	9.90 578	26	0.09 422	9.89 152	10	**10**
51	9.79 746	16	9.90 604	26	0.09 396	9.89 142	10	9
52	9.79 762	16	9.90 630	26	0.09 370	9.89 132	10	8
53	9.79 778	15	9.90 656	26	0.09 344	9.89 122	10	7
54	9.79 793	16	9.90 682	26	0.09 318	9.89 112	11	6
55	9.79 809	16	9.90 708	26	0.09 292	9.89 101	10	**5**
56	9.79 825	15	9.90 734	25	0.09 266	9.89 091	10	4
57	9.79 840	16	9.90 759	26	0.09 241	9.89 081	10	3
58	9.79 856	16	9.90 785	26	0.09 215	9.89 071	11	2
59	9.79 872	15	9.90 811	26	0.09 189	9.89 060	10	1
60	9.79 887		9.90 837		0.09 163	9.89 050		**0**
c	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	26	25	17
2	5.2	5.0	3.4
3	7.8	7.5	5.1
4	10.4	10.0	6.8
5	13.0	12.5	8.5
6	15.6	15.0	10.2
7	18.2	17.5	11.9
8	20.8	20.0	13.6
9	23.4	22.5	15.3

	16	15	11
2	3.2	3.0	2.2
3	4.8	4.5	3.3
4	6.4	6.0	4.4
5	8.0	7.5	5.5
6	9.6	9.0	6.6
7	11.2	10.5	7.7
8	12.8	12.0	8.8
9	14.4	13.5	9.9

	10	9
2	2.0	1.8
3	3.0	2.7
4	4.0	3.6
5	5.0	4.5
6	6.0	5.4
7	7.0	6.3
8	8.0	7.2
9	9.0	8.1

From the top:

For **38°**+ or **218°**+, read as printed; for **128°**+ or **308°**+, read co-function.

From the bottom:

For **51°**+ or **231°**+, read as printed; for **141°**+ or **321°**+, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.79 887		9.90 837		0.09 163	9.89 050		60
1	9.79 903	16	9.90 863	26	0.09 137	9.89 040	10	59
2	9.79 918	15	9.90 889	26	0.09 111	9.89 030	10	58
3	9.79 934	16	9.90 914	25	0.09 086	9.89 020	10	57
4	9.79 950	16	9.90 940	26	0.09 060	9.89 009	11	56
5	9.79 965	15	9.90 966	26	0.09 034	9.88 999	10	55
6	9.79 981	16	9.90 992	26	0.09 008	9.88 989	10	54
7	9.79 996	15	9.91 018	26	0.08 982	9.88 978	11	53
8	9.80 012	16	9.91 043	25	0.08 957	9.88 968	10	52
9	9.80 027	15	9.91 069	26	0.08 931	9.88 958	10	51
10	9.80 043	16	9.91 095	26	0.08 905	9.88 948	10	50
11	9.80 058	15	9.91 121	26	0.08 879	9.88 937	11	49
12	9.80 074	16	9.91 147	26	0.08 853	9.88 927	10	48
13	9.80 089	15	9.91 172	25	0.08 828	9.88 917	10	47
14	9.80 105	16	9.91 198	26	0.08 802	9.88 906	11	46
15	9.80 120	15	9.91 224	26	0.08 776	9.88 896	10	45
16	9.80 136	16	9.91 250	26	0.08 750	9.88 886	10	44
17	9.80 151	15	9.91 276	26	0.08 724	9.88 875	11	43
18	9.80 166	15	9.91 301	25	0.08 699	9.88 865	10	42
19	9.80 182	16	9.91 327	26	0.08 673	9.88 855	10	41
20	9.80 197	15	9.91 353	26	0.08 647	9.88 844	11	40
21	9.80 213	16	9.91 379	26	0.08 621	9.88 834	10	39
22	9.80 228	15	9.91 404	25	0.08 596	9.88 824	10	38
23	9.80 244	16	9.91 430	26	0.08 570	9.88 813	11	37
24	9.80 259	15	9.91 456	26	0.08 544	9.88 803	10	36
25	9.80 274	15	9.91 482	26	0.08 518	9.88 793	10	35
26	9.80 290	16	9.91 507	25	0.08 493	9.88 782	11	34
27	9.80 305	15	9.91 533	26	0.08 467	9.88 772	10	33
28	9.80 320	15	9.91 559	26	0.08 441	9.88 761	11	32
29	9.80 336	16	9.91 585	26	0.08 415	9.88 751	10	31
30	9.80 351	15	9.91 610	25	0.08 390	9.88 741	10	30
31	9.80 366	15	9.91 636	26	0.08 364	9.88 730	11	29
32	9.80 382	16	9.91 662	26	0.08 338	9.88 720	10	28
33	9.80 397	15	9.91 688	26	0.08 312	9.88 709	11	27
34	9.80 412	15	9.91 713	25	0.08 287	9.88 699	10	26
35	9.80 428	16	9.91 739	26	0.08 261	9.88 688	11	25
36	9.80 443	15	9.91 765	26	0.08 235	9.88 678	10	24
37	9.80 458	15	9.91 791	26	0.08 209	9.88 668	10	23
38	9.80 473	15	9.91 816	25	0.08 184	9.88 657	11	22
39	9.80 489	16	9.91 842	26	0.08 158	9.88 647	10	21
40	9.80 504	15	9.91 868	26	0.08 132	9.88 636	11	20
41	9.80 519	15	9.91 893	25	0.08 107	9.88 626	10	19
42	9.80 534	15	9.91 919	26	0.08 081	9.88 615	11	18
43	9.80 550	16	9.91 945	26	0.08 055	9.88 605	10	17
44	9.80 565	15	9.91 971	26	0.08 029	9.88 594	11	16
45	9.80 580	15	9.91 996	25	0.08 004	9.88 584	10	15
46	9.80 595	15	9.92 022	26	0.07 978	9.88 573	11	14
47	9.80 610	15	9.92 048	26	0.07 952	9.88 563	10	13
48	9.80 625	15	9.92 073	25	0.07 927	9.88 552	11	12
49	9.80 641	16	9.92 099	26	0.07 901	9.88 542	10	11
50	9.80 656	15	9.92 125	26	0.07 875	9.88 531	11	10
51	9.80 671	15	9.92 150	25	0.07 850	9.88 521	10	9
52	9.80 686	15	9.92 176	26	0.07 824	9.88 510	11	8
53	9.80 701	15	9.92 202	26	0.07 798	9.88 499	11	7
54	9.80 716	15	9.92 227	25	0.07 773	9.88 489	10	6
55	9.80 731	15	9.92 253	26	0.07 747	9.88 478	11	5
56	9.80 746	15	9.92 279	26	0.07 721	9.88 468	10	4
57	9.80 762	16	9.92 304	25	0.07 696	9.88 457	11	3
58	9.80 777	15	9.92 330	26	0.07 670	9.88 447	10	2
59	9.80 792	15	9.92 356	26	0.07 644	9.88 436	11	1
60	9.80 807	15	9.92 381	25	0.07 619	9.88 425	11	0

Prop. Pts.

	26	25	16
2	5.2	5.0	3.2
3	7.8	7.5	4.8
4	10.4	10.0	6.4
5	13.0	12.5	8.0
6	15.6	15.0	9.6
7	18.2	17.5	11.2
8	20.8	20.0	12.8
9	23.4	22.5	14.4

	15	11	10
2	3.0	2.2	2.0
3	4.5	3.3	3.0
4	6.0	4.4	4.0
5	7.5	5.5	5.0
6	9.0	6.6	6.0
7	10.5	7.7	7.0
8	12.0	8.8	8.0
9	13.5	9.9	9.0

From the top:

For **39°+** or **219°+**, read as printed; for **129°+** or **309°+**, read co-function.

From the bottom:

For **50°+** or **230°+**, read as printed; for **140°+** or **320°+**, read co-function.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.80 807	15	9.92 381	26	0.07 619	9.88 425	10	**60**
1	9.80 822	15	9.92 407	26	0.07 593	9.88 415	11	59
2	9.80 837	15	9.92 433	25	0.07 567	9.88 404	10	58
3	9.80 852	15	9.92 458	26	0.07 542	9.88 394	11	57
4	9.80 867	15	9.92 484	26	0.07 516	9.88 383	11	56
5	9.80 882	15	9.92 510	25	0.07 490	9.88 372	10	**55**
6	9.80 897	15	9.92 535	26	0.07 465	9.88 362	11	54
7	9.80 912	15	9.92 561	26	0.07 439	9.88 351	11	53
8	9.80 927	15	9.92 587	25	0.07 413	9.88 340	10	52
9	9.80 942	15	9.92 612	26	0.07 388	9.88 330	11	51
10	9.80 957	15	9.92 638	25	0.07 362	9.88 319	11	**50**
11	9.80 972	15	9.92 663	26	0.07 337	9.88 308	10	49
12	9.80 987	15	9.92 689	26	0.07 311	9.88 298	11	48
13	9.81 002	15	9.92 715	25	0.07 285	9.88 287	11	47
14	9.81 017	15	9.92 740	26	0.07 260	9.88 276	10	46
15	9.81 032	15	9.92 766	26	0.07 234	9.88 266	11	**45**
16	9.81 047	14	9.92 792	25	0.07 208	9.88 255	11	44
17	9.81 061	15	9.92 817	26	0.07 183	9.88 244	10	43
18	9.81 076	15	9.92 843	25	0.07 157	9.88 234	11	42
19	9.81 091	15	9.92 868	26	0.07 132	9.88 223	11	41
20	9.81 106	15	9.92 894	26	0.07 106	9.88 212	11	**40**
21	9.81 121	15	9.92 920	25	0.07 080	9.88 201	10	39
22	9.81 136	15	9.92 945	26	0.07 055	9.88 191	11	38
23	9.81 151	15	9.92 971	25	0.07 029	9.88 180	11	37
24	9.81 166	14	9.92 996	26	0.07 004	9.88 169	11	36
25	9.81 180	15	9.93 022	26	0.06 978	9.88 158	10	**35**
26	9.81 195	15	9.93 048	25	0.06 952	9.88 148	11	34
27	9.81 210	15	9.93 073	26	0.06 927	9.88 137	11	33
28	9.81 225	15	9.93 099	25	0.06 901	9.88 126	11	32
29	9.81 240	14	9.93 124	26	0.06 876	9.88 115	10	31
30	9.81 254	15	9.93 150	25	0.06 850	9.88 105	11	**30**
31	9.81 269	15	9.93 175	26	0.06 825	9.88 094	11	29
32	9.81 284	15	9.93 201	26	0.06 799	9.88 083	11	28
33	9.81 299	15	9.93 227	25	0.06 773	9.88 072	11	27
34	9.81 314	14	9.93 252	26	0.06 748	9.88 061	10	26
35	9.81 328	15	9.93 278	25	0.06 722	9.88 051	11	**25**
36	9.81 343	15	9.93 303	26	0.06 697	9.88 040	11	24
37	9.81 358	14	9.93 329	25	0.06 671	9.88 029	11	23
38	9.81 372	15	9.93 354	26	0.06 646	9.88 018	11	22
39	9.81 387	15	9.93 380	26	0.06 620	9.88 007	11	21
40	9.81 402	15	9.93 406	25	0.06 594	9.87 996	11	**20**
41	9.81 417	14	9.93 431	26	0.06 569	9.87 985	10	19
42	9.81 431	15	9.93 457	25	0.06 543	9.87 975	11	18
43	9.81 446	15	9.93 482	26	0.06 518	9.87 964	11	17
44	9.81 461	14	9.93 508	25	0.06 492	9.87 953	11	16
45	9.81 475	15	9.93 533	26	0.06 467	9.87 942	11	**15**
46	9.81 490	15	9.93 559	25	0.06 441	9.87 931	11	14
47	9.81 505	14	9.93 584	26	0.06 416	9.87 920	11	13
48	9.81 519	15	9.93 610	26	0.06 390	9.87 909	11	12
49	9.81 534	15	9.93 636	25	0.06 364	9.87 898	11	11
50	9.81 549	14	9.93 661	26	0.06 339	9.87 887	10	**10**
51	9.81 563	15	9.93 687	25	0.06 313	9.87 877	11	9
52	9.81 578	14	9.93 712	26	0.06 288	9.87 866	11	8
53	9.81 592	15	9.93 738	25	0.06 262	9.87 855	11	7
54	9.81 607	15	9.93 763	26	0.06 237	9.87 844	11	6
55	9.81 622	14	9.93 789	25	0.06 211	9.87 833	11	**5**
56	9.81 636	15	9.93 814	26	0.06 186	9.87 822	11	4
57	9.81 651	14	9.93 840	25	0.06 160	9.87 811	11	3
58	9.81 665	15	9.93 865	26	0.06 135	9.87 800	11	2
59	9.81 680	14	9.93 891	25	0.06 109	9.87 789	11	1
60	9.81 694		9.93 916		0.06 084	9.87 778		**0**
′	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	26	**25**	**15**
2	5.2	5.0	3.0
3	7.8	7.5	4.5
4	10.4	10.0	6.0
5	13.0	12.5	7.5
6	15.6	15.0	9.0
7	18.2	17.5	10.5
8	20.8	20.0	12.0
9	23.4	22.5	13.5

	14	**11**	**10**
2	2.8	2.2	2.0
3	4.2	3.3	3.0
4	5.6	4.4	4.0
5	7.0	5.5	5.0
6	8.4	6.6	6.0
7	9.8	7.7	7.0
8	11.2	8.8	8.0
9	12.6	9.9	9.0

From the top:

For **40°+** or **220°+**, read as printed; for **130°+** or **310°+**, read co-function.

From the bottom:

For **49°+** or **229°+**, read as printed; for **139°+** or **319°+**, read co-function.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.81 694	15	9.93 916	26	0.06 084	9.87 778	11	**60**
1	9.81 709	14	9.93 942	25	0.06 058	9.87 767	11	59
2	9.81 723	15	9.93 967	26	0.06 033	9.87 756	11	58
3	9.81 738	14	9.93 993	25	0.06 007	9.87 745	11	57
4	9.81 752	15	9.94 018	26	0.05 982	9.87 734	11	56
5	9.81 767	14	9.94 044	25	0.05 956	9.87 723	11	**55**
6	9.81 781	15	9.94 069	26	0.05 931	9.87 712	11	54
7	9.81 796	14	9.94 095	25	0.05 905	9.87 701	11	53
8	9.81 810	15	9.94 120	26	0.05 880	9.87 690	11	52
9	9.81 825	14	9.94 146	25	0.05 854	9.87 679	11	51
10	9.81 839	15	9.94 171	26	0.05 829	9.87 668	11	**50**
11	9.81 854	14	9.94 197	25	0.05 803	9.87 657	11	49
12	9.81 868	14	9.94 222	26	0.05 778	9.87 646	11	48
13	9.81 882	15	9.94 248	25	0.05 752	9.87 635	11	47
14	9.81 897	14	9.94 273	26	0.05 727	9.87 624	11	46
15	9.81 911	15	9.94 299	25	0.05 701	9.87 613	12	**45**
16	9.81 926	14	9.94 324	26	0.05 676	9.87 601	11	44
17	9.81 940	15	9.94 350	25	0.05 650	9.87 590	11	43
18	9.81 955	14	9.94 375	26	0.05 625	9.87 579	11	42
19	9.81 969	14	9.94 401	25	0.05 599	9.87 568	11	41
20	9.81 983	15	9.94 426	26	0.05 574	9.87 557	11	**40**
21	9.81 998	14	9.94 452	25	0.05 548	9.87 546	11	39
22	9.82 012	14	9.94 477	26	0.05 523	9.87 535	11	38
23	9.82 026	15	9.94 503	25	0.05 497	9.87 524	11	37
24	9.82 041	14	9.94 528	26	0.05 472	9.87 513	12	36
25	9.82 055	14	9.94 554	25	0.05 446	9.87 501	11	**35**
26	9.82 069	15	9.94 579	25	0.05 421	9.87 490	11	34
27	9.82 084	14	9.94 604	26	0.05 396	9.87 479	11	33
28	9.82 098	14	9.94 630	25	0.05 370	9.87 468	11	32
29	9.82 112	14	9.94 655	26	0.05 345	9.87 457	11	31
30	9.82 126	15	9.94 681	25	0.05 319	9.87 446	12	**30**
31	9.82 141	14	9.94 706	26	0.05 294	9.87 434	11	29
32	9.82 155	14	9.94 732	25	0.05 268	9.87 423	11	28
33	9.82 169	15	9.94 757	26	0.05 243	9.87 412	11	27
34	9.82 184	14	9.94 783	25	0.05 217	9.87 401	11	26
35	9.82 198	14	9.94 808	26	0.05 192	9.87 390	12	**25**
36	9.82 212	14	9.94 834	25	0.05 166	9.87 378	11	24
37	9.82 226	14	9.94 859	25	0.05 141	9.87 367	11	23
38	9.82 240	15	9.94 884	26	0.05 116	9.87 356	11	22
39	9.82 255	14	9.94 910	25	0.05 090	9.87 345	11	21
40	9.82 269	14	9.94 935	26	0.05 065	9.87 334	12	**20**
41	9.82 283	14	9.94 961	25	0.05 039	9.87 322	11	19
42	9.82 297	14	9.94 986	26	0.05 014	9.87 311	11	18
43	9.82 311	15	9.95 012	25	0.04 988	9.87 300	12	17
44	9.82 326	14	9.95 037	25	0.04 963	9.87 288	11	16
45	9.82 340	14	9.95 062	26	0.04 938	9.87 277	11	**15**
46	9.82 354	14	9.95 088	25	0.04 912	9.87 266	11	14
47	9.82 368	14	9.95 113	26	0.04 887	9.87 255	12	13
48	9.82 382	14	9.95 139	25	0.04 861	9.87 243	11	12
49	9.82 396	14	9.95 164	26	0.04 836	9.87 232	11	11
50	9.82 410	14	9.95 190	25	0.04 810	9.87 221	12	**10**
51	9.82 424	15	9.95 215	25	0.04 785	9.87 209	11	9
52	9.82 439	14	9.95 240	26	0.04 760	9.87 198	11	8
53	9.82 453	14	9.95 266	25	0.04 734	9.87 187	12	7
54	9.82 467	14	9.95 291	26	0.04 709	9.87 175	11	6
55	9.82 481	14	9.95 317	25	0.04 683	9.87 164	11	**5**
56	9.82 495	14	9.95 342	26	0.04 658	9.87 153	12	4
57	9.82 509	14	9.95 368	25	0.04 632	9.87 141	11	3
58	9.82 523	14	9.95 393	25	0.04 607	9.87 130	11	2
59	9.82 537	14	9.95 418	26	0.04 582	9.87 119	12	1
60	9.82 551		9.95 444		0.04 556	9.87 107		**0**

Prop. Pts.

	26	25	15
2	5.2	5.0	3.0
3	7.8	7.5	4.5
4	10.4	10.0	6.0
5	13.0	12.5	7.5
6	15.6	15.0	9.0
7	18.2	17.5	10.5
8	20.8	20.0	12.0
9	23.4	22.5	13.5

	14	12	11
2	2.8	2.4	2.2
3	4.2	3.6	3.3
4	5.6	4.8	4.4
5	7.0	6.0	5.5
6	8.4	7.2	6.6
7	9.8	8.4	7.7
8	11.2	9.6	8.8
9	12.6	10.8	9.9

From the top:

For **41°**+ or **221°**+, read as printed; for **131°**+ or **311°**+, read co-function.

From the bottom:

For **48°**+ or **228°**+, read as printed; for **138°**+ or **318°**+, read co-function.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.82 551	14	9.95 444	25	0.04 556	9.87 107	11	**60**
1	9.82 565	14	9.95 469	26	0.04 531	9.87 096	11	59
2	9.82 579	14	9.95 495	25	0.04 505	9.87 085	12	58
3	9.82 593	14	9.95 520	25	0.04 480	9.87 073	11	57
4	9.82 607	14	9.95 545	26	0.04 455	9.87 062	12	56
5	9.82 621	14	9.95 571	25	0.04 429	9.87 050	11	**55**
6	9.82 635	14	9.95 596	26	0.04 404	9.87 039	11	54
7	9.82 649	14	9.95 622	25	0.04 378	9.87 028	12	53
8	9.82 663	14	9.95 647	25	0.04 353	9.87 016	11	52
9	9.82 677	14	9.95 672	26	0.04 328	9.87 005	12	51
10	9.82 691	14	9.95 698	25	0.04 302	9.86 993	11	**50**
11	9.82 705	14	9.95 723	25	0.04 277	9.86 982	12	49
12	9.82 719	14	9.95 748	26	0.04 252	9.86 970	11	48
13	9.82 733	14	9.95 774	25	0.04 226	9.86 959	12	47
14	9.82 747	14	9.95 799	26	0.04 201	9.86 947	11	46
15	9.82 761	14	9.95 825	25	0.04 175	9.86 936	12	**45**
16	9.82 775	13	9.95 850	25	0.04 150	9.86 924	11	44
17	9.82 788	14	9.95 875	26	0.04 125	9.86 913	11	43
18	9.82 802	14	9.95 901	25	0.04 099	9.86 902	12	42
19	9.82 816	14	9.95 926	26	0.04 074	9.86 890	11	41
20	9.82 830	14	9.95 952	25	0.04 048	9.86 879	12	**40**
21	9.82 844	14	9.95 977	25	0.04 023	9.86 867	12	39
22	9.82 858	14	9.96 002	26	0.03 998	9.86 855	11	38
23	9.82 872	13	9.96 028	25	0.03 972	9.86 844	12	37
24	9.82 885	14	9.96 053	25	0.03 947	9.86 832	11	36
25	9.82 899	14	9.96 078	26	0.03 922	9.86 821	12	**35**
26	9.82 913	14	9.96 104	25	0.03 896	9.86 809	11	34
27	9.82 927	14	9.96 129	26	0.03 871	9.86 798	12	33
28	9.82 941	14	9.96 155	25	0.03 845	9.86 786	11	32
29	9.82 955	13	9.96 180	25	0.03 820	9.86 775	12	31
30	9.82 968	14	9.96 205	26	0.03 795	9.86 763	11	**30**
31	9.82 982	14	9.96 231	25	0.03 769	9.86 752	12	29
32	9.82 996	14	9.96 256	25	0.03 744	9.86 740	12	28
33	9.83 010	13	9.96 281	26	0.03 719	9.86 728	11	27
34	9.83 023	14	9.96 307	25	0.03 693	9.86 717	12	26
35	9.83 037	14	9.96 332	25	0 03 668	9.86 705	11	**25**
36	9.83 051	14	9.96 357	26	0.03 643	9.86 694	12	24
37	9.83 065	13	9.96 383	25	0.03 617	9.86 682	12	23
38	9.83 078	14	9.96 408	25	0.03 592	9.86 670	11	22
39	9.83 092	14	9.96 433	26	0.03 567	9.86 659	12	21
40	9.83 106	14	9.96 459	25	0.03 541	9.86 647	12	**20**
41	9.83 120	13	9.96 484	26	0.03 516	9.86 635	11	19
42	9.83 133	14	9.96 510	25	0.03 490	9.86 624	12	18
43	9.83 147	14	9.96 535	25	0.03 465	9.86 612	12	17
44	9.83 161	13	9.96 560	26	0.03 440	9.86 600	11	16
45	9.83 174	14	9.96 586	25	0.03 414	9.86 589	12	**15**
46	9.83 188	14	9.96 611	25	0.03 389	9.86 577	12	14
47	9.83 202	13	9.96 636	26	0.03 364	9.86 565	11	13
48	9.83 215	14	9.96 662	25	0.03 338	9.86 554	12	12
49	9.83 229	13	9.96 687	25	0.03 313	9.86 542	12	11
50	9.83 242	14	9.96 712	26	0.03 288	9.86 530	12	**10**
51	9.83 256	14	9.96 738	25	0.03 262	9.86 518	11	9
52	9.83 270	13	9.96 763	25	0.03 237	9.86 507	12	8
53	9.83 283	14	9.96 788	26	0.03 212	9.86 495	12	7
54	9.83 297	13	9.96 814	25	0.03 186	9.86 483	11	6
55	9.83 310	14	9.96 839	25	0.03 161	9.86 472	12	**5**
56	9.83 324	14	9.96 864	26	0.93 136	9.86 460	12	4
57	9.83 338	13	9.96 890	25	0.03 110	9.86 448	12	3
58	9.83 351	14	9.96 915	25	0.03 085	9.86 436	11	2
59	9.83 365	13	9.96 940	26	0.03 060	9.86 425	12	1
60	9.83 378		9.96 966		0.03 034	9.86 413		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	26	25	14
2	5.2	5.0	2.8
3	7.8	7.5	4.2
4	10.4	10.0	5.6
5	13.0	12.5	7.0
6	15.6	15.0	8.4
7	18.2	17.5	9.8
8	20.8	20.0	11.2
9	23.4	22.5	12.6

	13	12	11
2	2.6	2.4	2.2
3	3.9	3.6	3.3
4	5.2	4.8	4.4
5	6.5	6.0	5.5
6	7.8	7.2	6.6
7	9.1	8.4	7.7
8	10.4	9.6	8.8
9	11.7	10.8	9.9

From the top:

For **42°**+ or **222°**+, read as printed; for **132°**+ or **312°**+, read co-function.

From the bottom:

For **47°**+ or **227°**+, read as printed; for **137°**+ or **317°**+, read co-function.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.83 378	14	9.96 966	25	0.03 034	9.86 413	12	**60**
1	9.83 392	13	9.96 991	25	0.03 009	9.86 401	12	59
2	9.83 405	14	9.97 016	26	0.02 984	9.86 389	12	58
3	9.83 419	13	9.97 042	25	0.02 958	9.86 377	11	57
4	9.83 432	14	9.97 067	25	0.02 933	9.86 366	12	56
5	9.83 446	13	9.97 092	26	0.02 908	9.86 354	12	**55**
6	9.83 459	14	9.97 118	25	0.02 882	9.86 342	12	54
7	9.83 473	13	9.97 143	25	0.02 857	9.86 330	12	53
8	9.83 486	14	9.97 168	25	0.02 832	9.86 318	12	52
9	9.83 500	13	9.97 193	26	0.02 807	9.86 306	11	51
10	9.83 513	14	9.97 219	25	0.02 781	9.86 295	12	**50**
11	9.83 527	13	9.97 244	25	0.02 756	9.86 283	12	49
12	9.83 540	14	9.97 269	26	0.02 731	9.86 271	12	48
13	9.83 554	13	9.97 295	25	0.02 705	9.86 259	12	47
14	9.83 567	14	9.97 320	25	0.02 680	9.86 247	12	46
15	9.83 581	13	9.97 345	26	0.02 655	9.86 235	12	**45**
16	9.83 594	14	9.97 371	25	0.02 629	9.86 223	12	44
17	9.83 608	13	9.97 396	25	0.02 604	9.86 211	11	43
18	9.83 621	13	9.97 421	26	0.02 579	9.86 200	12	42
19	9.83 634	14	9.97 447	25	0.02 553	9.86 188	12	41
20	9.83 648	13	9.97 472	25	0.02 528	9.86 176	12	**40**
21	9.83 661	13	9.97 497	26	0.02 503	9.86 164	12	39
22	9.83 674	14	9.97 523	25	0.02 477	9.86 152	12	38
23	9.83 688	13	9.97 548	25	0.02 452	9.86 140	12	37
24	9.83 701	14	9.97 573	25	0.02 427	9.86 128	12	36
25	9.83 715	13	9.97 598	26	0.02 402	9.86 116	12	**35**
26	9.83 728	13	9.97 624	25	0.02 376	9.86 104	12	34
27	9.83 741	14	9.97 649	25	0.02 351	9.86 092	12	33
28	9.83 755	13	9.97 674	26	0.02 326	9.86 080	12	32
29	9.83 768	13	9.97 700	25	0.02 300	9.86 068	12	31
30	9.83 781	14	9.97 725	25	0 02 275	9.86 056	12	**30**
31	9.83 795	13	9.97 750	26	0.02 250	9.86 044	12	29
32	9.83 808	13	9.97 776	25	0.02 224	9.86 032	12	28
33	9.83 821	13	9.97 801	25	0.02 199	9.86 020	12	27
34	9.83 834	14	9.97 826	25	0.02 174	9.86 008	12	26
35	9.83 848	13	9.97 851	26	0.02 149	9.85 996	12	**25**
36	9.83 861	13	9.97 877	25	0.02 123	9.85 984	12	24
37	9.83 874	13	9.97 902	25	0.02 098	9.85 972	12	23
38	9.83 887	14	9.97 927	26	0.02 073	9.85 960	12	22
39	9.83 901	13	9.97 953	25	0.02 047	9.85 948	12	21
40	9.83 914	13	9.97 978	25	0.02 022	9.85 936	12	**20**
41	9.83 927	13	9.98 003	26	0.01 997	9.85 924	12	19
42	9.83 940	14	9.98 029	25	0.01 971	9.85 912	12	18
43	9.83 954	13	9.98 054	25	0.01 946	9.85 900	12	17
44	9.83 967	13	9.98 079	25	0.01 921	9.85 888	12	16
45	9.83 980	13	9.98 104	26	0.01 896	9.85 876	12	**15**
46	9.83 993	13	9.98 130	25	0.01 870	9.85 864	13	14
47	9.84 006	14	9.98 155	25	0.01 845	9.85 851	12	13
48	9.84 020	13	9.98 180	26	0.01 820	9.85 839	12	12
49	9.84 033	13	9.98 206	25	0.01 794	8.85 827	12	11
50	9.84 046	13	9.98 231	25	0.01 769	9.85 815	12	**10**
51	9.84 059	13	9.98 256	25	0.01 744	9.85 803	12	9
52	9.84 072	13	9.98 281	26	0.01 719	9.85 791	12	8
53	9.84 085	13	9.98 307	25	0.01 693	9.85 779	13	7
54	9.84 098	14	9.98 332	25	0.01 668	9.85 766	12	6
55	9.84 112	13	9.98 357	26	0.01 643	9.85 754	12	**5**
56	9.84 125	13	9.98 383	25	0.01 617	9.85 742	12	4
57	9.84 138	13	9.98 408	25	0.01 592	9.85 730	12	3
58	9.84 151	13	9.98 433	25	0.01 567	9.85 718	12	2
59	9.84 164	13	9.98 458	26	0.01 542	9.85 706	13	1
60	9.84 177		9.98 484		0.01 516	9.85 693		**0**
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	26	25	14
2	5.2	5.0	2.8
3	7.8	7.5	4.2
4	10.4	10.0	5.6
5	13.0	12.5	7.0
6	15.6	15.0	8.4
7	18.2	17.5	9.8
8	20.8	20.0	11.2
9	23.4	22.5	12.6

	13	12	11
2	2.6	2.4	2.2
3	3.9	3.6	3.3
4	5.2	4.8	4.4
5	6.5	6.0	5.5
6	7.8	7.2	6.6
7	9.1	8.4	7.7
8	10.4	9.6	8.8
9	11.7	10.8	9.9

From the top :

For **43°+** or **223°+**, read as printed; for **133°+** or **313°+**, read co-function.

From the bottom :

For **46°+** or **226°+**, read as printed; for **136°+** or **316°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.84 177	13	9.98 484	25	0.01 516	9.85 693	12	60
1	9.84 190	13	9.98 509	25	0.01 491	9.85 681	12	59
2	9.84 203	13	9.98 534	26	0.01 466	9.85 669	12	58
3	9.84 216	13	9.98 560	25	0.01 440	9.85 657	12	57
4	9.84 229	13	9.98 585	25	0.01 415	9.85 645	13	56
5	9.84 242	13	9.98 610	25	0.01 390	9.85 632	12	55
6	9.84 255	14	9.98 635	26	0.01 365	9.85 620	12	54
7	9.84 269	13	9.98 661	25	0.01 339	9.85 608	12	53
8	9.84 282	13	9.98 686	25	0.01 314	9.85 596	13	52
9	9.84 295	13	9.98 711	26	0.01 289	9.85 583	12	51
10	9.84 308	13	9.98 737	25	0.01 263	9.85 571	12	50
11	9.84 321	13	9.98 762	25	0.01 238	9.85 559	12	49
12	9.84 334	13	9.98 787	25	0.01 213	9.85 547	13	48
13	9.84 347	13	9.98 812	26	0.01 188	9.85 534	12	47
14	9.84 360	13	9.98 838	25	0.01 162	9.85 522	12	46
15	9.84 373	12	9.98 863	25	0.01 137	9.85 510	13	45
16	9.84 385	13	9.98 888	25	0.01 112	9.85 497	12	44
17	9.84 398	13	9.98 913	26	0.01 087	9.85 485	12	43
18	9.84 411	13	9.98 939	25	0.01 061	9.85 473	13	42
19	9.84 424	13	9.98 964	25	0.01 036	9.85 460	12	41
20	9.84 437	13	9.98 989	26	0.01 011	9.85 448	12	40
21	9.84 450	13	9.99 015	25	0.00 985	9.85 436	13	39
22	9.84 463	13	9.99 040	25	0.00 960	9.85 423	12	38
23	9.84 476	13	9.99 065	25	0.00 935	9.85 411	12	37
24	9.84 489	13	9.99 090	26	0.00 910	9.85 399	13	36
25	9.84 502	13	9.99 116	25	0.00 884	9.85 386	12	35
26	9.84 515	13	9.99 141	25	0.00 859	9.85 374	13	34
27	9.84 528	12	9.99 166	25	0.00 834	9.85 361	12	33
28	9.84 540	13	9.99 191	26	0.00 809	9.85 349	12	32
29	9.84 553	13	9.99 217	25	0.00 783	9.85 337	13	31
30	9.84 566	13	9.99 242	25	0.00 758	9.85 324	12	30
31	9.84 579	13	9.99 267	26	0.00 733	9.85 312	13	29
32	9.84 592	13	9.99 293	25	0.00 707	9.85 299	12	28
33	9.84 605	13	9.99 318	25	0.00 682	9.85 287	13	27
34	9.84 618	12	9.99 343	25	0.00 657	9.85 274	12	26
35	9.84 630	13	9.99 368	26	0.00 632	9.85 262	12	25
36	9.84 643	13	9.99 394	25	0.00 606	9.85 250	13	24
37	9.84 656	13	9.99 419	25	0.00 581	9.85 237	12	23
38	9.84 669	13	9.99 444	25	0.00 556	9.85 225	13	22
39	9.84 682	12	9.99 469	26	0.00 531	9.85 212	12	21
40	9.84 694	13	9.99 495	25	0.00 505	9.85 200	13	20
41	9.84 707	13	9.99 520	25	0.00 480	9.85 187	12	19
42	9.84 720	13	9.99 545	25	0.00 455	9.85 175	13	18
43	9.84 733	12	9.99 570	26	0.00 430	9.85 162	12	17
44	9.84 745	13	9.99 596	25	0.00 404	9.85 150	13	16
45	9.84 758	13	9.99 621	25	0.00 379	9.85 137	12	15
46	9.84 771	13	9.99 646	26	0.00 354	9.85 125	13	14
47	9.84 784	12	9.99 672	25	0.00 328	9.85 112	12	13
48	9.84 796	13	9.99 697	25	0.00 303	9.85 100	13	12
49	9.84 809	13	9.99 722	25	0.00 278	9.85 087	13	11
50	9.84 822	13	9.99 747	26	0.00 253	9.85 074	12	10
51	9.84 835	12	9.99 773	25	0.00 227	9.85 062	13	9
52	9.84 847	13	9.99 798	25	0.00 202	9.85 049	12	8
53	9.84 860	13	9.99 823	25	0.00 177	9.85 037	13	7
54	9.84 873	12	9.99 848	26	0.00 152	9.85 024	12	6
55	9.84 885	13	9.99 874	25	0.00 126	9.85 012	13	5
56	9.84 898	13	9.99 899	25	0.00 101	9.84 999	13	4
57	9.84 911	12	9.99 924	25	0.00 076	9.84 986	12	3
58	9.84 923	13	9.99 949	26	0.00 051	9.84 974	13	2
59	9.84 936	13	9.99 975	25	0.00 025	9.84 961	12	1
60	9.84 949		0.00 000		0.00 000	9.84 949		0
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	26	25	14
2	5.2	5.0	2.8
3	7.8	7.5	4.2
4	10.4	10.0	5.6
5	13.0	12.5	7.0
6	15.6	15.0	8.4
7	18.2	17.5	9.8
8	20.8	20.0	11.2
9	23.4	22.5	12.6

	13	12
2	2.6	2.4
3	3.9	3.6
4	5.2	4.8
5	6.5	6.0
6	7.8	7.2
7	9.1	8.4
8	10.4	9.6
9	11.7	10.8

From the top:

For **44°**+ or **224°**+, read as printed; for **134°**+ or **314°**+, read co-function.

From the bottom:

For **45°**+ or **225°**+, read as printed; for **135°**+ or **315°**+, read co-function.

Prop. Pts.

Degrees						Minutes		Seconds	
0°	0.00000 00	60°	1.04719 76	120°	2.09439 51	0′	0.00000 00	0″	0.00000 00
1	0.01745 33	61	1.06465 08	121	2.11184 84	1	0.00029 09	1	0.00000 48
2	0.03490 66	62	1.08210 41	122	2.12930 17	2	0.00058 18	2	0.00000 97
3	0.05235 99	63	1.09955 74	123	2.14675 50	3	0.00087 27	3	0.00001 45
4	0.06981 32	64	1.11701 07	124	2.16420 83	4	0.00116 36	4	0.00001 94
5	0.08726 65	65	1.13446 40	125	2.18166 16	5	0.00145 44	5	0.00002 42
6	0.10471 98	66	1.15191 73	126	2.19911 49	6	0.00174 53	6	0.00002 91
7	0.12217 30	67	1.16937 06	127	2.21656 82	7	0.00203 62	7	0.00003 39
8	0.13962 63	68	1.18682 39	128	2.23402 14	8	0.00232 71	8	0.00003 88
9	0.15707 96	69	1.20427 72	129	2.25147 47	9	0.00261 80	9	0.00004 36
10	0.17453 29	70	1.22173 05	130	2.26892 80	10	0.00290 89	10	0.00004 85
11	0.19198 62	71	1.23918 38	131	2.28638 13	11	0.00319 98	11	0.00005 33
12	0.20943 95	72	1.25663 71	132	2.30383 46	12	0.00349 07	12	0.00005 82
13	0.22689 28	73	1.27409 04	133	2.32128 79	13	0.00378 15	13	0.00006 30
14	0.24434 61	74	1.29154 36	134	2.33874 12	14	0.00407 24	14	0.00006 79
15	0.26179 94	75	1.30899 69	135	2.35619 45	15	0.00436 33	15	0.00007 27
16	0.27925 27	76	1.32645 02	136	2.37364 78	16	0.00465 42	16	0.00007 76
17	0.29670 60	77	1.34390 35	137	2.39110 11	17	0.00494 51	17	0.00008 24
18	0.31415 93	78	1.36135 68	138	2.40855 44	18	0.00523 60	18	0.00008 73
19	0.33161 26	79	1.37881 01	139	2.42600 77	19	0.00552 69	19	0.00009 21
20	0.34906 59	80	1.39626 34	140	2.44346 10	20	0.00581 78	20	0.00009 70
21	0.36651 91	81	1.41371 67	141	2.46091 42	21	0.00610 87	21	0.00010 18
22	0.38397 24	82	1.43117 00	142	2.47836 75	22	0.00639 95	22	0.00010 67
23	0.40142 57	83	1.44862 33	143	2.49582 08	23	0.00669 04	23	0.00011 15
24	0.41887 90	84	1.46607 66	144	2.51327 41	24	0.00698 13	24	0.00011 64
25	0.43633 23	85	1.48352 99	145	2.53072 74	25	0.00727 22	25	0.00012 12
26	0.45378 56	86	1.50098 32	146	2.54818 07	26	0.00756 31	26	0.00012 61
27	0.47123 89	87	1.51843 64	147	2.56563 40	27	0.00785 40	27	0.00013 09
28	0.48869 22	88	1.53588 97	148	2.58308 73	28	0.00814 49	28	0.00013 57
29	0.50614 55	89	1.55334 30	149	2.60054 06	29	0.00843 58	29	0.00014 06
30	0.52359 88	90	1.57079 63	150	2.61799 39	30	0.00872 66	30	0.00014 54
31	0.54105 21	91	1.58824 96	151	2.63544 72	31	0.00901 75	31	0.00015 03
32	0.55850 54	92	1.60570 29	152	2.65290 05	32	0.00930 84	32	0.00015 51
33	0.57595 87	93	1.62315 62	153	2.67035 38	33	0.00959 93	33	0.00016 00
34	0.59341 19	94	1.64060 95	154	2.68780 70	34	0.00989 02	34	0.00016 48
35	0.61086 52	95	1.65806 28	155	2.70526 03	35	0.01018 11	35	0.00016 97
36	0.62831 85	96	1.67551 61	156	2.72271 36	36	0.01047 20	36	0.00017 45
37	0.64577 18	97	1.69296 94	157	2.74016 69	37	0.01076 29	37	0.00017 94
38	0.66322 51	98	1.71042 27	158	2.75762 02	38	0.01105 38	38	0.00018 42
39	0.68067 84	99	1.72787 60	159	2.77507 35	39	0.01134 46	39	0.00018 91
40	0.69813 17	100	1.74532 93	160	2.79252 68	40	0.01163 55	40	0.00019 39
41	0.71558 50	101	1.76278 25	161	2.80998 01	41	0.01192 64	41	0.00019 88
42	0.73303 83	102	1.78023 58	162	2.82743 34	42	0.01221 73	42	0.00020 36
43	0.75049 16	103	1.79768 91	163	2.84488 67	43	0.01250 82	43	0.00020 85
44	0.76794 49	104	1.81514 24	164	2.86234 00	44	0.01279 91	44	0.00021 33
45	0.78539 82	105	1.83259 57	165	2.87979 33	45	0.01309 00	45	0.00021 82
46	0.80285 15	106	1.85004 90	166	2.89724 66	46	0.01338 09	46	0.00022 30
47	0.82030 47	107	1.86750 23	167	2.91469 99	47	0.01367 17	47	0.00022 79
48	0.83775 80	108	1.88495 56	168	2.93215 31	48	0.01396 26	48	0.00023 27
49	0.85521 13	109	1.90240 89	169	2.94960 64	49	0.01425 35	49	0.00023 76
50	0.87266 46	110	1.91986 22	170	2.96705 97	50	0.01454 44	50	0.00024 24
51	0.89011 79	111	1.93731 55	171	2.98451 30	51	0.01483 53	51	0.00024 73
52	0.90757 12	112	1.95476 88	172	3.00196 63	52	0.01512 62	52	0.00025 21
53	0.92502 45	113	1.97222 21	173	3.01941 96	53	0.01541 71	53	0.00025 70
54	0.94247 78	114	1.98967 53	174	3.03687 29	54	0.01570 80	54	0.00026 18
55	0.95993 11	115	2.00712 86	175	3.05432 62	55	0.01599 89	55	0.00026 66
56	0.97738 44	116	2.02458 19	176	3.07177 95	56	0.01628 97	56	0.00027 15
57	0.99483 77	117	2.04203 52	177	3.08923 28	57	0.01658 06	57	0.00027 63
58	1.01229 10	118	2.05948 85	178	3.10668 61	58	0.01687 15	58	0.00028 12
59	1.20974 43	119	2.07694 18	179	3.12413 94	59	0.01716 24	59	0.00028 60

x Radians	Sin x	Cos x	Tan x	Equivalent of x
.00	.00000	1.0000	.00000	0° 00′.0
.01	.01000	.99995	.01000	0° 34′.4
.02	.02000	.99980	.02000	1° 08′.8
.03	.03000	.99955	.03001	1° 43′.1
.04	.03999	.99920	.04002	2° 17′.5
.05	.04998	.99875	.05004	2° 51′.9
.06	.05996	.99820	.06007	3° 26′.3
.07	.06994	.99755	.07011	4° 00′.6
.08	.07991	.99680	.08017	4° 35′.0
.09	.08988	.99595	.09024	5° 09′.4
.10	.09983	.99500	.10033	5° 43′.8
.11	.10978	.99396	.11045	6° 18′.2
.12	.11971	.99281	.12058	6° 52′.5
.13	.12963	.99156	.13074	7° 26′.9
.14	.13954	.99022	.14092	8° 01′.3
.15	.14944	.98877	.15114	8° 35′.7
.16	.15932	.98723	.16138	9° 10′.0
.17	.16918	.98558	.17166	9° 44′.4
.18	.17903	.98384	.18197	10° 18′.8
.19	.18886	.98200	.19232	10° 53′.2
.20	.19867	.98007	.20271	11° 27′.5
.21	.20846	.97803	.21314	12° 01′.9
.22	.21823	.97590	.22362	12° 36′.3
.23	.22798	.97367	.23414	13° 10′.7
.24	.23770	.97134	.24472	13° 45′.1
.25	.24740	.96891	.25534	14° 19′.4
.26	.25708	.96639	.26602	14° 53′.8
.27	.26673	.96377	.27676	15° 28′.2
.28	.27636	.96106	.28755	16° 02′.6
.29	.28595	.95824	.29841	16° 36′.9
.30	.29552	.95534	.30934	17° 11′.3
.31	.30506	.95233	.32033	17° 45′.7
.32	.31457	.94924	.33139	18° 20′.1
.33	.32404	.94604	.34252	18° 54′.5
.34	.33349	.94275	.35374	19° 28′.8
.35	.34290	.93937	.36503	20° 03′.2
.36	.35227	.93590	.37640	20° 37′.6
.37	.36162	.93233	.38786	21° 12′.0
.38	.37092	.92866	.39941	21° 46′.3
.39	.38019	.92491	.41106	22° 20′.7
.40	.38942	.92106	.42279	22° 55′.1
.41	.39861	.91712	.43463	23° 29′.5
.42	.40776	.91309	.44657	24° 03′.9
.43	.41687	.90897	.45862	24° 38′.2
.44	.42594	.90475	.47078	25° 12′.6
.45	.43497	.90045	.48305	25° 47′.0
.46	.44395	.89605	.49545	26° 21′.4
.47	.45289	.89157	.50795	26° 55′.7
.48	.46178	.88699	.52061	27° 30′.1
.49	.47063	.88233	.53339	28° 04′.5

x Radians	Sin x	Cos x	Tan x	Equivalent of x
.50	.47943	.87758	.54630	28° 38′.9
.51	.48818	.87274	.55936	29° 13′.3
.52	.49688	.86782	.57256	29° 47′.6
.53	.50553	.86281	.58592	30° 22′.0
.54	.51414	.85771	.59943	30° 56′.4
.55	.52269	.85252	.61311	31° 30′.8
.56	.53119	.84726	.62695	32° 05′.1
.57	.53963	.84190	.64097	32° 39′.5
.58	.54802	.83646	.65517	33° 13′.9
.59	.55636	.83094	.66956	33° 48′.3
.60	.56464	.82534	.68414	34° 22′.6
.61	.57287	.81965	.69892	34° 57′.0
.62	.58104	.81388	.71391	35° 31′.4
.63	.58914	.80803	.72911	36° 05′.8
.64	.59720	.80210	.74454	36° 40′.2
.65	.60519	.79608	.76020	37° 14′.5
.66	.61312	.78999	.77610	37° 48′.9
.67	.62099	.78382	.79225	38° 23′.3
.68	.62879	.77757	.80866	38° 57′.7
.69	.63654	.77125	.82533	39° 32′.0
.70	.64422	.76484	.84229	40° 06′.4
.71	.65183	.75836	.85953	40° 40′.8
.72	.65938	.75181	.87707	41° 15′.2
.73	.66687	.74517	.89492	41° 49′.6
.74	.67429	.73847	.91309	42° 23′.9
.75	.68164	.73169	.93160	42° 58′.3
.76	.68892	.72484	.95055	43° 32′.7
.77	.69614	.71791	.96967	44° 07′.1
.78	.70328	.71091	.98926	44° 41′.4
.79	.71035	.70385	1.0092	45° 15′.8
.80	.71736	.69671	1.0296	45° 50′.2
.81	.72429	.68950	1.0505	46° 24′.6
.82	.73115	.68222	1.0717	46° 59′.0
.83	.73793	.67488	1.0934	47° 33′.3
.84	.74464	.66746	1.1156	48° 07′.7
.85	.75128	.65998	1.1383	48° 42′.1
.86	.75784	.65244	1.1616	49° 16′.5
.87	.76433	.64483	1.1853	49° 50′.8
.88	.77074	.63715	1.2097	50° 25′.2
.89	.77707	.62941	1.2346	50° 59′.6
.90	.78333	.62161	1.2602	51° 34′.0
.91	.78950	.61375	1.2864	52° 08′.3
.92	.79560	.60582	1.3133	52° 42′.7
.93	.80162	.59783	1.3409	53° 17′.1
.94	.80756	.58979	1.3692	53° 51′.5
.95	.81342	.58168	1.3984	54° 25′.9
.96	.81919	.57352	1.4284	55° 00′.2
.97	.82489	.56530	1.4592	55° 34′.6
.98	.83050	.55702	1.4910	56° 09′.0
.99	.83603	.54869	1.5237	56° 43′.4

x Radians	Sin x	Cos x	Tan x	Equivalent of x
1.00	.84147	.54030	1.5574	57° 17′.7
1.01	.84683	.53186	1.5922	57° 52′.1
1.02	.85211	.52337	1.6281	58° 26′.5
1.03	.85730	.51482	1.6652	59° 00′.9
1.04	.86240	.50622	1.7036	59° 35′.3
1.05	.86742	.49757	1.7433	60° 09′.6
1.06	.87236	.48887	1.7844	60° 44′.0
1.07	.87720	.48012	1.8270	61° 18′.4
1.08	.88196	.47133	1.8712	61° 52′.8
1.09	.88663	.46249	1.9171	62° 27′.1
1.10	.89121	.45360	1.9648	63° 01′.5
1.11	.89570	.44466	2.0143	63° 35′.9
1.12	.90010	.43568	2.0660	64° 10′.3
1.13	.90441	.42666	2.1198	64° 44′.7
1.14	.90863	.41759	2.1759	65° 19′.0
1.15	.91276	.40849	2.2345	65° 53′.4
1.16	.91680	.39934	2.2958	66° 27′.8
1.17	.92075	.39015	2.3600	67° 02′.2
1.18	.92461	.38092	2.4273	67° 36′.5
1.19	.92837	.37166	2.4979	68° 10′.9
1.20	.93204	.36236	2.5722	68° 45′.3
1.21	.93562	.35302	2.6503	69° 19′.7
1.22	.93910	.34365	2.7328	69° 54′.1
1.23	.94249	.33424	2.8198	70° 28′.4
1.24	.94578	.32480	2.9119	71° 02′.8
1.25	.94898	.31532	3.0096	71° 37′.2
1.26	.95209	.30582	3.1133	72° 11′.6
1.27	.95510	.29628	3.2236	72° 45′.9
1.28	.95802	.28672	3.3413	73° 20′.3
1.29	.96084	.27712	3.4672	73° 54′.7
1.30	.96356	.26750	3.6021	74° 29′.1

x Radians	Sin x	Cos x	Tan x	Equivalent of x
1.30	.96356	.26750	3.6021	74° 29′.1
1.31	.96618	.25785	3.7470	75° 03′.4
1.32	.96872	.24818	3.9033	75° 37′.8
1.33	.97115	.23848	4.0723	76° 12′.2
1.34	.97348	.22875	4.2556	76° 46′.6
1.35	.97572	.21901	4.4552	77° 21′.0
1.36	.97786	.20924	4.6734	77° 55′.3
1.37	.97991	.19945	4.9131	78° 29′.7
1.38	.98185	.18964	5.1774	79° 04′.1
1.39	.98370	.17981	5.4707	79° 38′.5
1.40	.98545	.16997	5.7979	80° 12′.8
1.41	.98710	.16010	6.1654	80° 47′.2
1.42	.98865	.15023	6.5811	81° 21′.6
1.43	.99010	.14033	7.0555	81° 56′.0
1.44	.99146	.13042	7.6018	82° 30′.4
1.45	.99271	.12050	8.2381	83° 04′.7
1.46	.99387	.11057	8.9886	83° 39′.1
1.47	.99492	.10063	9.8874	84° 13′.5
1.48	.99588	.09067	10.983	84° 47′.9
1.49	.99674	.08071	12.350	85° 22′.2
1.50	.99749	.07074	14.101	85° 56′.6
1.51	.99815	.06076	16.428	86° 31′.0
1.52	.99871	.05077	19.670	87° 05′.4
1.53	.99917	.04079	24.498	87° 39′.8
1.54	.99953	.03079	32.461	88° 14′.1
1.55	.99978	.02079	48.078	88° 48′.5
1.56	.99994	.01080	92.621	89° 22′.9
1.57	1.0000	.00080	1255.8	89° 57′.3
1.58	.99996	-.00920	-108.65	90° 31′.6
1.59	.99982	-.01920	-52.067	91° 06′.0
1.60	.99957	-.02920	-34.233	91° 40′.4

π radians = 180° $\qquad\qquad$ $\pi = 3.14159265$

1 radian = 57° 17′ 44″.806 = 57.° 2957795

3600″ = 60′ = 1° = .01745329 radian

TABLE Va — RADIANS TO DEGREES

	Radians	Tenths	Hundredths	Thousandths	Ten-thousandths
1	57°17′44″.8	5°43′46″.5	0°34′22″.6	0° 3′26″.3	0° 0′20″.6
2	114°35′29″.6	11°27′33″.0	1° 8′45″.3	0° 6′52″.5	0° 0′41″.3
3	171°53′14″.4	17°11′19″.4	1°43′07″.9	0°10′18″.8	0° 1′01″.9
4	229°10′59″.2	22°55′05″.9	2°17′30″.6	0°13′45″.1	0° 1′22″.5
5	286°28′44″.0	28°38′52″.4	2°51′53″.2	0°17′11″.3	0° 1′43″.1
6	343°46′28″.8	34°22′38″.9	3°26′15″.9	0°20′37″.6	0° 2′03″.8
7	401° 4′13″.6	40° 6′25″.4	4° 0′38″.5	0°24′03″.9	0° 2′24″.4
8	458°21′58″.4	45°50′11″.8	4°35′01″.2	0°27′30″.1	0° 2′45″.0
9	515°39′43″.3	51°33′58″.3	5° 9′23″.8	0°30′56″.4	0° 3′05″.6

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
1.00	1.0000	1.00000	3.16228	1.00000	1.00000	2.15443	4.64159	1.00000
1.01	1.0201	1.00499	3.17805	1.03030	1.00332	2.16159	4.65701	.990099
1.02	1.0404	1.00995	3.19374	1.06121	1.00662	2.16870	4.67233	.980392
1.03	1.0609	1.01489	3.20936	1.09273	1.00990	2.17577	4.68755	.970874
1.04	1.0816	1.01980	3.22490	1.12486	1.01316	2.18279	4.70267	.961538
1.05	1.1025	1.02470	3.24037	1.15762	1.01640	2.18976	4.71769	.952381
1.06	1.1236	1.02956	3.25576	1.19102	1.01961	2.19669	4.73262	.943396
1.07	1.1449	1.03441	3.27109	1.22504	1.02281	2.20358	4.74746	.934579
1.08	1.1664	1.03923	3.28634	1.25971	1.02599	2.21042	4.76220	.925926
1.09	1.1881	1.04403	3.30151	1.29503	1.02914	2.21722	4.77686	.917431
1.10	1.2100	1.04881	3.31662	1.33100	1.03228	2.22398	4.79142	.909091
1.11	1.2321	1.05357	3.33167	1.36763	1.03540	2.23070	4.80590	.900901
1.12	1.2544	1.05830	3.34664	1.40493	1.03850	2.23738	4.82028	.892857
1.13	1.2769	1.06301	3.36155	1.44290	1.04158	2.24402	4.83459	.884956
1.14	1.2996	1.06771	3.37639	1.48154	1.04464	2.25062	4.84881	.877193
1.15	1.3225	1.07238	3.39116	1.52088	1.04769	2.25718	4.86294	.869565
1.16	1.3456	1.07703	3.40588	1.56090	1.05072	2.26370	4.87700	.862069
1.17	1.3689	1.08167	3.42053	1.60161	1.05373	2.27019	4.89097	.854701
1.18	1.3924	1.08628	3.43511	1.64303	1.05672	2.27664	4.90487	.847458
1.19	1.4161	1.09087	3.44964	1.68516	1.05970	2.28305	4.91868	.840336
1.20	1.4400	1.09545	3.46410	1.72800	1.06266	2.28943	4.93242	.833333
1.21	1.4641	1.10000	3.47851	1.77156	1.06560	2.29577	4.94609	.826446
1.22	1.4884	1.10454	3.49285	1.81585	1.06853	2.30208	4.95968	.819672
1.23	1.5129	1.10905	3.50714	1.86087	1.07144	2.30835	4.97319	.813008
1.24	1.5376	1.11355	3.52136	1.90662	1.07434	2.31459	4.98663	.806452
1.25	1.5625	1.11803	3.53553	1.95312	1.07722	2.32079	5.00000	.800000
1.26	1.5876	1.12250	3.54965	2.00038	1.08008	2.32697	5.01330	.793651
1.27	1.6129	1.12694	3.56371	2.04838	1.08293	2.33311	5.02653	.787402
1.28	1.6384	1.13137	3.57771	2.09715	1.08577	2.33921	5.03968	.781250
1.29	1.6641	1.13578	3.59166	2.14669	1.08859	2.34529	5.05277	.775194
1.30	1.6900	1.14018	3.60555	2.19700	1.09139	2.35133	5.06580	.769231
1.31	1.7161	1.14455	3.61939	2.24809	1.09418	2.35735	5.07875	.763359
1.32	1.7424	1.14891	3.63318	2.29997	1.09696	2.36333	5.09164	.757576
1.33	1.7689	1.15326	3.64692	2.35264	1.09972	2.36928	5.10447	.751880
1.34	1.7956	1.15758	3.66060	2.40610	1.10247	2.37521	5.11723	.746269
1.35	1.8225	1.16190	3.67423	2.46038	1.10521	2.38110	5.12993	.740741
1.36	1.8496	1.16619	3.68782	2.51546	1.10793	2.38697	5.14256	.735294
1.37	1.8769	1.17047	3.70135	2.57135	1.11064	2.39280	5.15514	.729927
1.38	1.9044	1.17473	3.71484	2.62807	1.11334	2.39861	5.16765	.724638
1.39	1.9321	1.17898	3.72827	2.68562	1.11602	2.40439	5.18010	.719424
1.40	1.9600	1.18322	3.74166	2.74400	1.11869	2.41014	5.19249	.714286
1.41	1.9881	1.18743	3.75500	2.80322	1.12135	2.41587	5.20483	.709220
1.42	2.0164	1.19164	3.76829	2.86329	1.12399	2.42156	5.21710	.704225
1.43	2.0449	1.19583	3.78153	2.92421	1.12662	2.42724	5.22932	.699301
1.44	2.0736	1.20000	3.79473	2.98598	1.12924	2.43288	5.24148	.694444
1.45	2.1025	1.20416	3.80789	3.04862	1.13185	2.43850	5.25359	.689655
1.46	2.1316	1.20830	3.82099	3.11214	1.13445	2.44409	5.26564	.684932
1.47	2.1609	1.21244	3.83406	3.17652	1.13703	2.44966	5.27763	.680272
1.48	2.1904	1.21655	3.84708	3.24179	1.13960	2.45520	5.28957	.675676
1.49	2.2201	1.22066	3.86005	3.30795	1.14216	2.46072	5.30146	.671141
1.50	2.2500	1.22474	3.87298	3.37500	1.14471	2.46621	5.31329	.666667
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
1.50	2.2500	1.22474	3.87298	3.37500	1.14471	2.46621	5.31329	.666667
1.51	2.2801	1.22882	3.88587	3.44295	1.14725	2.47168	5.32507	.662252
1.52	2.3104	1.23288	3.89872	3.51181	1.14978	2.47712	5.33680	.657895
1.53	2.3409	1.23693	3.91152	3.58158	1.15230	2.48255	5.34848	.653595
1.54	2.3716	1.24097	3.92428	3.65226	1.15480	2.48794	5.36011	.649351
1.55	2.4025	1.24499	3.93700	3.72388	1.15729	2.49332	5.37169	.645161
1.56	2.4336	1.24900	3.94968	3.79642	1.15978	2.49867	5.38321	.641026
1.57	2.4649	1.25300	3.96232	3.86989	1.16225	2.50399	5.39469	.636943
1.58	2.4964	1.25698	3.97492	3.94431	1.16471	2.50930	5.40612	.632911
1.59	2.5281	1.26095	3.98748	4.01968	1.16717	2.51458	5.41750	.628931
1.60	2.5600	1.26491	4.00000	4.09600	1.16961	2.51984	5.42884	.625000
1.61	2.5921	1.26886	4.01248	4.17328	1.17204	2.52508	5.44012	.621118
1.62	2.6244	1.27279	4.02492	4.25153	1.17446	2.53030	5.45136	.617284
1.63	2.6569	1.27671	4.03733	4.33075	1.17687	2.53549	5.46256	.613497
1.64	2.6896	1.28062	4.04969	4.41094	1.17927	2.54067	5.47370	.609756
1.65	2.7225	1.28452	4.06202	4.49212	1.18167	2.54582	5.48481	.606061
1.66	2.7556	1.28841	4.07431	4.57430	1.18405	2.55095	5.49586	.602410
1.67	2.7889	1.29228	4.08656	4.65746	1.18642	2.55607	5.50688	.598802
1.68	2.8224	1.29615	4.09878	4.74163	1.18878	2.56116	5.51785	.595238
1.69	2.8561	1.30000	4.11096	4.82681	1.19114	2.56623	5.52877	.591716
1.70	2.8900	1.30384	4.12311	4.91300	1.19348	2.57128	5.53966	.588235
1.71	2.9241	1.30767	4.13521	5.00021	1.19582	2.57631	5.55050	.584795
1.72	2.9584	1.31149	4.14729	5.08845	1.19815	2.58133	5.56130	.581395
1.73	2.9929	1.31529	4.15933	5.17772	1.20046	2.58632	5.57205	.578035
1.74	3.0276	1.31909	4.17133	5.26802	1.20277	2.59129	5.58277	.574713
1.75	3.0625	1.32288	4.18330	5.35938	1.20507	2.59625	5.59344	.571429
1.76	3.0976	1.32665	4.19524	5.45178	1.20736	2.60118	5.60408	.568182
1.77	3.1329	1.33041	4.20714	5.54523	1.20964	2.60610	5.61467	.564972
1.78	3.1684	1.33417	4.21900	5.63975	1.21192	2.61100	5.62523	.561798
1.79	3.2041	1.33791	4.23084	5.73534	1.21418	2.61588	5.63574	.558659
1.80	3.2400	1.34164	4.24264	5.83200	1.21644	2.62074	5.64622	.555556
1.81	3.2761	1.34536	4.25441	5.92974	1.21869	2.62559	5.65665	.552486
1.82	3.3124	1.34907	4.26615	6.02857	1.22093	2.63041	5.66705	.549451
1.83	3.3489	1.35277	4.27785	6.12849	1.22316	2.63522	5.67741	.546448
1.84	3.3856	1.35647	4.28952	6.22950	1.22539	2.64001	5.68773	.543478
1.85	3.4225	1.36015	4.30116	6.33162	1.22760	2.64479	5.69802	.540541
1.86	3.4596	1.36382	4.31277	6.43486	1.22981	2.64954	5.70827	.537634
1.87	3.4969	1.36748	4.32435	6.53920	1.23201	2.65428	5.71848	.534759
1.88	3.5344	1.37113	4.33590	6.64467	1.23420	2.65901	5.72865	.531915
1.89	3.5721	1.37477	4.34741	6.75127	1.23639	2.66371	5.73879	.529101
1.90	3.6100	1.37840	4.35890	6.85900	1.23856	2.66840	5.74890	.526316
1.91	3.6481	1.38203	4.37035	6.96787	1.24073	2.67307	5.75897	.523560
1.92	3.6864	1.38564	4.38178	7.07789	1.24289	2.67773	5.76900	.520833
1.93	3.7249	1.38924	4.39318	7.18906	1.24505	2.68237	5.77900	.518135
1.94	3.7636	1.39284	4.40454	7.30138	1.24719	2.68700	5.78896	.515464
1.95	3.8025	1.39642	4.41588	7.41488	1.24933	2.69161	5.79889	.512821
1.96	3.8416	1.40000	4.42719	7.52954	1.25146	2.69620	5.80879	.510204
1.97	3.8809	1.40357	4.43847	7.64537	1.25359	2.70078	5.81865	.507614
1.98	3.9204	1.40712	4.44972	7.76239	1.25571	2.70534	5.82848	.505051
1.99	3.9601	1.41067	4.46094	7.88060	1.25782	2.70989	5.83827	.502513
2.00	4.0000	1.41421	4.47214	8.00000	1.25992	2.71442	5.84804	.500000
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
2.00	4.0000	1.41421	4 47214	8.00000	1.25992	2.71442	5.84804	.500000
2.01	4.0401	1.41774	4 48330	8.12060	1.26202	2.71893	5.85777	.497512
2.02	4.0804	1.42127	4.49444	8.24241	1.26411	2.72344	5.86746	.495050
2.03	4.1209	1.42478	4.50555	8.36543	1.26619	2.72792	5.87713	.492611
2.04	4.1616	1.42829	4.51664	8.48966	1.26827	2.73239	5.88677	.490196
2.05	4.2025	1.43178	4.52769	8.61512	1.27033	2.73685	5.89637	.487805
2.06	4.2436	1.43527	4.53872	8.74182	1.27240	2.74129	5.90594	.485437
2.07	4.2849	1.43875	4.54973	8.86974	1.27445	2.74572	5.91548	.483092
2.08	4.3264	1.44222	4.56070	8.99891	1.27650	2.75014	5.92499	.480769
2.09	4.3681	1.44568	4.57165	9.12933	1.27854	2.75454	5.93447	.478469
2.10	4.4100	1.44914	4 58258	9 26100	1.28058	2.75892	5.94392	.476190
2.11	4.4521	1.45258	4.59347	9.39393	1.28261	2.76330	5.95334	.473934
2.12	4.4944	1.45602	4.60435	9.52813	1.28463	2.76766	5.96273	.471698
2.13	4.5369	1.45945	4.61519	9.66360	1.28665	2.77200	5.97209	.469484
2.14	4.5796	1.46287	4.62601	9.80034	1.28866	2.77633	5.98142	.467290
2.15	4.6225	1.46629	4.63681	9.93838	1.29066	2.78065	5.99073	.465116
2.16	4.6656	1.46969	4.64758	10.0777	1.29266	2.78495	6.00000	.462963
2.17	4.7089	1.47309	4.65833	10.2183	1.29465	2.78924	6.00925	.460829
2.18	4.7524	1.47648	4.66905	10.3602	1.29664	2.79352	6.01846	.458716
2.19	4.7961	1.47986	4.67974	10.5035	1.29862	2.79779	6.02765	.456621
2.20	4.8400	1.48324	4.69042	10.6480	1.30059	2.80204	6.03681	.454545
2.21	4.8841	1.48661	4.70106	10.7939	1.30256	2.80628	6.04594	.452489
2.22	4.9284	1.48997	4.71169	10.9410	1.30452	2.81050	6.05505	.450450
2.23	4.9729	1.49332	4.72229	11.0896	1.30648	2.81472	6.06413	.448430
2.24	5.0176	1.49666	4.73286	11.2394	1.30843	2.81892	6.07318	.446429
2.25	5.0625	1.50000	4.74342	11.3906	1.31037	2.82311	6.08220	.444444
2.26	5.1076	1.50333	4.75395	11.5432	1.31231	2.82728	6.09120	.442478
2.27	5.1529	1.50665	4.76445	11.6971	1.31424	2.83145	6.10017	.440529
2.28	5.1984	1.50997	4.77493	11.8524	1.31617	2.83560	6.10911	.438596
2.29	5.2441	1.51327	4.78539	12.0090	1.31809	2.83974	6.11803	.436681
2.30	5.2900	1 51658	4.79583	12.1670	1.32001	2.84387	6.12693	.434783
2.31	5.3361	1.51987	4.80625	12.3264	1.32192	2.84798	6.13579	.432900
2.32	5.3824	1.52315	4.81664	12.4872	1.32382	2.85209	6.14463	.431034
2.33	5.4289	1.52643	4.82701	12.6493	1.32572	2.85618	6.15345	.429185
2.34	5.4756	1.52971	4.83735	12.8129	1.32761	2.86026	6.16224	.427350
2.35	5.5225	1.53297	4.84768	12.9779	1.32950	2.86433	6.17101	.425532
2.36	5.5696	1.53623	4.85798	13.1443	1.33139	2.86838	6.17975	.423729
2.37	5.6169	1.53948	4.86826	13.3121	1.33326	2.87243	6.18846	.421941
2.38	5.6644	1.54272	4.87852	13.4813	1.33514	2.87646	6.19715	.420168
2.39	5.7121	1.54596	4.88876	13.6519	1.33700	2.88049	6.20582	.418410
2.40	5.7600	1.54919	4.89898	13.8240	1.33887	2.88450	6.21447	.416667
2.41	5.8081	1.55242	4.90918	13.9975	1.34072	2.88850	6.22308	.414938
2.42	5.8564	1.55563	4.91935	14.1725	1.34257	2.89249	6.23168	.413223
2.43	5.9049	1.55885	4.92950	14.3489	1.34442	2.89647	6.24025	.411523
2.44	5.9536	1.56205	4.93964	14.5268	1.34626	2.90044	6.24880	.409836
2.45	6.0025	1.56525	4.94975	14.7061	1.34810	2.90439	6.25732	.408163
2.46	6.0516	1.56844	4.95984	14.8869	1.34993	2.90834	6.26583	.406504
2.47	6.1009	1.57162	4.96991	15.0692	1.35176	2.91227	6.27431	.404858
2.48	6.1504	1.57480	4.97996	15.2530	1.35358	2.91620	6.28276	.403226
2.49	6.2001	1.57797	4.98999	15.4382	1.35540	2.92011	6.29119	.401606
2.50	6.2500	1.58114	5.00000	15.6250	1.35721	2.92402	6.29961	.400000
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
2.50	6.2500	1.58114	5.00000	15.6250	1.35721	2.92402	6.29961	.400000
2.51	6.3001	1.58430	5.00999	15.8133	1.35902	2.92791	6.30799	.398406
2.52	6.3504	1.58745	5.01996	16.0030	1.36082	2.93179	6.31636	.396825
2.53	6.4009	1.59060	5.02991	16.1943	1.36262	2.93567	6.32470	.395257
2.54	6.4516	1.59374	5.03984	16.3871	1.36441	2.93953	6.33303	.393701
2.55	6.5025	1.59687	5.04975	16.5814	1.36620	2.94338	6.34133	.392157
2.56	6.5536	1.60000	5.05964	16.7772	1.36798	2.94723	6.34960	.390625
2.57	6.6049	1.60312	5.06952	16.9746	1.36976	2.95106	6.35786	.389105
2.58	6.6564	1.60624	5.07937	17.1735	1.37153	2.95488	6.36610	.387597
2.59	6.7081	1.60935	5.08920	17.3740	1.37330	2.95869	6.37431	.386100
2.60	6.7600	1.61245	5.09902	17.5760	1.37507	2.96250	6.38250	.384615
2.61	6.8121	1.61555	5.10882	17.7796	1.37683	2.96629	6.39068	.383142
2.62	6.8644	1.61864	5.11859	17.9847	1.37859	2.97007	6.39883	.381679
2.63	6.9169	1.62173	5.12835	18.1914	1.38034	2.97385	6.40696	.380228
2.64	6.9696	1.62481	5.13809	18.3997	1.38208	2.97761	6.41507	.378788
2.65	7.0225	1.62788	5.14782	18.6096	1.38383	2.98137	6.42316	.377358
2.66	7.0756	1.63095	5.15752	18.8211	1.38557	2.98511	6.43123	.375940
2.67	7.1289	1.63401	5.16720	19.0342	1.38730	2.98885	6.43928	.374532
2.68	7.1824	1.63707	5.17687	19.2488	1.38903	2.99257	6.44731	.373134
2.69	7.2361	1.64012	5.18652	19.4651	1.39076	2.99629	6.45531	.371747
2.70	7.2900	1.64317	5.19615	19.6830	1.39248	3.00000	6.46330	.370370
2.71	7.3441	1.64621	5.20577	19.9025	1.39419	3.00370	6.47127	.369004
2.72	7.3984	1.64924	5.21536	20.1236	1.39591	3.00739	6.47922	.367647
2.73	7.4529	1.65227	5.22494	20.3464	1.39761	3.01107	6.48715	.366300
2.74	7.5076	1.65529	5.23450	20.5708	1.39932	3.01474	6.49507	.364964
2.75	7.5625	1.65831	5.24404	20.7969	1.40102	3.01841	6.50296	.363636
2.76	7.6176	1.66132	5.25357	21.0246	1.40272	3.02206	6.51083	.362319
2.77	7.6729	1.66433	5.26308	21.2539	1.40441	3.02570	6.51868	.361011
2.78	7.7284	1.66733	5.27257	21.4850	1.40610	3.02934	6.52652	.359712
2.79	7.7841	1.67033	5.28205	21.7176	1.40778	3.03297	6.53434	.358423
2.80	7.8400	1.67332	5.29150	21.9520	1.40946	3.03659	6.54213	.357143
2.81	7.8961	1.67631	5.30094	22.1880	1.41114	3.04020	6.54991	.355872
2.82	7.9524	1.67929	5.31037	22.4258	1.41281	3.04380	6.55767	.354610
2.83	8.0089	1.68226	5.31977	22.6652	1.41448	3.04740	6.56541	.353357
2.84	8.0656	1.68523	5.32917	22.9063	1.41614	3.05098	6.57314	.352113
2.85	8.1225	1.68819	5.33854	23.1491	1.41780	3.05456	6.58084	.350877
2.86	8.1796	1.69115	5.34790	23.3937	1.41946	3.05813	6.58853	.349650
2.87	8.2369	1.69411	5.35724	23.6399	1.42111	3.06169	6.59620	.348432
2.88	8.2944	1.69706	5.36656	23.8879	1.42276	3.06524	6.60385	.347222
2.89	8.3521	1.70000	5.37587	24.1376	1.42440	3.06878	6.61149	.346021
2.90	8.4100	1.70294	5.38516	24.3890	1.42604	3.07232	6.61911	.344828
2.91	8.4681	1.70587	5.39444	24.6422	1.42768	3.07584	6.62671	.343643
2.92	8.5264	1.70880	5.40370	24.8971	1.42931	3.07936	6.63429	.342466
2.93	8.5849	1.71172	5.41295	25.1538	1.43094	3.08287	6.64185	.341297
2.94	8.6436	1.71464	5.42218	25.4122	1.43257	3.08638	6.64940	.340136
2.95	8.7025	1.71756	5.43139	25.6724	1.43419	3.08987	6.65693	.338983
2.96	8.7616	1.72047	5.44059	25.9343	1.43581	3.09336	6.66444	.337838
2.97	8.8209	1.72337	5.44977	26.1981	1.43743	3.09684	6.67194	.336700
2.98	8.8804	1.72627	5.45894	26.4636	1.43904	3.10031	6.67942	.335570
2.99	8.9401	1.72916	5.46809	26.7309	1.44065	3.10378	6.68688	.334448
3.00	9.0000	1.73205	5.47723	27.0000	1.44225	3.10723	6.69433	.333333
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
3.00	9.0000	1.73205	5.47723	27.0000	1.44225	3.10723	6.69433	.333333
3.01	9.0601	1.73494	5.48635	27.2709	1.44385	3.11068	6.70176	.332226
3.02	9.1204	1.73781	5.49545	27.5436	1.44545	3.11412	6.70917	.331126
3.03	9.1809	1.74069	5.50454	27.8181	1.44704	3.11756	6.71657	.330033
3.04	9.2416	1.74356	5.51362	28.0945	1.44863	3.12098	6.72395	.328947
3.05	9.3025	1.74642	5.52268	28.3726	1.45022	3.12440	6.73132	.327869
3.06	9.3636	1.74929	5.53173	28.6526	1.45180	3.12781	6.73866	.326797
3.07	9.4249	1.75214	5.54076	28.9344	1.45338	3.13121	6.74600	.325733
3.08	9.4864	1.75499	5.54977	29.2181	1.45496	3.13461	6.75331	.324675
3.09	9.5481	1.75784	5.55878	29.5036	1.45653	3.13800	6.76061	.323625
3.10	9.6100	1.76068	5.56776	29.7910	1.45810	3.14138	6.76790	.322581
3.11	9.6721	1.76352	5.57674	30.0802	1.45967	3.14475	6.77517	.321543
3.12	9.7344	1.76635	5.58570	30.3713	1.46123	3.14812	6.78242	.320513
3.13	9.7969	1.76918	5.59464	30.6643	1.46279	3.15148	6.78966	.319489
3.14	9.8596	1.77200	5.60357	30.9591	1.46434	3.15483	6.79688	.318471
3.15	9.9225	1.77482	5.61249	31.2559	1.46590	3.15818	6.80409	.317460
3.16	9.9856	1.77764	5.62139	31.5545	1.46745	3.16152	6.81128	.316456
3.17	10.0489	1.78045	5.63028	31.8550	1.46899	3.16485	6.81846	.315457
3.18	10.1124	1.78326	5.63915	32.1574	1.47054	3.16817	6.82562	.314465
3.19	10.1761	1.78606	5.64801	32.4618	1.47208	3.17149	6.83277	.313480
3.20	10.2400	1.78885	5.65685	32.7680	1.47361	3.17480	6.83990	.312500
3.21	10.3041	1.79165	5.66569	33.0762	1.47515	3.17811	6.84702	.311526
3.22	10.3684	1.79444	5.67450	33.3862	1.47668	3.18140	6.85412	.310559
3.23	10.4329	1.79722	5.68331	33.6983	1.47820	3.18469	6.86121	.309598
3.24	10.4976	1.80000	5.69210	34.0122	1.47973	3.18798	6.86829	.308642
3.25	10.5625	1.80278	5.70088	34.3281	1.48125	3.19125	6.87534	.307692
3.26	10.6276	1.80555	5.70964	34.6460	1.48277	3.19452	6.88239	.306748
3.27	10.6929	1.80831	5.71839	34.9658	1.48428	3.19778	6.88942	.305810
3.28	10.7584	1.81108	5.72713	35.2876	1.48579	3.20104	6.89643	.304878
3.29	10.8241	1.81384	5.73585	35.6113	1.48730	3.20429	6.90344	.303951
3.30	10.8900	1.81659	5.74456	35.9370	1.48881	3.20753	6.91042	.303030
3.31	10.9561	1.81934	5.75326	36.2647	1.49031	3.21077	6.91740	.302115
3.32	11.0224	1.82209	5.76194	36.5944	1.49181	3.21400	6.92436	.301205
3.33	11.0889	1.82483	5.77062	36.9260	1.49330	3.21722	6.93130	.300300
3.34	11.1556	1.82757	5.77927	37.2597	1.49480	3.22044	6.93823	.299401
3.35	11.2225	1.83030	5.78792	37.5954	1.49629	3.22365	6.94515	.298507
3.36	11.2896	1.83303	5.79655	37.9331	1.49777	3.22686	6.95205	.297619
3.37	11.3569	1.83576	5.80517	38.2728	1.49926	3.23006	6.95894	.296736
3.38	11.4244	1.83848	5.81378	38.6145	1.50074	3.23325	6.96582	.295858
3.39	11.4921	1.84120	5.82237	38.9582	1.50222	3.23643	6.97268	.294985
3.40	11.5600	1.84391	5.83095	39.3040	1.50369	3.23961	6.97953	.294118
3.41	11.6281	1.84662	5.83952	39.6518	1.50517	3.24278	6.98637	.293255
3.42	11.6964	1.84932	5.84808	40.0017	1.50664	3.24595	6.99319	.292398
3.43	11.7649	1.85203	5.85662	40.3536	1.50810	3.24911	7.00000	.291545
3.44	11.8336	1.85472	5.86515	40.7076	1.50957	3.25227	7.00680	.290698
3.45	11.9025	1.85742	5.87367	41.0636	1.51103	3.25542	7.01358	.289855
3.46	11.9716	1.86011	5.88218	41.4217	1.51249	3.25856	7.02035	.289017
3.47	12.0409	1.86279	5.89067	41.7819	1.51394	3.26169	7.02711	.288184
3.48	12.1104	1.86548	5.89915	42.1442	1.51540	3.26482	7.03385	.287356
3.49	12.1801	1.86815	5.90762	42.5085	1.51685	3.26795	7.04058	.286533
3.50	12.2500	1.87083	5.91608	42.8750	1.51829	3.27107	7.04730	.285714
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
3.50	12.2500	1.87083	5.91608	42.8750	1.51829	3.27107	7.04730	.285714
3.51	12.3201	1.87350	5.92453	43.2436	1.51974	3.27418	7.05400	.284900
3.52	12.3904	1.87617	5.93296	43.6142	1.52118	3.27729	7.06070	.284091
3.53	12.4609	1.87883	5.94138	43.9870	1.52262	3.28039	7.06738	.283286
3.54	12.5316	1.88149	5.94979	44.3619	1.52406	3.28348	7.07404	.282486
3.55	12.6025	1.88414	5.95819	44.7389	1.52549	3.28657	7.08070	.281690
3.56	12.6736	1.88680	5.96657	45.1180	1.52692	3.28965	7.08734	.280899
3.57	12.7449	1.88944	5.97495	45.4993	1.52835	3.29273	7.09397	.280112
3.58	12.8164	1.89209	5.98331	45.8827	1.52978	3.29580	7.10059	.279330
3.59	12.8881	1.89473	5.99166	46.2683	1.53120	3.29887	7.10719	.278552
3.60	12.9600	1.89737	6.00000	46.6560	1.53262	3.30193	7.11379	.277778
3.61	13.0321	1.90000	6.00833	47.0459	1.53404	3.30498	7.12037	.277008
3.62	13.1044	1.90263	6.01664	47.4379	1.53545	3.30803	7.12694	.276243
3.63	13.1769	1.90526	6.02495	47.8321	1.53686	3.31107	7.13349	.275482
3.64	13.2496	1.90788	6.03324	48.2285	1.53827	3.31411	7.14004	.274725
3.65	13.3225	1.91050	6.04152	48.6271	1.53968	3.31714	7.14657	.273973
3.66	13.3956	1.91311	6.04979	49.0279	1.54109	3.32017	7.15309	.273224
3.67	13.4689	1.91572	6.05805	49.4309	1.54249	3.32319	7.15960	.272480
3.68	13.5424	1.91833	6.06630	49.8360	1.54389	3.32621	7.16610	.271739
3.69	13.6161	1.92094	6.07454	50.2434	1.54529	3.32922	7.17258	.271003
3.70	13.6900	1.92354	6.08276	50.6530	1.54668	3.33222	7.17905	.270270
3.71	13.7641	1.92614	6.09098	51.0648	1.54807	3.33522	7.18552	.269542
3.72	13.8384	1.92873	6.09918	51.4788	1.54946	3.33822	7.19197	.268817
3.73	13.9129	1.93132	6.10737	51.8951	1.55085	3.34120	7.19840	.268097
3.74	13.9876	1.93391	6.11555	52.3136	1.55223	3.34419	7.20483	.267380
3.75	14.0625	1.93649	6.12372	52.7344	1.55362	3.34716	7.21125	.266667
3.76	14.1376	1.93907	6.13188	53.1574	1.55500	3.35014	7.21765	.265957
3.77	14.2129	1.94165	6.14003	53.5826	1.55637	3.35310	7.22405	.265252
3.78	14.2884	1.94422	6.14817	54.0102	1.55775	3.35607	7.23043	.264550
3.79	14.3641	1.94679	6.15630	54.4399	1.55912	3.35902	7.23680	.263852
3.80	14.4400	1.94936	6.16441	54.8720	1.56049	3.36198	7.24316	.263158
3.81	14.5161	1.95192	6.17252	55.3063	1.56186	3.36492	7.24950	.262467
3.82	14.5924	1.95448	6.18061	55.7430	1.56322	3.36786	7.25584	.261780
3.83	14.6689	1.95704	6.18870	56.1819	1.56459	3.37080	7.26217	.261097
3.84	14.7456	1.95959	6.19677	56.6231	1.56595	3.37373	7.26848	.260417
3.85	14.8225	1.96214	6.20484	57.0666	1.56731	3.37666	7.27479	.259740
3.86	14.8996	1.96469	6.21289	57.5125	1.56866	3.37958	7.28108	.259067
3.87	14.9769	1.96723	6.22093	57.9606	1.57001	3.38249	7.28736	.258398
3.88	15.0544	1.96977	6.22896	58.4111	1.57137	3.38540	7.29363	.257732
3.89	15.1321	1.97231	6.23699	58.8639	1.57271	3.38831	7.29989	.257069
3.90	15.2100	1.97484	6.24500	59.3190	1.57406	3.39121	7.30614	.256410
3.91	15.2881	1.97737	6.25300	59.7765	1.57541	3.39411	7.31238	.255754
3.92	15.3664	1.97990	6.26099	60.2363	1.57675	3.39700	7.31861	.255102
3.93	15.4449	1.98242	6.26897	60.6985	1.57809	3.39988	7.32483	.254453
3.94	15.5236	1.98494	6.27694	61.1630	1.57942	3.40277	7.33104	.253807
3.95	15.6025	1.98746	6.28490	61.6299	1.58076	3.40564	7.33723	.253165
3.96	15.6816	1.98997	6.29285	62.0991	1.58209	3.40851	7.34342	.252525
3.97	15.7609	1.99249	6.30079	62.5708	1.58342	3.41138	7.34960	.251889
3.98	15.8404	1.99499	6.30872	63.0448	1.58475	3.41424	7.35576	.251256
3.99	15.9201	1.99750	6.31664	63.5212	1.58608	3.41710	7.36192	.250627
4.00	16.0000	2.00000	6.32456	64.0000	1.58740	3.41995	7.36806	.250000
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
4.00	16.0000	2.00000	6.32456	64.0000	1.58740	3.41995	7.36806	.250000
4.01	16.0801	2.00250	6.33246	64.4812	1.58872	3.42280	7.37420	.249377
4.02	16.1604	2.00499	6.34035	64.9648	1.59004	3.42564	7.38032	.248756
4.03	16.2409	2.00749	6.34823	65.4508	1.59136	3.42848	7.38644	.248139
4.04	16.3216	2.00998	6.35610	65.9393	1.59267	3.43131	7.39254	.247525
4.05	16.4025	2.01246	6.36396	66.4301	1.59399	3.43414	7.39864	.246914
4.06	16.4836	2.01494	6.37181	66.9234	1.59530	3.43697	7.40472	.246305
4.07	16.5649	2.01742	6.37966	67.4191	1.59661	3.43979	7.41080	.245700
4.08	16.6464	2.01990	6.38749	67.9173	1.59791	3.44260	7.41686	.245098
4.09	16.7281	2.02237	6.39531	68.4179	1.59922	3.44541	7.42291	.244499
4.10	16.8100	2.02485	6.40312	68.9210	1.60052	3.44822	7.42896	.243902
4.11	16.8921	2.02731	6.41093	69.4265	1.60182	3.45102	7.43499	.243309
4.12	16.9744	2.02978	6.41872	69.9345	1.60312	3.45382	7.44102	.242718
4.13	17.0569	2.03224	6.42651	70.4450	1.60441	3.45661	7.44703	.242131
4.14	17.1396	2.03470	6.43428	70.9579	1.60571	3.45939	7.45304	.241546
4.15	17.2225	2.03715	6.44205	71.4734	1.60700	3.46218	7.45904	.240964
4.16	17.3056	2.03961	6.44981	71.9913	1.60829	3.46496	7.46502	.240385
4.17	17.3889	2.04206	6.45755	72.5117	1.60958	3.46773	7.47100	.239808
4.18	17.4724	2.04450	6.46529	73.0346	1.61086	3.47050	7.47697	.239234
4.19	17.5561	2.04695	6.47302	73.5601	1.61215	3.47327	7.48292	.238663
4.20	17.6400	2.04939	6.48074	74.0880	1.61343	3.47603	7.48887	.238095
4.21	17.7241	2.05183	6.48845	74.6185	1.61471	3.47878	7.49481	.237530
4.22	17.8084	2.05426	6.49615	75.1514	1.61599	3.48154	7.50074	.236967
4.23	17.8929	2.05670	6.50384	75.6870	1.61726	3.48428	7.50666	.236407
4.24	17.9776	2.05913	6.51153	76.2250	1.61853	3.48703	7.51257	.235849
4.25	18.0625	2.06155	6.51920	76.7656	1.61981	3.48977	7.51847	.235294
4.26	18.1476	2.06398	6.52687	77.3088	1.62108	3.49250	7.52437	.234742
4.27	18.2329	2.06640	6.53452	77.8545	1.62234	3.49523	7.53025	.234192
4.28	18.3184	2.06882	6.54217	78.4028	1.62361	3.49796	7.53612	.233645
4.29	18.4041	2.07123	6.54981	78.9536	1.62487	3.50068	7.54199	.233100
4.30	18.4900	2.07364	6.55744	79.5070	1.62613	3.50340	7.54784	.232558
4.31	18.5761	2.07605	6.56506	80.0630	1.62739	3.50611	7.55369	.232019
4.32	18.6624	2.07846	6.57267	80.6216	1.62865	3.50882	7.55953	.231481
4.33	18.7489	2.08087	6.58027	81.1827	1.62991	3.51153	7.56535	.230947
4.34	18.8356	2.08327	6.58787	81.7465	1.63116	3.51423	7.57117	.230415
4.35	18.9225	2.08567	6.59545	82.3129	1.63241	3.51692	7.57698	.229885
4.36	19.0096	2.08806	6.60303	82.8819	1.63366	3.51962	7.58279	.229358
4.37	19.0969	2.09045	6.61060	83.4535	1.63491	3.52231	7.58858	.228833
4.38	19.1844	2.09284	6.61816	84.0277	1.63619	3.52499	7.59436	.228311
4.39	19.2721	2.09523	6.62571	84.6045	1.63740	3.52767	7.60014	.227790
4.40	19.3600	2.09762	6.63325	85.1840	1.63864	3.53035	7.60590	.227273
4.41	19.4481	2.10000	6.64078	85.7661	1.63988	3.53302	7.61166	.226757
4.42	19.5364	2.10238	6.64831	86.3509	1.64112	3.53569	7.61741	.226244
4.43	19.6249	2.10476	6.65582	86.9383	1.64236	3.53835	7.62315	.225734
4.44	19.7136	2.10713	6.66333	87.5284	1.64359	3.54101	7.62888	.225225
4.45	19.8025	2.10950	6.67083	88.1211	1.64483	3.54367	7.63461	.224719
4.46	19.8916	2.11187	6.67832	88.7165	1.64606	3.54632	7.64032	.224215
4.47	19.9809	2.11424	6.68581	89.3146	1.64729	3.54897	7.64603	.223714
4.48	20.0704	2.11660	6.69328	89.9154	1.64851	3.55162	7.65172	.223214
4.49	20.1601	2.11896	6.70075	90.5188	1.64974	3.55426	7.65741	.222717
4.50	20.2500	2.12132	6.70820	91.1250	1.65096	3.55689	7.66309	.222222
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
4.50	20.2500	2.12132	6.70820	91.1250	1.65096	3.55689	7.66309	.222222
4.51	20.3401	2.12368	6.71565	91.7339	1.65219	3.55953	7.66877	.221729
4.52	20.4304	2.12603	6.72309	92.3454	1.65341	3.56215	7.67443	.221239
4.53	20.5209	2.12838	6.73053	92.9597	1.65462	3.56478	7.68009	.220751
4.54	20.6116	2.13073	6.73795	93.5767	1.65584	3.56740	7.68573	.220264
4.55	20.7025	2.13307	6.74537	94.1964	1.65706	3.57002	7.69137	.219780
4.56	20.7936	2.13542	6.75278	94.8188	1.65827	3.57263	7.69700	.219298
4.57	20.8849	2.13776	6.76018	95.4440	1.65948	3.57524	7.70262	.218818
4.58	20.9764	2.14009	6.76757	96.0719	1.66069	3.57785	7.70824	.218341
4.59	21.0681	2.14243	6.77495	96.7026	1.66190	3.58045	7.71384	.217865
4.60	21.1600	2.14476	6.78233	97.3360	1.66310	3.58305	7.71944	.217391
4.61	21.2521	2.14709	6.78970	97.9722	1.66431	3.58564	7.72503	.216920
4.62	21.3444	2.14942	6.79706	98.6111	1.66551	3.58823	7.73061	.216450
4.63	21.4369	2.15174	6.80441	99.2528	1.66671	3.59082	7.73619	.215983
4.64	21.5296	2.15407	6.81175	99.8973	1.66791	3.59340	7.74175	.215517
4.65	21.6225	2.15639	6.81909	100.545	1.66911	3.59598	7.74731	.215054
4.66	21.7156	2.15870	6.82642	101.195	1.67030	3.59856	7.75286	.214592
4.67	21.8089	2.16102	6.83374	101.848	1.67150	3.60113	7.75840	.214133
4.68	21.9024	2.16333	6.84105	102.503	1.67269	3.60370	7.76394	.213675
4.69	21.9961	2.16564	6.84836	103.162	1.67388	3.60626	7.76946	.213220
4.70	22.0900	2.16795	6.85565	103.823	1.67507	3.60883	7.77498	.212766
4.71	22.1841	2.17025	6.86294	104.487	1.67626	3.61138	7.78049	.212314
4.72	22.2784	2.17256	6.87023	105.154	1.67744	3.61394	7.78599	.211864
4.73	22.3729	2.17486	6.87750	105.824	1.67863	3.61649	7.79149	.211416
4.74	22.4676	2.17715	6.88477	106.496	1.67981	3.61903	7.79697	.210970
4.75	22.5625	2.17945	6.89202	107.172	1.68099	3.62158	7.80245	.210526
4.76	22.6576	2.18174	6.89928	107.850	1.68217	3.62412	7.80793	.210084
4.77	22.7529	2.18403	6.90652	108.531	1.68334	3.62665	7.81339	.209644
4.78	22.8484	2.18632	6.91375	109.215	1.68452	3.62919	7.81885	.209205
4.79	22.9441	2.18861	6.92098	109.902	1.68569	3.63172	7.82429	.208768
4.80	23.0400	2.19089	6.92820	110.592	1.68687	3.63424	7.82974	.208333
4.81	23.1361	2.19317	6.93542	111.285	1.68804	3.63676	7.83517	.207900
4.82	23.2324	2.19545	6.94262	111.980	1.68920	3.63928	7.84059	.207469
4.83	23.3289	2.19773	6.94982	112.679	1.69037	3.64180	7.84601	.207039
4.84	23.4256	2.20000	6.95701	113.380	1.69154	3.64431	7.85142	.206612
4.85	23.5225	2.20227	6.96419	114.084	1.69270	3.64682	7.85683	.206186
4.86	23.6196	2.20454	6.97137	114.791	1.69386	3.64932	7.86222	.205761
4.87	23.7169	2.20681	6.97854	115.501	1.69503	3.65182	7.86761	.205339
4.88	23.8144	2.20907	6.98570	116.214	1.69619	3.65432	7.87299	.204918
4.89	23.9121	2.21133	6.99285	116.930	1.69734	3.65681	7.87837	.204499
4.90	24.0100	2.21359	7.00000	117.649	1.69850	3.65931	7.88374	.204082
4.91	24.1081	2.21585	7.00714	118.371	1.69965	3.66179	7.88909	.203666
4.92	24.2064	2.21811	7.01427	119.095	1.70081	3.66428	7.89445	.203252
4.93	24.3049	2.22036	7.02140	119.823	1.70196	3.66676	7.89979	.202840
4.94	24.4036	2.22261	7.02851	120.554	1.70311	3.66924	7.90513	.202429
4.95	24.5025	2.22486	7.03562	121.287	1.70426	3.67171	7.91046	.202020
4.96	24.6016	2.22711	7.04273	122.024	1.70540	3.67418	7.91578	.201613
4.97	24.7009	2.22935	7.04982	122.763	1.70655	3.67665	7.92110	.201207
4.98	24.8004	2.23159	7.05691	123.506	1.70769	3.67911	7.92641	.200803
4.99	24.9001	2.23383	7.06399	124.251	1.70884	3.68157	7.93171	.200401
5.00	25.0000	2.23607	7.07107	125.000	1.70998	3.68403	7.93701	.200000
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
5.00	25.0000	2.23607	7.07107	125.000	1.70998	3.68403	7.93701	.200000
5.01	25.1001	2.23830	7.07814	125.752	1.71112	3.68649	7.94229	.199601
5.02	25.2004	2.24054	7.08520	126.506	1.71225	3.68894	7.94757	.199203
5.03	25.3009	2.24277	7.09225	127.264	1.71339	3.69138	7.95285	.198807
5.04	25.4016	2.24499	7.09930	128.024	1.71452	3.69383	7.95811	.198413
5.05	25.5025	2.24722	7.10634	128.788	1.71566	3.69627	7.96337	.198020
5.06	25.6036	2.24944	7.11337	129.554	1.71679	3.69871	7.96863	.197628
5.07	25.7049	2.25167	7.12039	130.324	1.71792	3.70114	7.97387	.197239
5.08	25.8064	2.25389	7.12741	131.097	1.71905	3.70357	7.97911	.196850
5.09	25.9081	2.25610	7.13442	131.872	1.72017	3.70600	7.98434	.196464
5.10	26.0100	2.25832	7.14143	132.651	1.72130	3.70843	7.98957	.196078
5.11	26.1121	2.26053	7.14843	133.433	1.72242	3.71085	7.99479	.195695
5.12	26.2144	2.26274	7.15542	134.218	1.72355	3.71327	8.00000	.195312
5.13	26.3169	2.26495	7.16240	135.006	1.72467	3.71569	8.00520	.194932
5.14	26.4196	2.26716	7.16938	135.797	1.72579	3.71810	8.01040	.194553
5.15	26.5225	2.26936	7.17635	136.591	1.72691	3.72051	8.01559	.194175
5.16	26.6256	2.27156	7.18331	137.388	1.72802	3.72292	8.02078	.193798
5.17	26.7289	2.27376	7.19027	138.188	1.72914	3.72532	8.02596	.193424
5.18	26.8324	2.27596	7.19722	138.992	1.73025	3.72772	8.03113	.193050
5.19	26.9361	2.27816	7.20417	139.798	1.73137	3.73012	8.03629	.192678
5.20	27.0400	2.28035	7.21110	140.608	1.73248	3.73251	8.04145	.192308
5.21	27.1441	2.28254	7.21803	141.421	1.73359	3.73490	8.04660	.191939
5.22	27.2484	2.28473	7.22496	142.237	1.73470	3.73729	8.05175	.191571
5.23	27.3529	2.28692	7.23187	143.056	1.73580	3.73968	8.05689	.191205
5.24	27.4576	2.28910	7.23878	143.878	1.73691	3.74206	8.06202	.190840
5.25	27.5625	2.29129	7.24569	144.703	1.73801	3.74443	8.06714	.190476
5.26	27.6676	2.29347	7.25259	145.532	1.73912	3.74681	8.07226	.190114
5.27	27.7729	2.29565	7.25948	146.363	1.74022	3.74918	8.07737	.189753
5.28	27.8784	2.29783	7.26636	147.198	1.74132	3.75155	8.08248	.189394
5.29	27.9841	2.30000	7.27324	148.036	1.74242	3.75392	8.08758	.189036
5.30	28.0900	2.30217	7.28011	148.877	1.74351	3.75629	8.09267	.188679
5.31	28.1961	2.30434	7.28697	149.721	1.74461	3.75865	8.09776	.188324
5.32	28.3024	2.30651	7.29383	150.569	1.74570	3.76101	8.10284	.187970
5.33	28.4089	2.30868	7.30068	151.419	1.74680	3.76336	8.10791	.187617
5.34	28.5156	2.31084	7.30753	152.273	1.74789	3.76571	8.11298	.187266
5.35	28.6225	2.31301	7.31437	153.130	1.74898	3.76806	8.11804	.186916
5.36	28.7296	2.31517	7.32120	153.991	1.75007	3.77041	8.12310	.186567
5.37	28.8369	2.31733	7.32803	154.854	1.75116	3.77275	8.12814	.186220
5.38	28.9444	2.31948	7.33485	155.721	1.75224	3.77509	8.13319	.185874
5.39	29.0521	2.32164	7.34166	156.591	1.75333	3.77743	8.13822	.185529
5.40	29.1600	2.32379	7.34847	157.464	1.75441	3.77976	8.14325	.185185
5.41	29.2681	2.32594	7.35527	158.340	1.75549	3.78209	8.14828	.184843
5.42	29.3764	2.32809	7.36206	159.220	1.75657	3.78442	8.15329	.184502
5.43	29.4849	2.33024	7.36885	160.103	1.75765	3.78675	8.15831	.184162
5.44	29.5936	2.33238	7.37564	160.989	1.75873	3.78907	8.16331	.183824
5.45	29.7025	2.33452	7.38241	161.879	1.75981	3.79139	8.16831	.183486
5.46	29.8116	2.33666	7.38918	162.771	1.76088	3.79371	8.17330	.183150
5.47	29.9209	2.33880	7.39594	163.667	1.76196	3.79603	8.17829	.182815
5.48	30.0304	2.34094	7.40270	164.567	1.76303	3.79834	8.18327	.182482
5.49	30.1401	2.34307	7.40945	165.469	1.76410	3.80065	8.18824	.182149
5.50	30.2500	2.34521	7.41620	166.375	1.76517	3.80295	8.19321	.181818
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
5.50	30.2500	2.34521	7.41620	166.375	1.76517	3.80295	8.19321	.181818
5.51	30.3601	2.34734	7.42294	167.284	1.76624	3.80526	8.19818	.181488
5.52	30.4704	2.34947	7.42967	168.197	1.76731	3.80756	8.20313	.181159
5.53	30.5809	2.35160	7.43640	169.112	1.76838	3.80985	8.20808	.180832
5.54	30.6916	2.35372	7.44312	170.031	1.76944	3.81215	8.21303	.180505
5.55	30.8025	2.35584	7.44983	170.954	1.77051	3.81444	8.21797	.180180
5.56	30.9136	2.35797	7.45654	171.880	1.77157	3.81673	8.22290	.179856
5.57	31.0249	2.36008	7.46324	172.809	1.77263	3.81902	8.22783	.179533
5.58	31.1364	2.36220	7.46994	173.741	1.77369	3.82130	8.23275	.179211
5.59	31.2481	2.36432	7.47663	174.677	1.77475	3.82358	8.23766	.178891
5.60	31.3600	2.36643	7.48331	175.616	1.77581	3.82586	8.24257	.178571
5.61	31.4721	2.36854	7.48999	176.558	1.77686	3.82814	8.24747	.178253
5.62	31.5844	2.37065	7.49667	177.504	1.77792	3.83041	8.25237	.177936
5.63	31.6969	2.37276	7.50333	178.454	1.77897	3.83268	8.25726	.177620
5.64	31.8096	2.37487	7.50999	179.406	1.78003	3.83495	8.26215	.177305
5.65	31.9225	2.37697	7.51665	180.362	1.78108	3.83722	8.26703	.176991
5.66	32.0356	2.37908	7.52330	181.321	1.78213	3.83948	8.27190	.176678
5.67	32.1489	2.38118	7.52994	182.284	1.78318	3.84174	8.27677	.176367
5.68	32.2624	2.38328	7.53658	183.250	1.78422	3.84399	8.28164	.176056
5.69	32.3761	2.38537	7.54321	184.220	1.78527	3.84625	8.28649	.175747
5.70	32.4900	2.38747	7.54983	185.193	1.78632	3.84850	8.29134	.175439
5.71	32.6041	2.38956	7.55645	186.169	1.78736	3.85075	8.29619	.175131
5.72	32.7184	2.39165	7.56307	187.149	1.78840	3.85300	8.30103	.174825
5.73	32.8329	2.39374	7.56968	188.133	1.78944	3.85524	8.30587	.174520
5.74	32.9476	2.39583	7.57628	189.119	1.79048	3.85748	8.31069	.174216
5.75	33.0625	2.39792	7.58288	190.109	1.79152	3.85972	8.31552	.173913
5.76	33.1776	2.40000	7.58947	191.103	1.79256	3.86196	8.32034	.173611
5.77	33.2929	2.40208	7.59605	192.100	1.79360	3.86419	8.32515	.173310
5.78	33.4084	2.40416	7.60263	193.101	1.79463	3.86642	8.32995	.173010
5.79	33.5241	2.40624	7.60920	194.105	1.79567	3.86865	8.33476	.172712
5.80	33.6400	2.40832	7.61577	195.112	1.79670	3.87088	8.33955	.172414
5.81	33.7561	2.41039	7.62234	196.123	1.79773	3.87310	8.34434	.172117
5.82	33.8724	2.41247	7.62889	197.137	1.79876	3.87532	8.34913	.171821
5.83	33.9889	2.41454	7.63544	198.155	1.79979	3.87754	8.35390	.171527
5.84	34.1056	2.41661	7.64199	199.177	1.80082	3.87975	8.35868	.171233
5.85	34.2225	2.41868	7.64853	200.202	1.80185	3.88197	8.36345	.170940
5.86	34.3396	2.42074	7.65506	201.230	1.80288	3.88418	8.36821	.170649
5.87	34.4569	2.42281	7.66159	202.262	1.80390	3.88639	8.37297	.170358
5.88	34.5744	2.42487	7.66812	203.297	1.80492	3.88859	8.37772	.170068
5.89	34.6921	2.42693	7.67463	204.336	1.80595	3.89080	8.38247	.169779
5.90	34.8100	2.42899	7.68115	205.379	1.80697	3.89300	8.38721	.169492
5.91	34.9281	2.43105	7.68765	206.425	1.80799	3.89519	8.39194	.169205
5.92	35.0464	2.43311	7.69415	207.475	1.80901	3.89739	8.39667	.168919
5.93	35.1649	2.43516	7.70065	208.528	1.81003	3.89958	8.40140	.168634
5.94	35.2836	2.43721	7.70714	209.585	1.81104	3.90177	8.40612	.168350
5.95	35.4025	2.43926	7.71362	210.645	1.81206	3.90396	8.41083	.168067
5.96	35.5216	2.44131	7.72010	211.709	1.81307	3.90615	8.41554	.167785
5.97	35.6409	2.44336	7.72658	212.776	1.81409	3.90833	8.42025	.167504
5.98	35.7604	2.44540	7.73305	213.847	1.81510	3.91051	8.42494	.167224
5.99	35.8801	2.44745	7.73951	214.922	1.81611	3.91269	8.42964	.166945
6.00	36.0000	2.44949	7.74597	216.000	1.81712	3.91487	8.43433	.166667
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
6.00	36.0000	2.44949	7.74597	216.000	1.81712	3.91487	8.43433	.166667
6.01	36.1201	2.45153	7.75242	217.082	1.81813	3.91704	8.43901	.166389
6.02	36.2404	2.45357	7.75887	218.167	1.81914	3.91921	8.44369	.166113
6.03	36.3609	2.45561	7.76531	219.256	1.82014	3.92138	8.44836	.165837
6.04	36.4816	2.45764	7.77174	220.349	1.82115	3.92355	8.45303	.165563
6.05	36.6025	2.45967	7.77817	221.445	1.82215	3.92571	8.45769	.165289
6.06	36.7236	2.46171	7.78460	222.545	1.82316	3.92787	8.46235	.165017
6.07	36.8449	2.46374	7.79102	223.649	1.82416	3.93003	8.46700	.164745
6.08	36.9664	2.46577	7.79744	224.756	1.82516	3.93219	8.47165	.164474
6.09	37.0881	2.46779	7.80385	225.867	1.82616	3.93434	8.47629	.164204
6.10	37.2100	2.46982	7.81025	226.981	1.82716	3.93650	8.48093	.163934
6.11	37.3321	2.47184	7.81665	228.099	1.82816	3.93865	8.48556	.163666
6.12	37.4544	2.47386	7.82304	229.221	1.82915	3.94079	8.49018	.163399
6.13	37.5769	2.47588	7.82943	230.346	1.83015	3.94294	8.49481	.163132
6.14	37.6996	2.47790	7.83582	231.476	1.83115	3.94508	8.49942	.162866
6.15	37.8225	2.47992	7.84219	232.608	1.83214	3.94722	8.50403	.162602
6.16	37.9456	2.48193	7.84857	233.745	1.83313	3.94936	8.50864	.162338
6.17	38.0689	2.48395	7.85493	234.885	1.83412	3.95150	8.51324	.162075
6.18	38.1924	2.48596	7.86130	236.029	1.83511	3.95363	8.51784	.161812
6.19	38.3161	2.48797	7.86766	237.177	1.83610	3.95576	8.52243	.161551
6.20	38.4400	2.48998	7.87401	238.328	1.83709	3.95789	8.52702	.161290
6.21	38.5641	2.49199	7.88036	239.483	1.83808	3.96002	8.53160	.161031
6.22	38.6884	2.49399	7.88670	240.642	1.83906	3.96214	8.53618	.160772
6.23	38.8129	2.49600	7.89303	241.804	1.84005	3.96427	8.54075	.160514
6.24	38.9376	2.49800	7.89937	242.971	1.84103	3.96638	8.54532	.160256
6.25	39.0625	2.50000	7.90569	244.141	1.84202	3.96850	8.54988	.160000
6.26	39.1876	2.50200	7.91202	245.314	1.84300	3.97062	8.55444	.159744
6.27	39.3129	2.50400	7.91833	246.492	1.84398	3.97273	8.55899	.159490
6.28	39.4384	2.50599	7.92465	247.673	1.84496	3.97484	8.56354	.159236
6.29	39.5641	2.50799	7.93095	248.858	1.84594	3.97695	8.56808	.158983
6.30	39.6900	2.50998	7.93725	250.047	1.84691	3.97906	8.57262	.158730
6.31	39.8161	2.51197	7.94355	251.240	1.84789	3.98116	8.57715	.158479
6.32	39.9424	2.51396	7.94984	252.436	1.84887	3.98326	8.58168	.158228
6.33	40.0689	2.51595	7.95613	253.636	1.84984	3.98536	8.58620	.157978
6.34	40.1956	2.51794	7.96241	254.840	1.85082	3.98746	8.59072	.157729
6.35	40.3225	2.51992	7.96869	256.048	1.85179	3.98956	8.59524	.157480
6.36	40.4496	2.52190	7.97496	257.259	1.85276	3.99165	8.59975	.157233
6.37	40.5769	2.52389	7.98123	258.475	1.85373	3.99374	8.60425	.156986
6.38	40.7044	2.52587	7.98749	259.694	1.85470	3.99583	8.60875	.156740
6.39	40.8321	2.52784	7.99375	260.917	1.85567	3.99792	8.61325	.156495
6.40	40.9600	2.52982	8.00000	262.144	1.85664	4.00000	8.61774	.156250
6.41	41.0881	2.53180	8.00625	263.375	1.85760	4.00208	8.62222	.156006
6.42	41.2164	2.53377	8.01249	264.609	1.85857	4.00416	8.62671	.155763
6.43	41.3449	2.53574	8.01873	265.848	1.85953	4.00624	8.63118	.155521
6.44	41.4736	2.53772	8.02496	267.090	1.86050	4.00832	8.63566	.155280
6.45	41.6025	2.53969	8.03119	268.336	1.86146	4.01039	8.64012	.155039
6.46	41.7316	2.54165	8.03741	269.586	1.86242	4.01246	8.64459	.154799
6.47	41.8609	2.54362	8.04363	270.840	1.86338	4.01453	8.64904	.154560
6.48	41.9904	2.54558	8.04984	272.098	1.86434	4.01660	8.65350	.154321
6.49	42.1201	2.54755	8.05605	273.359	1.86530	4.01866	8.65795	.154083
6.50	42.2500	2.54951	8.06226	274.625	1.86626	4.02073	8.66239	.153846
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
6.50	42.2500	2.54951	8.06226	274.625	1.86626	4.02073	8.66239	.153846
6.51	42.3801	2.55147	8.06846	275.894	1.86721	4.02279	8.66683	.153610
6.52	42.5104	2.55343	8.07465	277.168	1.86817	4.02485	8.67127	.153374
6.53	42.6409	2.55539	8.08084	278.445	1.86912	4.02690	8.67570	.153139
6.54	42.7716	2.55734	8.08703	279.726	1.87008	4.02896	8.68012	.152905
6.55	42.9025	2.55930	8.09321	281.011	1.87103	4.03101	8.68455	.152672
6.56	43.0336	2.56125	8.09938	282.300	1.87198	4.03306	8.68896	.152439
6.57	43.1649	2.56320	8.10555	283.593	1.87293	4.03511	8.69338	.152207
6.58	43.2964	2.56515	8.11172	284.890	1.87388	4.03715	8.69778	.151976
6.59	43.4281	2.56710	8.11788	286.191	1.87483	4.03920	8.70219	.151745
6.60	43.5600	2.56905	8.12404	287.496	1.87578	4.04124	8.70659	.151515
6.61	43.6921	2.57099	8.13019	288.805	1.87672	4.04328	8.71098	.151286
6.62	43.8244	2.57294	8.13634	290.118	1.87767	4.04532	8.71537	.151057
6.63	43.9569	2.57488	8.14248	291.434	1.87862	4.04735	8.71976	.150830
6.64	44.0896	2.57682	8.14862	292.755	1.87956	4.04939	8.72414	.150602
6.65	44.2225	2.57876	8.15475	294.080	1.88050	4.05142	8.72852	.150376
6.66	44.3556	2.58070	8.16088	295.408	1.88144	4.05345	8.73289	.150150
6.67	44.4889	2.58263	8.16701	296.741	1.88239	4.05548	8.73726	.149925
6.68	44.6224	2.58457	8.17313	298.078	1.88333	4.05750	8.74162	.149701
6.69	44.7561	2.58650	8.17924	299.418	1.88427	4.05953	8.74598	.149477
6.70	44.8900	2.58844	8.18535	300.763	1.88520	4.06155	8.75034	.149254
6.71	45.0241	2.59037	8.19146	302.112	1.88614	4.06357	8.75469	.149031
6.72	45.1584	2.59230	8.19756	303.464	1.88708	4.06559	8.75904	.148810
6.73	45.2929	2.59422	8.20366	304.821	1.88801	4.06760	8.76338	.148588
6.74	45.4276	2.59615	8.20975	306.182	1.88895	4.06961	8.76772	.148368
6.75	45.5625	2.59808	8.21584	307.547	1.88988	4.07163	8.77205	.148148
6.76	45.6976	2.60000	8.22192	308.916	1.89081	4.07364	8.77638	.147929
6.77	45.8329	2.60192	8.22800	310.289	1.89175	4.07564	8.78071	.147710
6.78	45.9684	2.60384	8.23408	311.666	1.89268	4.07765	8.78503	.147493
6.79	46.1041	2.60576	8.24015	313.047	1.89361	4.07965	8.78935	.147275
6.80	46.2400	2.60768	8.24621	314.432	1.89454	4.08166	8.79366	.147059
6.81	46.3761	2.60960	8.25227	315.821	1.89546	4.08365	8.79797	.146843
6.82	46.5124	2.61151	8.25833	317.215	1.89639	4.08565	8.80227	.146628
6.83	46.6489	2.61343	8.26438	318.612	1.89732	4.08765	8.80657	.146413
6.84	46.7856	2.61534	8.27043	320.014	1.89824	4.08964	8.81087	.146199
6.85	46.9225	2.61725	8.27647	321.419	1.89917	4.09163	8.81516	.145985
6.86	47.0596	2.61916	8.28251	322.829	1.90009	4.09362	8.81945	.145773
6.87	47.1969	2.62107	8.28855	324.243	1.90102	4.09561	8.82373	.145560
6.88	47.3344	2.62298	8.29458	325.661	1.90194	4.09760	8.82801	.145349
6.89	47.4721	2.62488	8.30060	327.083	1.90286	4.09958	8.83228	.145138
6.90	47.6100	2.62679	8.30662	328.509	1.90378	4.10157	8.83656	.144928
6.91	47.7481	2.62869	8.31264	329.939	1.90470	4.10355	8.84082	.144718
6.92	47.8864	2.63059	8.31865	331.374	1.90562	4.10552	8.84509	.144509
6.93	48.0249	2.63249	8.32466	332.813	1.90653	4.10750	8.84934	.144300
6.94	48.1636	2.63439	8.33067	334.255	1.90745	4.10948	8.85360	.144092
6.95	48.3025	2.63629	8.33667	335.702	1.90837	4.11145	8.85785	.143885
6.96	48.4416	2.63818	8.34266	337.154	1.90928	4.11342	8.86210	.143678
6.97	48.5809	2.64008	8.34865	338.609	1.91019	4.11539	8.86634	.143472
6.98	48.7204	2.64197	8.35464	340.068	1.91111	4.11736	8.87058	.143266
6.99	48.8601	2.64386	8.36062	341.532	1.91202	4.11932	8.87481	.143062
7.00	49.0000	2.64575	8.36660	343.000	1.91293	4.12129	8.87904	.142857
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
7.00	49.0000	2.64575	8.36660	343.000	1.91293	4.12129	8.87904	.142857
7.01	49.1401	2.64764	8.37257	344.472	1.91384	4.12325	8.88327	.142653
7.02	49.2804	2.64953	8.37854	345.948	1.91475	4.12521	8.88749	.142450
7.03	49.4209	2.65141	8.38451	347.429	1.91566	4.12716	8.89171	.142248
7.04	49.5616	2.65330	8.39047	348.914	1.91657	4.12912	8.89592	.142045
7.05	49.7025	2.65518	8.39643	350.403	1.91747	4.13107	8.90013	.141844
7.06	49.8436	2.65707	8.40238	351.896	1.91838	4.13303	8.90434	.141643
7.07	49.9849	2.65895	8.40833	353.393	1.91929	4.13498	8.90854	.141443
7.08	50.1264	2.66083	8.41427	354.895	1.92019	4.13693	8.91274	.141243
7.09	50.2681	2.66271	8.42021	356.401	1.92109	4.13887	8.91693	.141044
7.10	50.4100	2.66458	8.42615	357.911	1.92200	4.14082	8.92112	.140845
7.11	50.5521	2.66646	8.43208	359.425	1.92290	4.14276	8.92531	.140647
7.12	50.6944	2.66833	8.43801	360.944	1.92380	4.14470	8.92949	.140449
7.13	50.8369	2.67021	8.44393	362.467	1.92470	4.14664	8.93367	.140252
7.14	50.9796	2.67208	8.44985	363.994	1.92560	4.14858	8.93784	.140056
7.15	51.1225	2.67395	8.45577	365.526	1.92650	4.15052	8.94201	.139860
7.16	51.2656	2.67582	8.46168	367.062	1.92740	4.15245	8.94618	.139665
7.17	51.4089	2.67769	8.46759	368.602	1.92829	4.15438	8.95034	.139470
7.18	51.5524	2.67955	8.47349	370.146	1.92919	4.15631	8.95450	.139276
7.19	51.6961	2.68142	8.47939	371.695	1.93008	4.15824	8.95866	.139082
7.20	51.8400	2.68328	8.48528	373.248	1.93098	4.16017	8.96281	.138889
7.21	51.9841	2.68514	8.49117	374.805	1.93187	4.16209	8.96696	.138696
7.22	52.1284	2.68701	8.49706	376.367	1.93277	4.16402	8.97110	.138504
7.23	52.2729	2.68887	8.50294	377.933	1.93366	4.16594	8.97524	.138313
7.24	52.4176	2.69072	8.50882	379.503	1.93455	4.16786	8.97938	.138122
7.25	52.5625	2.69258	8.51469	381.078	1.93544	4.16978	8.98351	.137931
7.26	52.7076	2.69444	8.52056	382.657	1.93633	4.17169	8.98764	.137741
7.27	52.8529	2.69629	8.52643	384.241	1.93722	4.17361	8.99176	.137552
7.28	52.9984	2.69815	8.53229	385.828	1.93810	4.17552	8.99588	.137363
7.29	53.1441	2.70000	8.53815	387.420	1.93899	4.17743	9.00000	.137174
7.30	53.2900	2.70185	8.54400	389.017	1.93988	4.17934	9.00411	.136986
7.31	53.4361	2.70370	8.54985	390.618	1.94076	4.18125	9.00822	.136799
7.32	53.5824	2.70555	8.55570	392.223	1.94165	4.18315	9.01233	.136612
7.33	53.7289	2.70740	8.56154	393.833	1.94253	4.18506	9.01643	.136426
7.34	53.8756	2.70924	8.56738	395.447	1.94341	4.18696	9.02053	.136240
7.35	54.0225	2.71109	8.57321	397.065	1.94430	4.18886	9.02462	.136054
7.36	54.1696	2.71293	8.57904	398.688	1.94518	4.19076	9.02871	.135870
7.37	54.3169	2.71477	8.58487	400.316	1.94606	4.19266	9.03280	.135685
7.38	54.4644	2.71662	8.59069	401.947	1.94694	4.19455	9.03689	.135501
7.39	54.6121	2.71846	8.59651	403.583	1.94782	4.19644	9.04097	.135318
7.40	54.7600	2.72029	8.60233	405.224	1.94870	4.19834	9.04504	.135135
7.41	54.9081	2.72213	8.60814	406.869	1.94957	4.20023	9.04911	.134953
7.42	55.0564	2.72397	8.61394	408.518	1.95045	4.20212	9.05318	.134771
7.43	55.2049	2.72580	8.61974	410.172	1.95132	4.20400	9.05725	.134590
7.44	55.3536	2.72764	8.62554	411.831	1.95220	4.20589	9.06131	.134409
7.45	55.5025	2.72947	8.63134	413.494	1.95307	4.20777	9.06537	.134228
7.46	55.6516	2.73130	8.63713	415.161	1.95395	4.20965	9.06942	.134048
7.47	55.8009	2.73313	8.64292	416.833	1.95482	4.21153	9.07347	.133869
7.48	55.9504	2.73496	8.64870	418.509	1.95569	4.21341	9.07752	.133690
7.49	56.1001	2.73679	8.65448	420.190	1.95656	4.21529	9.08156	.133511
7.50	56.2500	2.73861	8.66025	421.875	1.94743	4.21716	9.08560	.133333
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
7.50	56.2500	2.73861	8.66025	421.875	1.95743	4.21716	9.08560	.133333
7.51	56.4001	2.74044	8.66603	423.565	1.95830	4.21904	9.08964	.133156
7.52	56.5504	2.74226	8.67179	425.259	1.95917	4.22091	9.09367	.132979
7.53	56.7009	2.74408	8.67756	426.958	1.96004	4.22278	9.09770	.132802
7.54	56.8516	2.74591	8.68332	428.661	1.96091	4.22465	9.10173	.132626
7.55	57.0025	2.74773	8.68907	430.369	1.96177	4.22651	9.10575	.132450
7.56	57.1536	2.74955	8.69483	432.081	1.96264	4.22838	9.10977	.132275
7.57	57.3049	2.75136	8.70057	433.798	1.96350	4.23024	9.11378	.132100
7.58	57.4564	2.75318	8.70632	435.520	1.96437	4.23210	9.11779	.131926
7.59	57.6081	2.75500	8.71206	437.245	1.96523	4.23396	9.12180	.131752
7.60	57.7600	2.75681	8.71780	438.976	1.96610	4.23582	9.12581	.131579
7.61	57.9121	2.75862	8.72353	440.711	1.96696	4.23768	9.12981	.131406
7.62	58.0644	2.76043	8.72926	442.451	1.96782	4.23954	9.13380	.131234
7.63	58.2169	2.76225	8.73499	444.195	1.96868	4.24139	9.13780	.131062
7.64	58.3696	2.76405	8.74071	445.944	1.96954	4.24324	9.14179	.130890
7.65	58.5225	2.76586	8.74643	447.697	1.97040	4.24509	9.14577	.130719
7.66	58.6756	2.76767	8.75214	449.455	1.97126	4.24694	9.14976	.130548
7.67	58.8289	2.76948	8.75785	451.218	1.97211	4.24879	9.15374	.130378
7.68	58.9824	2.77128	8.76356	452.985	1.97297	4.25063	9.15771	.130208
7.69	59.1361	2.77308	8.76926	454.757	1.97383	4.25248	9.16169	.130039
7.70	59.2900	2.77489	8.77496	456.533	1.97468	4.25432	9.16566	.129870
7.71	59.4441	2.77669	8.78066	458.314	1.97554	4.25616	9.16962	.129702
7.72	59.5984	2.77849	8.78635	460.100	1.97639	4.25800	9.17359	.129534
7.73	59.7529	2.78029	8.79204	461.890	1.97724	4.25984	9.17754	.129366
7.74	59.9076	2.78209	8.79773	463.685	1.97809	4.26167	9.18150	.129199
7.75	60.0625	2.78388	8.80341	465.484	1.97895	4.26351	9.18545	.129032
7.76	60.2176	2.78568	8.80909	467.289	1.97980	4.26534	9.18940	.128866
7.77	60.3729	2.78747	8.81476	469.097	1.98065	4.26717	9.19335	.128700
7.78	60.5284	2.78927	8.82043	470.911	1.98150	4.26900	9.19729	.128535
7.79	60.6841	2.79106	8.82610	472.729	1.98234	4.27083	9.20123	.128370
7.80	60.8400	2.79285	8.83176	474.552	1.98319	4.27266	9.20516	.128205
7.81	60.9961	2.79464	8.83742	476.380	1.98404	4.27448	9.20910	.128041
7.82	61.1524	2.79643	8.84308	478.212	1.98489	4.27631	9.21302	.127877
7.83	61.3089	2.79821	8.84873	480.049	1.98573	4.27813	9.21695	.127714
7.84	61.4656	2.80000	8.85438	481.890	1.98658	4.27995	9.22087	.127551
7.85	61.6225	2.80179	8.86002	483.737	1.98742	4.28177	9.22479	.127389
7.86	61.7796	2.80357	8.86566	485.588	1.98826	4.28359	9.22871	.127226
7.87	61.9369	2.80535	8.87130	487.443	1.98911	4.28540	9.23262	.127065
7.88	62.0944	2.80713	8.87694	489.304	1.98995	4.28722	9.23653	.126904
7.89	62.2521	2.80891	8.88257	491.169	1.99079	4.28903	9.24043	.126743
7.90	62.4100	2.81069	8.88819	493.039	1.99163	4.29084	9.24434	.126582
7.91	62.5681	2.81247	8.89382	494.914	1.99247	4.29265	9.24823	.126422
7.92	62.7264	2.81425	8.89944	496.793	1.99331	4.29446	9.25213	.126263
7.93	62.8849	2.81603	8.90505	498.677	1.99415	4.29627	9.25602	.126103
7.94	63.0436	2.81780	8.91067	500.566	1.99499	4.29807	9.25991	.125945
7.95	63.2025	2.81957	8.91628	502.460	1.99582	4.29987	9.26380	.125786
7.96	63.3616	2.82135	8.92188	504.358	1.99666	4.30168	9.26768	.125628
7.97	63.5209	2.82312	8.92749	506.262	1.99750	4.30348	9.27156	.125471
7.98	63.6804	2.82489	8.93308	508.170	1.99833	4.30528	9.27544	.125313
7.99	63.8401	2.82666	8.93868	510.082	1.99917	4.30707	9.27931	.125156
8.00	64.0000	2.82843	8.94427	512.000	2.00000	4.30887	9.28318	.125000
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
8.00	64.0000	2.82843	8.94427	512.000	2.00000	4.30887	9.28318	.125000
8.01	64.1601	2.83019	8.94986	513.922	2.00083	4.31066	9.28704	.124844
8.02	64.3204	2.83196	8.95545	515.850	2.00167	4.31246	9.29091	.124688
8.03	64.4809	2.83373	8.96103	517.782	2.00250	4.31425	9.29477	.124533
8.04	64.6416	2.83549	8.96660	519.718	2.00333	4.31604	9.29862	.124378
8.05	64.8025	2.83725	8.97218	521.660	2.00416	4.31783	9.30248	.124224
8.06	64.9636	2.83901	8.97775	523.607	2.00499	4.31961	9.30633	.124069
8.07	65.1249	2.84077	8.98332	525.558	2.00582	4.32140	9.31018	.123916
8.08	65.2864	2.84253	8.98888	527.514	2.00664	4.32318	9.31402	.123762
8.09	65.4481	2.84429	8.99444	529.475	2.00747	4.32497	9.31786	.123609
8.10	65.6100	2.84605	9.00000	531.441	2.00830	4.32675	9.32170	.123457
8.11	65.7721	2.84781	9.00555	533.412	2.00912	4.32853	9.32553	.123305
8.12	65.9344	2.84956	9.01110	535.387	2.00995	4.33031	9.32936	.123153
8.13	66.0969	2.85132	9.01665	537.368	2.01078	4.33208	9.33319	.123001
8.14	66.2596	2.85307	9.02219	539.353	2.01160	4.33386	9.33702	.122850
8.15	66.4225	2.85482	9.02774	541.343	2.01242	4.33563	9.34084	.122699
8.16	66.5856	2.85657	9.03327	543.338	2.01325	4.33741	9.34466	.122549
8.17	66.7489	2.85832	9.03881	545.339	2.01407	4.33918	9.34847	.122399
8.18	66.9124	2.86007	9.04434	547.343	2.01489	4.34095	9.35229	.122249
8.19	67.0761	2.86182	9.04986	549.353	2.01571	4.34271	9.35610	.122100
8.20	67.2400	2.86356	9.05539	551.368	2.01653	4.34448	9.35990	.121951
8.21	67.4041	2.86531	9.06091	553.388	2.01735	4.34625	9.36370	.121803
8.22	67.5684	2.86705	9.06642	555.412	2.01817	4.34801	9.36751	.121655
8.23	67.7329	2.86880	9.07193	557.442	2.01899	4.34977	9.37130	.121507
8.24	67.8976	2.87054	9.07744	559.476	2.01980	4.35153	9.37510	.121359
8.25	68.0625	2.87228	9.08295	561.516	2.02062	4.35329	9.37889	.121212
8.26	68.2276	2.87402	9.08845	563.560	2.02144	4.35505	9.38268	.121065
8.27	68.3929	2.87576	9.09395	565.609	2.02225	4.35681	9.38646	.120919
8.28	68.5584	2.87750	9.09945	567.664	2.02307	4.35856	9.39024	.120773
8.29	68.7241	2.87924	9.10494	569.723	2.02388	4.36032	9.39402	.120627
8.30	68.8900	2.88097	9.11043	571.787	2.02469	4.36207	9.39780	.120482
8.31	69.0561	2.88271	9.11592	573.856	2.02551	4.36382	9.40157	.120337
8.32	69.2224	2.88444	9.12140	575.930	2.02632	4.36557	9.40534	.120192
8.33	69.3889	2.88617	9.12688	578.010	2.02713	4.36732	9.40911	.120048
8.34	69.5556	2.88791	9.13236	580.094	2.02794	4.36907	9.41287	.119904
8.35	69.7225	2.88964	9.13783	582.183	2.02875	4.37081	9.41663	.119760
8.36	69.8896	2.89137	9.14330	584.277	2.02956	4.37256	9.42039	.119617
8.37	70.0569	2.89310	9.14877	586.376	2.03037	4.37430	9.42414	.119474
8.38	70.2244	2.89482	9.15423	588.480	2.03118	4.37604	9.42789	.119332
8.39	70.3921	2.89655	9.15969	590.590	2.03199	4.37778	9.43164	.119190
8.40	70.5600	2.89828	9.16515	592.704	2.03279	4.37952	9.43539	.119048
8.41	70.7281	2.90000	9.17061	594.823	2.03360	4.38126	9.43913	.118906
8.42	70.8964	2.90172	9.17606	596.948	2.03440	4.38299	9.44287	.118765
8.43	71.0649	2.90345	9.18150	599.077	2.03521	4.38473	9.44661	.118624
8.44	71.2336	2.90517	9.18695	601.212	2.03601	4.38646	9.45034	.118483
8.45	71.4025	2.90689	9.19239	603.351	2.03682	4.38819	9.45407	.118343
8.46	71.5716	2.90861	9.19783	605.496	2.03762	4.38992	9.45780	.118203
8.47	71.7409	2.91033	9.20326	607.645	2.03842	4.39165	9.46152	.118064
8.48	71.9104	2.91204	9.20869	609.800	2.03923	4.39338	9.46525	.117925
8.49	72.0801	2.91376	9.21412	611.960	2.04003	4.39510	9.46897	.117786
8.50	72.2500	2.91548	9.21954	614.125	2.04083	4.39683	9.47268	.117647
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
8.50	72.2500	2.91548	9.21954	614.125	2.04083	4.39683	9.47268	.117647
8.51	72.4201	2.91719	9.22497	616.295	2.04163	4.39855	9.47640	.117509
8.52	72.5904	2.91890	9.23038	618.470	2.04243	4.40028	9.48011	.117371
8.53	72.7609	2.92062	9.23580	620.650	2.04323	4.40200	9.48381	.117233
8.54	72.9316	2.92233	9.24121	622.836	2.04402	4.40372	9.48752	.117096
8.55	73.1025	2.92404	9.24662	625.026	2.04482	4.40543	9.49122	.116959
8.56	73.2736	2.92575	9.25203	627.222	2.04562	4.40715	9.49492	.116822
8.57	73.4449	2.92746	9.25743	629.423	2.04641	4.40887	9.49861	.116686
8.58	73.6164	2.92916	9.26283	631.629	2.04721	4.41058	9.50231	.116550
8.59	73.7881	2.93087	9.26823	633.840	2.04801	4.41229	9.50600	.116414
8.60	73.9600	2.93258	9.27362	636.056	2.04880	4.41400	9.50969	.116279
8.61	74.1321	2.93428	9.27901	638.277	2.04959	4.41571	9.51337	.116144
8.62	74.3044	2.93598	9.28440	640.504	2.05039	4.41742	9.51705	.116009
8.63	74.4769	2.93769	9.28978	642.736	2.05118	4.41913	9.52073	.115875
8.64	74.6496	2.93939	9.29516	644.973	2.05197	4.42084	9.52441	.115741
8.65	74.8225	2.94109	9.30054	647.215	2.05276	4.42254	9.52808	.115607
8.66	74.9956	2.94279	9.30591	649.462	2.05355	4.42425	9.53175	.115473
8.67	75.1689	2.94449	9.31128	651.714	2.05434	4.42595	9.53542	.115340
8.68	75.3424	2.94618	9.31665	653.972	2.05513	4.42765	9.53908	.115207
8.69	75.5161	2.94788	9.32202	656.235	2.05592	4.42935	9.54274	.115075
8.70	75.6900	2.94958	9.32738	658.503	2.05671	4.43105	9.54640	.114943
8.71	75.8641	2.95127	9.33274	660.776	2.05750	4.43274	9.55006	.114811
8.72	76.0384	2.95296	9.33809	663.055	2.05828	4.43444	9.55371	.114679
8.73	76.2129	2.95466	9.34345	665.339	2.05907	4.43613	9.55736	.114548
8.74	76.3876	2.95635	9.34880	667.628	2.05986	4.43783	9.56101	.114416
8.75	76.5625	2.95804	9.35414	669.922	2.06064	4.43952	9.56466	.114286
8.76	76.7376	2.95973	9.35949	672.221	2.06143	4.44121	9.56830	.114155
8.77	76.9129	2.96142	9.36483	674.526	2.06221	4.44290	9.57194	.114025
8.78	77.0884	2.96311	9.37017	676.836	2.06299	4.44459	9.57557	.113895
8.79	77.2641	2.96479	9.37550	679.151	2.06378	4.44627	9.57921	.113766
8.80	77.4400	2.96648	9.38083	681.472	2.06456	4.44796	9.58284	.113636
8.81	77.6161	2.96816	9.38616	683.798	2.06534	4.44964	9.58647	.113507
8.82	77.7924	2.96985	9.39149	686.129	2.06612	4.45133	9.59009	.113379
8.83	77.9689	2.97153	9.39681	688.465	2.06690	4.45301	9.59372	.113250
8.84	78.1456	2.97321	9.40213	690.807	2.06768	4.45469	9.59734	.113122
8.85	78.3225	2.97489	9.40744	693.154	2.06846	4.45637	9.60095	.112994
8.86	78.4996	2.97658	9.41276	695.506	2.06924	4.45805	9.60457	.112867
8.87	78.6769	2.97825	9.41807	697.864	2.07002	4.45972	9.60818	.112740
8.88	78.8544	2.97993	9.42338	700.227	2.07080	4.46140	9.61179	.112613
8.89	79.0321	2.98161	9.42868	702.595	2.07157	4.46307	9.61540	.112486
8.90	79.2100	2.98329	9.43398	704.969	2.07235	4.46475	9.61900	.112360
8.91	79.3881	2.98496	9.43928	707.348	2.07313	4.46642	9.62260	.112233
8.92	79.5664	2.98664	9.44458	709.732	2.07390	4.46809	9.62620	.112108
8.93	79.7449	2.98831	9.44987	712.122	2.07468	4.46976	9.62980	.111982
8.94	79.9236	2.98998	9.45516	714.517	2.07545	4.47142	9.63339	.111857
8.95	80.1025	2.99166	9.46044	716.917	2.07622	4.47309	9.63698	.111732
8.96	80.2816	2.99333	9.46573	719.323	2.07700	4.47476	9.64057	.111607
8.97	80.4609	2.99500	9.47101	721.734	2.07777	4.47642	9.64415	.111483
8.98	80.6404	2.99666	9.47629	724.151	2.07854	4.47808	9.64774	.111359
8.99	80.8201	2.99833	9.48156	726.573	2.07931	4.47974	9.65132	.111235
9.00	81.0000	3.00000	9.48683	729.000	2.08008	4.48140	9.65489	.111111
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
9.00	81.0000	3.00000	9.48683	729.000	2.08008	4.48140	9.65489	.111111
9.01	81.1801	3.00167	9.49210	731.433	2.08085	4.48306	9.65847	.110988
9.02	81.3604	3.00333	9.49737	733.871	2.08162	4.48472	9.66204	.110865
9.03	81.5409	3.00500	9.50263	736.314	2.08239	4.48638	9.66561	.110742
9.04	81.7216	3.00666	9.50789	738.763	2.08316	4.48803	9.66918	.110619
9.05	81.9025	3.00832	9.51315	741.218	2.08393	4.48969	9.67274	.110497
9.06	82.0836	3.00998	9.51840	743.677	2.08470	4.49134	9.67630	.110375
9.07	82.2649	3.01164	9.52365	746.143	2.08546	4.49299	9.67986	.110254
9.08	82.4464	3.01330	9.52890	748.613	2.08623	4.49464	9.68342	.110132
9.09	82.6281	3.01496	9.53415	751.089	2.08699	4.49629	9.68697	.110011
9.10	82.8100	3.01662	9.53939	753.571	2.08776	4.49794	9.69052	.109890
9.11	82.9921	3.01828	9.54463	756.058	2.08852	4.49959	9.69407	.109769
9.12	83.1744	3.01993	9.54987	758.551	2.08929	4.50123	9.69762	.109649
9.13	83.3569	3.02159	9.55510	761.048	2.09005	4.50288	9.70116	.109529
9.14	83.5396	3.02324	9.56033	763.552	2.09081	4.50452	9.70470	.109409
9.15	83.7225	3.02490	9.56556	766.061	2.09158	4.50616	9.70824	.109290
9.16	83.9056	3.02655	9.57079	768.575	2.09234	4.50781	9.71177	.109170
9.17	84.0889	3.02820	9.57601	771.095	2.09310	4.50945	9.71531	.109051
9.18	84.2724	3.02985	9.58123	773.621	2.09386	4.51108	9.71884	.108932
9.19	84.4561	3.03150	9.58645	776.152	2.09462	4.51272	9.72236	.108814
9.20	84.6400	3.03315	9.59166	778.688	2.09538	4.51436	9.72589	.108696
9.21	84.8241	3.03480	9.59687	781.230	2.09614	4.51599	9.72941	.108578
9.22	85.0084	3.03645	9.60208	783.777	2.09690	4.51763	9.73293	.108460
9.23	85.1929	3.03809	9.60729	786.330	2.09765	4.51926	9.73645	.108342
9.24	85.3776	3.03974	9.61249	788.889	2.09841	4.52089	9.73996	.108225
9.25	85.5625	3.04138	9.61769	791.453	2.09917	4.52252	9.74348	.108108
9.26	85.7476	3.04302	9.62289	794.023	2.09992	4.52415	9.74699	.107991
9.27	85.9329	3.04467	9.62808	796.598	2.10068	4.52578	9.75049	.107875
9.28	86.1184	3.04631	9.63328	799.179	2.10144	4.52740	9.75400	.107759
9.29	86.3041	3.04795	9.63846	801.765	2.10219	4.52903	9.75750	.107643
9.30	86.4900	3.04959	9.64365	804.357	2.10294	4.53065	9.76100	.107527
9.31	86.6761	3.05123	9.64883	806.954	2.10370	4.53228	9.76450	.107411
9.32	86.8624	3.05287	9.65401	809.558	2.10445	4.53390	9.76799	.107296
9.33	87.0489	3.05450	9.65919	812.166	2.10520	4.53552	9.77148	.107181
9.34	87.2356	3.05614	9.66437	814.781	2.10595	4.53714	9.77497	.107066
9.35	87.4225	3.05778	9.66954	817.400	2.10671	4.53876	9.77846	.106952
9.36	87.6096	3.05941	9.67471	820.026	2.10746	4.54038	9.78195	.106838
9.37	87.7969	3.06105	9.67988	822.657	2.10821	4.54199	9.78543	.106724
9.38	87.9844	3.06268	9.68504	825.294	2.10896	4.54361	9.78891	.106610
9.39	88.1721	3.06431	9.69020	827.936	2.10971	4.54522	9.79239	.106496
9.40	88.3600	3.06594	9.69536	830.584	2.11045	4.54684	9.79586	.106383
9.41	88.5481	3.06757	9.70052	833.238	2.11120	4.54845	9.79933	.106270
9.42	88.7364	3.06920	9.70567	835.897	2.11195	4.55006	9.80280	.106157
9.43	88.9249	3.07083	9.71082	838.562	2.11270	4.55167	9.80627	.106045
9.44	89.1136	3.07246	9.71597	841.232	2.11344	4.55328	9.80974	.105932
9.45	89.3025	3.07409	9.72111	843.909	2.11419	4.55488	9.81320	.105820
9.46	89.4916	3.07571	9.72625	846.591	2.11494	4.55649	9.81666	.105708
9.47	89.6809	3.07734	9.73139	849.278	2.11568	4.55809	9.82012	.105597
9.48	89.8704	3.07896	9.73653	851.971	2.11642	4.55970	9.82357	.105485
9.49	90.0601	3.08058	9.74166	854.670	2.11717	4.56130	9.82703	.105374
9.50	90.2500	3.08221	9.74679	857.375	2.11791	4.56290	9.83048	.105263
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
9.50	90.2500	3.08221	9.74679	857.375	2.11791	4.56290	9.83048	.105263
9.51	90.4401	3.08383	9.75192	860.085	2.11865	4.56450	9.83392	.105152
9.52	90.6304	3.08545	9.75705	862.801	2.11940	4.56610	9.83737	.105042
9.53	90.8209	3.08707	9.76217	865.523	2.12014	4.56770	9.84081	.104932
9.54	91.0116	3.08869	9.76729	868.251	2.12088	4.56930	9.84425	.104822
9.55	91.2025	3.09031	9.77241	870.984	2.12162	4.57089	9.84769	.104712
9.56	91.3936	3.09192	9.77753	873.723	2.12236	4.57249	9.85113	.104603
9.57	91.5849	3.09354	9.78264	876.467	2.12310	4.57408	9.85456	.104493
9.58	91.7764	3.09516	9.78775	879.218	2.12384	4.57567	9.85799	.104384
9.59	91.9681	3.09677	9.79285	881.974	2.12458	4.57727	9.86142	.104275
9.60	92.1600	3.09839	9.79796	884.736	2.12532	4.57886	9.86485	.104167
9.61	92.3521	3.10000	9.80306	887.504	2.12605	4.58045	9.86827	.104058
9.62	92.5444	3.10161	9.80816	890.277	2.12679	4.58204	9.87169	.103950
9.63	92.7369	3.10322	9.81326	893.056	2.12753	4.58362	9.87511	.103842
9.64	92.9296	3.10483	9.81835	895.841	2.12826	4.58521	9.87853	.103734
9.65	93.1225	3.10644	9.82344	898.632	2.12900	4.58679	9.88195	.103627
9.66	93.3156	3.10805	9.82853	901.429	2.12974	4.58838	9.88536	.103520
9.67	93.5089	3.10966	9.83362	904.231	2.13047	4.58996	9.88877	.103413
9.68	93.7024	3.11127	9.83870	907.039	2.13120	4.59154	9.89217	.103306
9.69	93.8961	3.11288	9.84378	909.853	2.13194	4.59312	9.89558	.103199
9.70	94.0900	3.11448	9.84886	912.673	2.13267	4.59470	9.89898	.103093
9.71	94.2841	3.11609	9.85393	915.499	2.13340	4.59628	9.90238	.102987
9.72	94.4784	3.11769	9.85901	918.330	2.13414	4.59786	9.90578	.102881
9.73	94.6729	3.11929	9.86408	921.167	2.13487	4.59943	9.90918	.102775
9.74	94.8676	3.12090	9.86914	924.010	2.13560	4.60101	9.91257	.102669
9.75	95.0625	3.12250	9.87421	926.859	2.13633	4.60258	9.91596	.102564
9.76	95.2576	3.12410	9.87927	929.714	2.13706	4.60416	9.91935	.102459
9.77	95.4529	3.12570	9.88433	932.575	2.13779	4.60573	9.92274	.102354
9.78	95.6484	3.12730	9.88939	935.441	2.13852	4.60730	9.92612	.102249
9.79	95.8441	3.12890	9.89444	938.314	2.13925	4.60887	9.92950	.102145
9.80	96.0400	3.13050	9.89949	941.192	2.13997	4.61044	9.93288	.102041
9.81	96.2361	3.13209	9.90454	944.076	2.14070	4.61200	9.93626	.101937
9.82	96.4324	3.13369	9.90959	946.966	2.14143	4.61357	9.93964	.101833
9.83	96.6289	3.13528	9.91464	949.862	2.14216	4.61514	9.94301	.101729
9.84	96.8256	3.13688	9.91968	952.764	2.14288	4.61670	9.94638	.101626
9.85	97.0225	3.13847	9.92472	955.672	2.14361	4.61826	9.94975	.101523
9.86	97.2196	3.14006	9.92975	958.585	2.14433	4.61983	9.95311	.101420
9.87	97.4169	3.14166	9.93479	961.505	2.14506	4.62139	9.95648	.101317
9.88	97.6144	3.14325	9.93982	964.430	2.14578	4.62295	9.95984	.101215
9.89	97.8121	3.14484	9.94485	967.362	2.14651	4.62451	9.96320	.101112
9.90	98.0100	3.14643	9.94987	970.299	2.14723	4.62607	9.96655	.101010
9.91	98.2081	3.14802	9.95490	973.242	2.14795	4.62762	9.96991	.100908
9.92	98.4064	3.14960	9.95992	976.191	2.14867	4.62918	9.97326	.100806
9.93	98.6049	3.15119	9.96494	979.147	2.14940	4.63073	9.97661	.100705
9.94	98.8036	3.15278	9.96995	982.108	2.15012	4.63229	9.97996	.100604
9.95	99.0025	3.15436	9.97497	985.075	2.15084	4.63384	9.98331	.100503
9.96	99.2016	3.15595	9.97998	988.048	2.15156	4.63539	9.98665	.100402
9.97	99.4009	3.15753	9.98499	991.027	2.15228	4.63694	9.98999	.100301
9.98	99.6004	3.15911	9.98999	994.012	2.15300	4.63849	9.99333	.100200
9.99	99.8001	3.16070	9.99500	997.003	2.15372	4.64004	9.99667	.100100
10.00	100.000	3.16228	10.0000	1000.00	2.15443	4.64159	10.0000	.100000
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

Table VII — Napierian or Natural Logarithms

Take tabular value − 10

N	0	1	2	3	4	5	6	7	8	9
0.0		5.395	6 088	6.493	6.781	7.004	7.187	7.341	7.474	7.592
0.1	7.697	7.793	7.880	7.960	8.034	8.103	8.167	8.228	8.285	8.339
0.2	8.391	8.439	8.486	8.530	8.573	8 614	8.653	8.691	8.727	8.762
0.3	8.796	8.829	8.861	8.891	8.921	8.950	8.978	9.006	9.032	9.058
0.4	9.084	9.108	9.132	9.156	9.179	9.201	9.223	9.245	9.266	9.287
0.5	9.307	9.327	9.346	9.365	9.384	9.402	9.420	9.438	9.455	9.472
0.6	9.489	9.506	9.522	9.538	9.554	9.569	9.584	9.600	9.614	9.629
0.7	9.643	9.658	9.671	9 685	9.699	9.712	9.726	9.739	9.752	9.764
0.8	9.777	9.789	9.802	9.814	9.826	9.837	9.849	9.861	9.872	9.883
0.9	9.895	9.906	9.917	9.927	9.938	9.949	9.959	9.970	9.980	9.990
1.0	0.00000	0995	1980	2956	3922	4879	5827	6766	7696	8618
1.1	9531	*0436	*1333	*2222	*3103	*3976	*4842	*5700	*6551	*7395
1.2	0.1 8232	9062	9885	*0701	*1511	*2314	*3111	*3902	*4686	*5464
1.3	0.2 6236	7003	7763	8518	9267	*0010	*0748	*1481	*2208	*2930
1.4	0.3 3647	4359	5066	5767	6464	7156	7844	8526	9204	9878
1.5	0.4 0547	1211	1871	2527	3178	3825	4469	5108	5742	6373
1.6	7000	7623	8243	8858	9470	*0078	*0682	*1282	*1879	*2473
1.7	0.5 3063	3649	4232	4812	5389	5962	6531	7098	7661	8222
1.8	8779	9333	9884	*0432	*0977	*1519	*2058	*2594	*3127	*3658
1.9	0.6 4185	4710	5233	5752	6269	6783	7294	7803	8310	8813
2.0	9315	9813	*0310	*0804	*1295	*1784	*2271	*2755	*3237	*3716
2.1	0.7 4194	4669	5142	5612	6081	6547	7011	7473	7932	8390
2.2	8846	9299	9751	*0200	*0648	*1093	*1536	*1978	*2418	*2855
2.3	0.8 3291	3725	4157	4587	5015	5442	5866	6289	6710	7129
2.4	7547	7963	8377	8789	9200	9609	*0016	*0422	*0826	*1228
2.5	0.9 1629	2028	2426	2822	3216	3609	4001	4391	4779	5166
2.6	5551	5935	6317	6698	7078	7456	7833	8208	8582	8954
2.7	9325	9695	*0063	*0430	*0796	*1160	*1523	*1885	*2245	*2604
2 8	1.0 2962	3318	3674	4028	4380	4732	5082	5431	5779	6126
2.9	6471	6815	7158	7500	7841	8181	8519	8856	9192	9527
3.0	9861	*0194	*0526	*0856	*1186	*1514	*1841	*2168	*2493	*2817
3.1	1.1 3140	3462	3783	4103	4422	4740	5057	5373	5688	6002
3.2	6315	6627	6938	7248	7557	7865	8173	8479	8784	9089
3.3	9392	9695	9996	*0297	*0597	*0896	*1194	*1491	*1788	*2083
3.4	1.2 2378	2671	2964	3256	3547	3837	4127	4415	4703	4990
3.5	5276	5562	5846	6130	6413	6695	6976	7257	7536	7815
3.6	8093	8371	8647	8923	9198	9473	9746	*0019	*0291	*0563
3.7	1.3 0833	1103	1372	1641	1909	2176	2442	2708	2972	3237
3.8	3500	3763	4025	4286	4547	4807	5067	5325	5584	5841
3.9	6098	6354	6609	6864	7118	7372	7624	7877	8128	8379
4.0	8629	8879	9128	9377	9624	9872	*0118	*0364	*0610	*0854
4.1	1.4 1099	1342	1585	1828	2070	2311	2552	2792	3031	3270
4.2	3508	3746	3984	4220	4456	4692	4927	5161	5395	5629
4.3	5862	6094	6326	6557	6787	7018	7247	7476	7705	7933
4.4	8160	8387	8614	8840	9065	9290	9515	9739	9962	*0185
4.5	1.5 0408	0630	0851	1072	1293	1513	1732	1951	2170	2388
4.6	2606	2823	3039	3256	3471	3687	3902	4116	4330	4543
4.7	4756	4969	5181	5393	5604	5814	6025	6235	6444	6653
4.8	6862	7070	7277	7485	7691	7898	8104	8309	8515	8719
4.9	8924	9127	9331	9534	9737	9939	*0141	*0342	*0543	*0744
5.0	1.6 0944	1144	1343	1542	1741	1939	2137	2334	2531	2728
N	0	1	2	3	4	5	6	7	8	9

N	0	1	2	3	4	5	6	7	8	9
5.0	1.6 0944	1144	1343	1542	1741	1939	2137	2334	2531	2728
5.1	2924	3120	3315	3511	3705	3900	4094	4287	4481	4673
5.2	4866	5058	5250	5441	5632	5823	6013	6203	6393	6582
5.3	6771	6959	7147	7335	7523	7710	7896	8083	8269	8455
5.4	8640	8825	9010	9194	9378	9562	9745	9928	*0111	*0293
5.5	1.7 0475	0656	0838	1019	1199	1380	1560	1740	1919	2098
5.6	2277	2455	2633	2811	2988	3166	3342	3519	3695	3871
5.7	4047	4222	4397	4572	4746	4920	5094	5267	5440	5613
5.8	5786	5958	6130	6302	6473	6644	6815	6985	7156	7326
5.9	7495	7665	7834	8002	8171	8339	8507	8675	8842	9009
6.0	9176	9342	9509	9675	9840	*0006	*0171	*0336	*0500	*0665
6.1	1.8 0829	0993	1156	1319	1482	1645	1808	1970	2132	2294
6.2	2455	2616	2777	2938	3098	3258	3418	3578	3737	3896
6.3	4055	4214	4372	4530	4688	4845	5003	5160	5317	5473
6.4	5630	5786	5942	6097	6253	6408	6563	6718	6872	7026
6.5	7180	7334	7487	7641	7794	7947	8099	8251	8403	8555
6.6	8707	8858	9010	9160	9311	9462	9612	9762	9912	*0061
6.7	1.9 0211	0360	0509	0658	0806	0954	1102	1250	1398	1545
6.8	1692	1839	1986	2132	2279	2425	2571	2716	2862	3007
6.9	3152	3297	3442	3586	3730	3874	4018	4162	4305	4448
7.0	4591	4734	4876	5019	5161	5303	5445	5586	5727	5869
7.1	6009	6150	6291	6431	6571	6711	6851	6991	7130	7269
7.2	7408	7547	7685	7824	7962	8100	8238	8376	8513	8650
7.3	8787	8924	9061	9198	9334	9470	9606	9742	9877	*0013
7.4	2.0 0148	0283	0418	0553	0687	0821	0956	1089	1223	1357
7.5	1490	1624	1757	1890	2022	2155	2287	2419	2551	2683
7.6	2815	2946	3078	3209	3340	3471	3601	3732	3862	3992
7.7	4122	4252	4381	4511	4640	4769	4898	5027	5156	5284
7.8	5412	5540	5668	5796	5924	6051	6179	6306	6433	6560
7.9	6686	6813	6939	7065	7191	7317	7443	7568	7694	7819
8.0	7944	8069	8194	8318	8443	8567	8691	8815	8939	9063
8.1	9186	9310	9433	9556	9679	9802	9924	*0047	*0169	*0291
8.2	2.1 0413	0535	0657	0779	0900	1021	1142	1263	1384	1505
8.3	1626	1746	1866	1986	2106	2226	2346	2465	2585	2704
8.4	2823	2942	3061	3180	3298	3417	3535	3653	3771	3889
8.5	4007	4124	4242	4359	4476	4593	4710	4827	4943	5060
8.6	5176	5292	5409	5524	5640	5756	5871	5987	6102	6217
8.7	6332	6447	6562	6677	6791	6905	7020	7134	7248	7361
8.8	7475	7589	7702	7816	7929	8042	8155	8267	8380	8493
8.9	8605	8717	8830	8942	9054	9165	9277	9389	9500	9611
9.0	9722	9834	9944	*0055	*0166	*0276	*0387	*0497	*0607	*0717
9.1	2.2 0827	0937	1047	1157	1266	1375	1485	1594	1703	1812
9.2	1920	2029	2138	2246	2354	2462	2570	2678	2786	2894
9.3	3001	3109	3216	3324	3431	3538	3645	3751	3858	3965
9.4	4071	4177	4284	4390	4496	4601	4707	4813	4918	5024
9.5	5129	5234	5339	5444	5549	5654	5759	5863	5968	6072
9.6	6176	6280	6384	6488	6592	6696	6799	6903	7006	7109
9.7	7213	7316	7419	7521	7624	7727	7829	7932	8034	8136
9.8	8238	8340	8442	8544	8646	8747	8849	8950	9051	9152
9.9	9253	9354	9455	9556	9657	9757	9858	9958	*0058	*0158
10.0	2.3 0259	0358	0458	0558	0658	0757	0857	0956	1055	1154
N	0	1	2	3	4	5	6	7	8	9

10	2.30259	**25**	3.21888	**40**	3.68888	**55**	4.00733	**70**	4.24850	**85**	4.44265
11	2.39790	26	3.25810	41	3.71357	56	4.02535	71	4.26268	86	4.45435
12	2.48491	27	3.29584	42	3.73767	57	4.04305	72	4.27667	87	4.46591
13	2.56495	28	3.33220	43	3.76120	58	4.06044	73	4.29046	88	4.47734
14	2.63906	29	3.36730	44	3.78419	59	4.07754	74	4.30107	89	4.48864
15	2.70805	**30**	3.40120	**45**	3.80666	**60**	4.09434	**75**	4.31749	**90**	4.49981
16	2.77259	31	3.43399	46	3.82864	61	4.11087	76	4.33073	91	4.51086
17	2.83321	32	3.46574	47	3.85015	62	4.12713	77	4.34381	92	4.52179
18	2.89037	33	3.49651	48	3.87120	63	4.14313	78	4.35671	93	4.53260
19	2.94444	34	3.52636	49	3.89182	64	4.15888	79	4.36945	94	4.54329
20	2.99573	**35**	3.55535	**50**	3.91202	**65**	4.17439	**80**	4.38203	**95**	4.55388
21	3.04452	36	3.58352	51	3.93183	66	4.18965	81	4.39445	96	4.56435
22	3.09104	37	3.61092	52	3.95124	67	4.20469	82	4.40672	97	4.57471
23	3.13549	38	3.63759	53	3.97029	68	4.21951	83	4.41884	98	4.58497
24	3.17805	39	3.66356	54	3.98898	69	4.23411	84	4.43082	99	4.59512

NAPIERIAN OR NATURAL LOGARITHMS — 100 TO 409

N	0	1	2	3	4	5	6	7	8	9
10	4.6 0517	1512	2497	3473	4439	5396	6344	7283	8213	9135
11	4.7 0048	0953	1850	2739	3620	4493	5359	6217	7068	7912
12	8749	9579	*0402	*1218	*2028	*2831	*3628	*4419	*5203	*5981
13	4.8 6753	7520	8280	9035	9784	*0527	*1265	*1998	*2725	*3447
14	4.9 4164	4876	5583	6284	6981	7673	8361	9043	9721	*0395
15	5.0 1064	1728	2388	3044	3695	4343	4986	5625	6260	6890
16	7517	8140	8760	9375	9987	*0595	*1199	*1799	*2396	*2990
17	5.1 3580	4166	4749	5329	5906	6479	7048	7615	8178	8739
18	9296	9850	*0401	*0949	*1494	*2036	*2575	*3111	*3644	*4175
19	5.2 4702	5227	5750	6269	6786	7300	7811	8320	8827	9330
20	9832	*0330	*0827	*1321	*1812	*2301	*2788	*3272	*3754	*4233
21	5.3 4711	5186	5659	6129	6598	7064	7528	7990	8450	8907
22	9363	9816	*0268	*0717	*1165	*1610	*2053	*2495	*2935	*3372
23	5.4 3808	4242	4674	5104	5532	5959	6383	6806	7227	7646
24	8064	8480	8894	9306	9717	*0126	*0533	*0939	*1343	*1745
25	5.5 2146	2545	2943	3339	3733	4126	4518	4908	5296	5683
26	6068	6452	6834	7215	7595	7973	8350	8725	9099	9471
27	9842	*0212	*0580	*0947	*1313	*1677	*2040	*2402	*2762	*3121
28	5.6 3479	3835	4191	4545	4897	5249	5599	5948	6296	6643
29	6988	7332	7675	8017	8358	8698	9036	9373	9709	*0044
30	5.7 0378	0711	1043	1373	1703	2031	2359	2685	3010	3334
31	3657	3979	4300	4620	4939	5257	5574	5890	6205	6519
32	6832	7144	7455	7765	8074	8383	8690	8996	9301	9606
33	9909	*0212	*0513	*0814	*1114	*1413	*1711	*2008	*2305	*2600
34	5.8 2895	3188	3481	3773	4064	4354	4644	4932	5220	5507
35	5793	6079	6363	6647	6930	7212	7493	7774	8053	8332
36	8610	8888	9164	9440	9715	9990	*0263	*0536	*0808	*1080
37	5.9 1350	1620	1889	2158	2426	2693	2959	3225	3489	3754
38	4017	4280	4542	4803	5064	5324	5584	5842	6101	6358
39	6615	6871	7126	7381	7635	7889	8141	8394	8645	8896
40	9146	9396	9645	9894	*0141	*0389	*0635	*0881	*1127	*1372
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Above 409, use the formula $\log_e 10n = \log_e n + \log_e 10 = \log_e n + 2.30258509$.

[Characteristics of Logarithms omitted — determine by the usual rule from the value]

Radians	Degrees	Sine Value	Sine Log_{10}	Tangent Value	Tangent Log_{10}	Cotangent Value	Cotangent Log_{10}	Cosine Value	Cosine Log_{10}		
.1571	**9° 00′**	.1564	.1943	.1584	.1997	6.3138	.8003	.9877	.9946	**81° 00′**	1.4137
.1600	10	.1593	.2022	.1614	.2078	6.1970	.7922	.9872	.9944	50	1.4108
.1629	20	.1622	.2100	.1644	.2158	6.0844	.7842	.9868	.9942	40	1.4079
.1658	30	.1650	.2176	.1673	.2236	5.9758	.7764	.9863	.9940	30	1.4050
.1687	40	.1679	.2251	.1703	.2313	5.8708	.7687	.9858	.9938	20	1.4021
.1716	50	.1708	.2324	.1733	.2389	5.7694	.7611	.9853	.9936	10	1.3992
.1745	**10° 00′**	.1736	.2397	.1763	.2463	5.6713	.7537	.9848	.9934	**80° 00′**	1.3963
.1774	10	.1765	.2468	.1793	.2536	5.5764	.7464	.9843	.9931	50	1.3934
.1804	20	.1794	.2538	.1823	.2609	5.4845	.7391	.9838	.9929	40	1.3904
.1833	30	.1822	.2606	.1853	.2680	5.3955	.7320	.9833	.9927	30	1.3875
.1862	40	.1851	.2674	.1883	.2750	5.3093	.7250	.9827	.9924	20	1.3846
.1891	50	.1880	.2740	.1914	.2819	5.2257	.7181	.9822	.9922	10	1.3817
.1920	**11° 00′**	.1908	.2806	.1944	.2887	5.1446	.7113	.9816	.9919	**79° 00′**	1.3788
.1949	10	.1937	.2870	.1974	.2953	5.0658	.7047	.9811	.9917	50	1.3759
.1978	20	.1965	.2934	.2004	.3020	4.9894	.6980	.9805	.9914	40	1.3730
.2007	30	.1994	.2997	.2035	.3085	4.9152	.6915	.9799	.9912	30	1.3701
.2036	40	.2022	.3058	.2065	.3149	4.8430	.6851	.9793	.9909	20	1.3672
.2065	50	.2051	.3119	.2095	.3212	4.7729	.6788	.9787	.9907	10	1.3643
.2094	**12° 00′**	.2079	.3179	.2126	.3275	4.7046	.6725	.9781	.9904	**78° 00′**	1.3614
.2123	10	.2108	.3238	.2156	.3336	4.6382	.6664	.9775	.9901	50	1.3584
.2153	20	.2136	.3296	.2186	.3397	4.5736	.6603	.9769	.9899	40	1.3555
.2182	30	.2164	.3353	.2217	.3458	4.5107	.6542	.9763	.9896	30	1.3526
.2211	40	.2193	.3410	.2247	.3517	4.4494	.6483	.9757	.9893	20	1.3497
.2240	50	.2221	.3466	.2278	.3576	4.3897	.6424	.9750	.9890	10	1.3468
.2269	**13° 00′**	.2250	.3521	.2309	.3634	4.3315	.6366	.9744	.9887	**77° 00′**	1.3439
.2298	10	.2278	.3575	.2339	.3691	4.2747	.6309	.9737	.9884	50	1.3410
.2327	20	.2306	.3629	.2370	.3748	4.2193	.6252	.9730	.9881	40	1.3381
.2356	30	.2334	.3682	.2401	.3804	4.1653	.6196	.9724	.9878	30	1.3352
.2385	40	.2363	.3734	.2432	.3859	4.1126	.6141	.9717	.9875	20	1.3323
.2414	50	.2391	.3786	.2462	.3914	4.0611	.6086	.9710	.9872	10	1.3294
.2443	**14° 00′**	.2419	.3837	.2493	.3968	4.0108	.6032	.9703	.9869	**76° 00′**	1.3265
.2473	10	.2447	.3887	.2524	.4021	3.9617	.5979	.9696	.9866	50	1.3235
.2502	20	.2476	.3937	.2555	.4074	3.9136	.5926	.9689	.9863	40	1.3206
.2531	30	.2504	.3986	.2586	.4127	3.8667	.5873	.9681	.9859	30	1.3177
.2560	40	.2532	.4035	.2617	.4178	3.8208	.5822	.9674	.9856	20	1.3148
.2589	50	.2560	.4083	.2648	.4230	3.7760	.5770	.9667	.9853	10	1.3119
.2618	**15° 00′**	.2588	.4130	.2679	.4281	3.7321	.5719	.9659	.9849	**75° 00′**	1.3090
.2647	10	.2616	.4177	.2711	.4331	3.6891	.5669	.9652	.9846	50	1.3061
.2676	20	.2644	.4223	.2742	.4381	3.6470	.5619	.9644	.9843	40	1.3032
.2705	30	.2672	.4269	.2773	.4430	3.6059	.5570	.9636	.9839	30	1.3003
.2734	40	.2700	.4314	.2805	.4479	3.5656	.5521	.9628	.9836	20	1.2974
.2763	50	.2728	.4359	.2836	.4527	3.5261	.5473	.9621	.9832	10	1.2945
.2793	**16° 00′**	.2756	.4403	.2867	.4575	3.4874	.5425	.9613	.9828	**74° 00′**	1.2915
.2822	10	.2784	.4447	.2899	.4622	3.4495	.5378	.9605	.9825	50	1.2886
.2851	20	.2812	.4491	.2931	.4669	3.4124	.5331	.9596	.9821	40	1.2857
.2880	30	.2840	.4533	.2962	.4716	3.3759	.5284	.9588	.9817	30	1.2828
.2909	40	.2868	.4576	.2994	.4762	3.3402	.5238	.9580	.9814	20	1.2799
.2938	50	.2896	.4618	.3026	.4808	3.3052	.5192	.9572	.9810	10	1.2770
.2967	**17° 00′**	.2924	.4659	.3057	.4853	3.2709	.5147	.9563	.9806	**73° 00′**	1.2741
.2996	10	.2952	.4700	.3089	.4898	3.2371	.5102	.9555	.9802	50	1.2712
.3025	20	.2979	.4741	.3121	.4943	3.2041	.5057	.9546	.9798	40	1.2683
.3054	30	.3007	.4781	.3153	.4987	3.1716	.5013	.9537	.9794	30	1.2654
.3083	40	.3035	.4821	.3185	.5031	3.1397	.4969	.9528	.9790	20	1.2625
.3113	50	.3062	.4861	.3217	.5075	3.1084	.4925	.9520	.9786	10	1.2595
.3142	**18° 00′**	.3090	.4900	.3249	.5118	3.0777	.4882	.9511	.9782	**72° 00′**	1.2566
		Value	Log_{10}	Value	Log_{10}	Value	Log_{10}	Value	Log_{10}	Degrees	Radians
		Cosine		Cotangent		Tangent		Sine			

[Characteristics of Logarithms omitted — determine by the usual rule from the value]

Radians	Degrees	Sine Value	Sine Log_{10}	Tangent Value	Tangent Log_{10}	Cotangent Value	Cotangent Log_{10}	Cosine Value	Cosine Log_{10}		
.3142	**18° 00′**	.3090	.4900	.3249	.5118	3.0777	.4882	.9511	.9782	**72° 00′**	1.2566
.3171	10	.3118	.4939	.3281	.5161	3.0475	.4839	.9502	.9778	50	1.2537
.3200	20	.3145	.4977	.3314	.5203	3.0178	.4797	.9492	.9774	40	1.2508
.3229	30	.3173	.5015	.3346	.5245	2.9887	.4755	.9483	.9770	30	1.2479
.3258	40	.3201	.5052	.3378	.5287	2.9600	.4713	.9474	.9765	20	1.2450
.3287	50	.3228	.5090	.3411	.5329	2.9319	.4671	.9465	.9761	10	1.2421
.3316	**19° 00′**	.3256	.5126	.3443	.5370	2.9042	.4630	.9455	.9757	**71° 00′**	1.2392
.3345	10	.3283	.5163	.3476	.5411	2.8770	.4589	.9446	.9752	50	1.2363
.3374	20	.3311	.5199	.3508	.5451	2.8502	.4549	.9436	.9748	40	1.2334
.3403	30	.3338	.5235	.3541	.5491	2.8239	.4509	.9426	.9743	30	1.2305
.3432	40	.3365	.5270	.3574	.5531	2.7980	.4469	.9417	.9739	20	1.2275
.3462	50	.3393	.5306	.3607	.5571	2.7725	.4429	.9407	.9734	10	1.2246
.3491	**20° 00′**	.3420	.5341	.3640	.5611	2.7475	.4389	.9397	.9730	**70° 00′**	1.2217
.3520	10	.3448	.5375	.3673	.5650	2.7228	.4350	.9387	.9725	50	1.2188
.3549	20	.3475	.5409	.3706	.5689	2.6985	.4311	.9377	.9721	40	1.2159
.3578	30	.3502	.5443	.3739	.5727	2.6746	.4273	.9367	.9716	30	1.2130
.3607	40	.3529	.5477	.3772	.5766	2.6511	.4234	.9356	.9711	20	1.2101
.3636	50	.3557	.5510	.3805	.5804	2.6279	.4196	.9346	.9706	10	1.2072
.3665	**21° 00′**	.3584	.5543	.3839	.5842	2.6051	.4158	.9336	.9702	**69° 00′**	1.2043
.3694	10	.3611	.5576	.3872	.5879	2.5826	.4121	.9325	.9697	50	1.2014
.3723	20	.3638	.5609	.3906	.5917	2.5605	.4083	.9315	.9692	40	1.1985
.3752	30	.3665	.5641	.3939	.5954	2.5386	.4046	.9304	.9687	30	1.1956
.3782	40	.3692	.5673	.3973	.5991	2.5172	.4009	.9293	.9682	20	1.1926
.3811	50	.3719	.5704	.4006	.6028	2.4960	.3972	.9283	.9677	10	1.1897
.3840	**22° 00′**	.3746	.5736	.4040	.6064	2.4751	.3936	.9272	.9672	**68° 00′**	1.1868
.3869	10	.3773	.5767	.4074	.6100	2.4545	.3900	.9261	.9667	50	1.1839
.3898	20	.3800	.5798	.4108	.6136	2.4342	.3864	.9250	.9661	40	1.1810
.3927	30	.3827	.5828	.4142	.6172	2.4142	.3828	.9239	.9656	30	1.1781
.3956	40	.3854	.5859	.4176	.6208	2.3945	.3792	.9228	.9651	20	1.1752
.3985	50	.3881	.5889	.4210	.6243	2.3750	.3757	.9216	.9646	10	1.1723
.4014	**23° 00′**	.3907	.5919	.4245	.6279	2.3559	.3721	.9205	.9640	**67° 00′**	1.1694
.4043	10	.3934	.5948	.4279	.6314	2.3369	.3686	.9194	.9635	50	1.1665
.4072	20	.3961	.5978	.4314	.6348	2.3183	.3652	.9182	.9629	40	1.1636
.4102	30	.3987	.6007	.4348	.6383	2.2998	.3617	.9171	.9624	30	1.1606
.4131	40	.4014	.6036	.4383	.6417	2.2817	.3583	.9159	.9618	20	1.1577
.4160	50	.4041	.6065	.4417	.6452	2.2637	.3548	.9147	.9613	10	1.1548
.4189	**24° 00′**	.4067	.6093	.4452	.6486	2.2460	.3514	.9135	.9607	**66° 00′**	1.1519
.4218	10	.4094	.6121	.4487	.6520	2.2286	.3480	.9124	.9602	50	1.1490
.4247	20	.4120	.6149	.4522	.6553	2.2113	.3447	.9112	.9596	40	1.1461
.4276	30	.4147	.6177	.4557	.6587	2.1943	.3413	.9100	.9590	30	1.1432
.4305	40	.4173	.6205	.4592	.6620	2.1775	.3380	.9088	.9584	20	1.1403
.4334	50	.4200	.6232	.4628	.6654	2.1609	.3346	.9075	.9579	10	1.1374
.4363	**25° 00′**	.4226	.6259	.4663	.6687	2.1445	.3313	.9063	.9573	**65° 00′**	1.1345
.4392	10	.4253	.6286	.4699	.6720	2.1283	.3280	.9051	.9567	50	1.1316
.4422	20	.4279	.6313	.4734	.6752	2.1123	.3248	.9038	.9561	40	1.1286
.4451	30	.4305	.6340	.4770	.6785	2.0965	.3215	.9026	.9555	30	1.1257
.4480	40	.4331	.6366	.4806	.6817	2.0809	.3183	.9013	.9549	20	1.1228
.4509	50	.4358	.6392	.4841	.6850	2.0655	.3150	.9001	.9543	10	1.1199
.4538	**26° 00′**	.4384	.6418	.4877	.6882	2.0503	.3118	.8988	.9537	**64° 00′**	1.1170
.4567	10	.4410	.6444	.4913	.6914	2.0353	.3086	.8975	.9530	50	1.1141
.4596	20	.4436	.6470	.4950	.6946	2.0204	.3054	.8962	.9524	40	1.1112
.4625	30	.4462	.6495	.4986	.6977	2.0057	.3023	.8949	.9518	30	1.1083
.4654	40	.4488	.6521	.5022	.7009	1.9912	.2991	.8936	.9512	20	1.1054
.4683	50	.4514	.6546	.5059	.7040	1.9768	.2960	.8923	.9505	10	1.1025
.4712	**27° 00′**	.4540	.6570	.5095	.7072	1.9626	.2928	.8910	.9499	**63° 00′**	1.0996
		Value Cosine	Log_{10}	Value Cotangent	Log_{10}	Value Tangent	Log_{10}	Value Sine	Log_{10}	Degrees	Radians

[Characteristics of Logarithms omitted — determine by the usual rule from the value]

Radians	Degrees	Sine Value	Sine Log₁₀	Tangent Value	Tangent Log₁₀	Cotangent Value	Cotangent Log₁₀	Cosine Value	Cosine Log₁₀		
.4712	**27° 00′**	.4540	.6570	.5095	.7072	1.9626	.2928	.8910	.9499	**63° 00′**	1.0996
.4741	10	.4566	.6595	.5132	.7103	1.9486	.2897	.8897	.9492	50	1.0966
.4771	20	.4592	.6620	.5169	.7134	1.9347	.2866	.8884	.9486	40	1.0937
.4800	30	.4617	.6644	.5206	.7165	1.9210	.2835	.8870	.9479	30	1.0908
.4829	40	.4643	.6668	.5243	.7196	1.9074	.2804	.8857	.9473	20	1.0879
.4858	50	.4669	.6692	.5280	.7226	1.8940	.2774	.8843	.9466	10	1.0850
.4887	**28° 00′**	.4695	.6716	.5317	.7257	1.8807	.2743	.8829	.9459	**62° 00′**	1.0821
.4916	10	.4720	.6740	.5354	.7287	1.8676	.2713	.8816	.9453	50	1.0792
.4945	20	.4746	.6763	.5392	.7317	1.8546	.2683	.8802	.9446	40	1.0763
.4974	30	.4772	.6787	.5430	.7348	1.8418	.2652	.8788	.9439	30	1.0734
.5003	40	.4797	.6810	.5467	.7378	1.8291	.2622	.8774	.9432	20	1.0705
.5032	50	.4823	.6833	.5505	.7408	1.8165	.2592	.8760	.9425	10	1.0676
.5061	**29° 00′**	.4848	.6856	.5543	.7438	1.8040	.2562	.8746	.9418	**61° 00′**	1.0647
.5091	10	.4874	.6878	.5581	.7467	1.7917	.2533	.8732	.9411	50	1.0617
.5120	20	.4899	.6901	.5619	.7497	1.7796	.2503	.8718	.9404	40	1.0588
.5149	30	.4924	.6923	.5658	.7526	1.7675	.2474	.8704	.9397	30	1.0559
.5178	40	.4950	.6946	.5696	.7556	1.7556	.2444	.8689	.9390	20	1.0530
.5207	50	.4975	.6968	.5735	.7585	1.7437	.2415	.8675	.9383	10	1.0501
.5236	**30° 00′**	.5000	.6990	.5774	.7614	1.7321	.2386	.8660	.9375	**60° 00′**	1.0472
.5265	10	.5025	.7012	.5812	.7644	1.7205	.2356	.8646	.9368	50	1.0443
.5294	20	.5050	.7033	.5851	.7673	1.7090	.2327	.8631	.9361	40	1.0414
.5323	30	.5075	.7055	.5890	.7701	1.6977	.2299	.8616	.9353	30	1.0385
.5352	40	.5100	.7076	.5930	.7730	1.6864	.2270	.8601	.9346	20	1.0356
.5381	50	.5125	.7097	.5969	.7759	1.6753	.2241	.8587	.9338	10	1.0327
.5411	**31° 00′**	.5150	.7118	.6009	.7788	1.6643	.2212	.8572	.9331	**59° 00′**	1.0297
.5440	10	.5175	.7139	.6048	.7816	1.6534	.2184	.8557	.9323	50	1.0268
.5469	20	.5200	.7160	.6088	.7845	1.6426	.2155	.8542	.9315	40	1.0239
.5498	30	.5225	.7181	.6128	.7873	1.6319	.2127	.8526	.9308	30	1.0210
.5527	40	.5250	.7201	.6168	.7902	1.6212	.2098	.8511	.9300	20	1.0181
.5556	50	.5275	.7222	.6208	.7930	1.6107	.2070	.8496	.9292	10	1.0152
.5585	**32° 00′**	.5299	.7242	.6249	.7958	1.6003	.2042	.8480	.9284	**58° 00′**	1.0123
.5614	10	.5324	.7262	.6289	.7986	1.5900	.2014	.8465	.9276	50	1.0094
.5643	20	.5348	.7282	.6330	.8014	1.5798	.1986	.8450	.9268	40	1.0065
.5672	30	.5373	.7302	.6371	.8042	1.5697	.1958	.8434	.9260	30	1.0036
.5701	40	.5398	.7322	.6412	.8070	1.5597	.1930	.8418	.9252	20	1.0007
.5730	50	.5422	.7342	.6453	.8097	1.5497	.1903	.8403	.9244	10	.9977
.5760	**33° 00′**	.5446	.7361	.6494	.8125	1.5399	.1875	.8387	.9236	**57° 00′**	.9948
.5789	10	.5471	.7380	.6536	.8153	1.5301	.1847	.8371	.9228	50	.9919
.5818	20	.5495	.7400	.6577	.8180	1.5204	.1820	.8355	.9219	40	.9890
.5847	30	.5519	.7419	.6619	.8208	1.5108	.1792	.8339	.9211	30	.9861
.5876	40	.5544	.7438	.6661	.8235	1.5013	.1765	.8323	.9203	20	.9832
.5905	50	.5568	.7457	.6703	.8263	1.4919	.1737	.8307	.9194	10	.9803
.5934	**34° 00′**	.5592	.7476	.6745	.8290	1.4826	.1710	.8290	.9186	**56° 00′**	.9774
.5963	10	.5616	.7494	.6787	.8317	1.4733	.1683	.8274	.9177	50	.9745
.5992	20	.5640	.7513	.6830	.8344	1.4641	.1656	.8258	.9169	40	.9716
.6021	30	.5664	.7531	.6873	.8371	1.4550	.1629	.8241	.9160	30	.9687
.6050	40	.5688	.7550	.6916	.8398	1.4460	.1602	.8225	.91[illegible]	20	.9657
.6080	50	.5712	.7568	.6959	.8425	1.4370	.1575	.8208	.9142	10	.9628
.6109	**35° 00′**	.5736	.7586	.7002	.8452	1.4281	.1548	.8192	.9134	**55° 00′**	.9599
.6138	10	.5760	.7604	.7046	.8479	1.4193	.1521	.8175	.9125	50	.9570
.6167	20	.5783	.7622	.7089	.8506	1.4106	.1494	.8158	.9116	40	.9541
.6196	30	.5807	.7640	.7133	.8533	1.4019	.1467	.8141	.9107	30	.9512
.6225	40	.5831	.7657	.7177	.8559	1.3934	.1441	.8124	.9098	20	.9483
.6254	50	.5854	.7675	.7221	.8586	1.3848	.1414	.8107	.9089	10	.9454
.6283	**36° 00′**	.5878	.7692	.7265	.8613	1.3764	.1387	.8090	.9080	**54° 00′**	.9425
		Value	Log₁₀	Value	Log₁₀	Value	Log₁₀	Value	Log₁₀	Degrees	Radians
		Cosine		Cotangent		Tangent		Sine			

[Characteristics of Logarithms omitted — determine by the usual rule from the value]

Radians	Degrees	Sine Value	Sine Log₁₀	Tangent Value	Tangent Log₁₀	Cotangent Value	Cotangent Log₁₀	Cosine Value	Cosine Log₁₀		
.6283	**36° 00′**	.5878	.7692	.7265	.8613	1.3764	.1387	.8090	.9080	**54° 00′**	.9425
.6312	10	.5901	.7710	.7310	.8639	1.3680	.1361	.8073	.9070	50	.9396
.6341	20	.5925	.7727	.7355	.8666	1.3597	.1334	.8056	.9061	40	.9367
.6370	30	.5948	.7744	.7400	.8692	1.3514	.1308	.8039	.9052	30	.9338
.6400	40	.5972	.7761	.7445	.8718	1.3432	.1282	.8021	.9042	20	.9308
.6429	50	.5995	.7778	.7490	.8745	1.3351	.1255	.8004	.9033	10	.9279
.6458	**37° 00′**	.6018	.7795	.7536	.8771	1.3270	.1229	.7986	.9023	**53° 00′**	.9250
.6487	10	.6041	.7811	.7581	.8797	1.3190	.1203	.7969	.9014	50	.9221
.6516	20	.6065	.7828	.7627	.8824	1.3111	.1176	.7951	.9004	40	.9192
.6545	30	.6088	.7844	.7673	.8850	1.3032	.1150	.7934	.8995	30	.9163
.6574	40	.6111	.7861	.7720	.8876	1.2954	.1124	.7916	.8985	20	.9134
.6603	50	.6134	.7877	.7766	.8902	1.2876	.1098	.7898	.8975	10	.9105
.6632	**38° 00′**	.6157	.7893	.7813	.8928	1.2799	.1072	.7880	.8965	**52° 00′**	.9076
.6661	10	.6180	.7910	.7860	.8954	1.2723	.1046	.7862	.8955	50	.9047
.6690	20	.6202	.7926	.7907	.8980	1.2647	.1020	.7844	.8945	40	.9018
.6720	30	.6225	.7941	.7954	.9006	1.2572	.0994	.7826	.8935	30	.8988
.6749	40	.6248	.7957	.8002	.9032	1.2497	.0968	.7808	.8925	20	.8959
.6778	50	.6271	.7973	.8050	.9058	1.2423	.0942	.7790	.8915	10	.8930
.6807	**39° 00′**	.6293	.7989	.8098	.9084	1.2349	.0916	.7771	.8905	**51° 00′**	.8901
.6836	10	.6316	.8004	.8146	.9110	1.2276	.0890	.7753	.8895	50	.8872
.6865	20	.6338	.8020	.8195	.9135	1.2203	.0865	.7735	.8884	40	.8843
.6894	30	.6361	.8035	.8243	.9161	1.2131	.0839	.7716	.8874	30	.8814
.6923	40	.6383	.8050	.8292	.9187	1.2059	.0813	.7698	.8864	20	.8785
.6952	50	.6406	.8066	.8342	.9212	1.1988	.0788	.7679	.8853	10	.8756
.6981	**40° 00′**	.6428	.8081	.8391	.9238	1.1918	.0762	.7660	.8843	**50° 00′**	.8727
.7010	10	.6450	.8096	.8441	.9264	1.1847	.0736	.7642	.8832	50	.8698
.7039	20	.6472	.8111	.8491	.9289	1.1778	.0711	.7623	.8821	40	.8668
.7069	30	.6494	.8125	.8541	.9315	1.1708	.0685	.7604	.8810	30	.8639
.7098	40	.6517	.8140	.8591	.9341	1.1640	.0659	.7585	.8800	20	.8610
.7127	50	.6539	.8155	.8642	.9366	1.1571	.0634	.7566	.8789	10	.8581
.7156	**41° 00′**	.6561	.8169	.8693	.9392	1.1504	.0608	.7547	.8778	**49° 00′**	.8552
.7185	10	.6583	.8184	.8744	.9417	1.1436	.0583	.7528	.8767	50	.8523
.7214	20	.6604	.8198	.8796	.9443	1.1369	.0557	.7509	.8756	40	.8494
.7243	30	.6626	.8213	.8847	.9468	1.1303	.0532	.7490	.8745	30	.8465
.7272	40	.6648	.8227	.8899	.9494	1.1237	.0506	.7470	.8733	20	.8436
.7301	50	.6670	.8241	.8952	.9519	1.1171	.0481	.7451	.8722	10	.8407
.7330	**42° 00′**	.6691	.8255	.9004	.9544	1.1106	.0456	.7431	.8711	**48° 00′**	.8378
.7359	10	.6713	.8269	.9057	.9570	1.1041	.0430	.7412	.8699	50	.8348
.7389	20	.6734	.8283	.9110	.9595	1.0977	.0405	.7392	.8688	40	.8319
.7418	30	.6756	.8297	.9163	.9621	1.0913	.0379	.7373	.8676	30	.8290
.7447	40	.6777	.8311	.9217	.9646	1.0850	.0354	.7353	.8665	20	.8261
.7476	50	.6799	.8324	.9271	.9671	1.0786	.0329	.7333	.8653	10	.8232
.7505	**43° 00′**	.6820	.8338	.9325	.9697	1.0724	.0303	.7314	.8641	**47° 00′**	.8203
.7534	10	.6841	.8351	.9380	.9722	1.0661	.0278	.7294	.8629	50	.8174
.7563	20	.6862	.8365	.9435	.9747	1.0599	.0253	.7274	.8618	40	.8145
.7592	30	.6884	.8378	.9490	.9772	1.0538	.0228	.7254	.8606	30	.8116
.7621	40	.6905	.8391	.9545	.9798	1.0477	.0202	.7234	.8594	20	.8087
.7650	50	.6926	.8405	.9601	.9823	1.0416	.0177	.7214	.8582	10	.8058
.7679	**44° 00′**	.6947	.8418	.9657	.9848	1.0355	.0152	.7193	.8569	**46° 00′**	.8029
.7709	10	.6967	.8431	.9713	.9874	1.0295	.0126	.7173	.8557	50	.7999
.7738	20	.6988	.8444	.9770	.9899	1.0235	.0101	.7153	.8545	40	.7970
.7767	30	.7009	.8457	.9827	.9924	1.0176	.0076	.7133	.8532	30	.7941
.7796	40	.7030	.8469	.9884	.9949	1.0117	.0051	.7112	.8520	20	.7912
.7825	50	.7050	.8482	.9942	.9975	1.0058	.0025	.7092	.8507	10	.7883
.7854	**45° 00′**	.7071	.8495	1.0000	.0000	1.0000	.0000	.7071	.8495	**45° 00′**	.7854
		Value Cosine	Log₁₀	Value Cotangent	Log₁₀	Value Tangent	Log₁₀	Value Sine	Log₁₀	Degrees	Radians